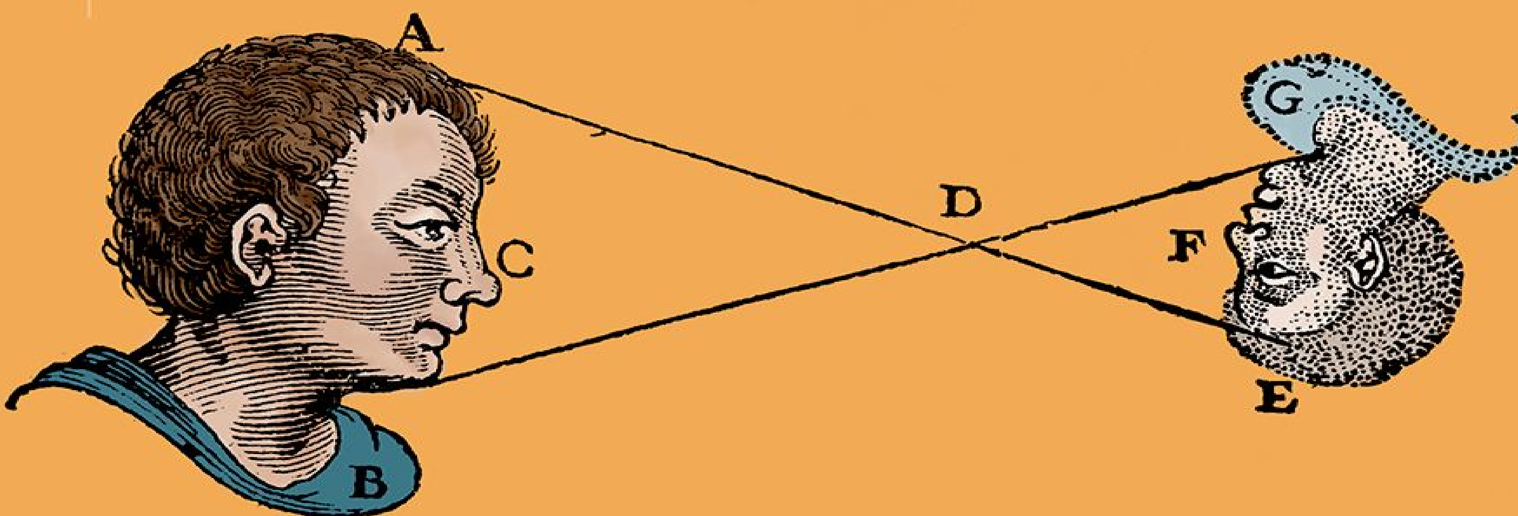
OXFORD



DESCARTES'S METHOD

the formation of the subject *of* science

TAREK R. DIKA

Descartes’s Method

In *Descartes's Method*, Tarek Dika offers a comprehensive and authoritative examination of Descartes's early *Rules for the Direction of the Mind*. Dika argues powerfully that the *Rules* constitutes a development rather than—as others have argued—a rejection of the scholastic theory of *habitus* and that, contrary to a "Uniformity Thesis" popular in the literature, there is significant flexibility in the application of the method in this text. Also notable is the excellent use Dika makes of the so-called "Cambridge manuscript," a recently discovered version of the *Rules* that differs significantly from other extant versions of this text. The discussion in *Descartes's Method* is well-sourced throughout and exhibits an impressive command of Descartes's writings as well as of the relevant context, including the historical background required to understand properly illustrative treatments in the *Rules* of mathematics and optics.

Tad Schmaltz, University of Michigan

Many scholars have tried to identify Descartes's method with this or that technique, without convincing results. In the most comprehensive study ever written on this topic, Tarek Dika shows that, far from being a uniform procedure, Descartes's method is an acquired habit (*habitus*) closely related to Aristotelian *phronesis*. The change of focus is illuminating. The width of views, the clarity of exposition, the breadth of the underlying culture, the search for the highest precision on each point: this is undoubtedly a great book.

Denis Kambouchner, University of Paris I Panthéon-Sorbonne

No question in Descartes's thought is more difficult than his method. The *Rules for the Direction of the Mind*, an unfinished treatise on method left unpublished in Descartes's lifetime has given rise to numerous inconsistent interpretations, nor is it easy to see how it is reflected in his later work. Tarek Dika's new book takes account of startling new manuscript discoveries about the *Rules* and offers a bold and unified new reading of the text and the project of method in Descartes's thought. It changes the way in which we see Descartes, and will open up new conversations about this canonical figure in the history of philosophy and science.

Daniel Garber, Princeton University

Descartes's Method

The Formation of the Subject of Science

TAREK R. DIKA

OXFORD
UNIVERSITY PRESS

OXFORD UNIVERSITY PRESS

Great Clarendon Street, Oxford, OX2 6DP, United Kingdom

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

First Edition published in 2023

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2022941376

ISBN 978–0–19–286986–9
ebook ISBN 978–0–19–269694–6

DOI: 10.1093/oso/9780192869869.001.0001

Printed and bound in the UK by TJ Books Limited

For my loving parents,
Amira and Rifaat Dika,
My wife, Constance, and our beautiful daughter, Saba

C’est à nous de procurer par nos propres efforts cette perfection de l’*ingenium* que nous invite la Méthode Cartésienne. Là est sa véritable signification et son véritable prix. Et c’est pourquoi Descartes proteste qu’elle consiste “plus en pratique qu’en théorie.” Lorsqu’il nous parle de “bien conduire notre raison,” il ne croit ni souhaitable ni possible de nous révéler des chemins tout tracés qu’il suffirait de suivre pour arriver infailliblement et comme automatiquement au but. Il nous recommande de “former” notre esprit au contact des choses, de le “nourrir de vérités,” de le cultiver en l’exerçant. Il a en vue, non pas “une clef de l’art d’inventer,” mais une éducation et, pour ainsi dire, un “dressage” de notre faculté inventive. Autrement dit, la méthode de Descartes, c’est un ensemble d’habitudes à prendre par chacun de nous, à l’exemple de Descartes, d’après des moyens analogues à ceux dont il a personnellement ressenti l’efficacité. […] Et c’est l’habitude, ou plutôt l’ensemble d’habitudes ainsi prises, c’est cela, et non pas une liste de formules rigides machinalement applicables, qui fait l’essence de la méthode cartésienne.

Jean Laporte

Contents

Acknowledgments

It took me nearly ten years to write this book. A combination of naiveté and enthusiasm blinded me to how demanding it would be. But I am happy to have written it, and even happier to thank the individuals and institutions who supported me along the winding road to completion.

The research for this book was conducted at the Johns Hopkins University, the University of Michigan, and the University of Notre Dame. A three-year postdoctoral fellowship at the University of Michigan Society of Fellows permitted me to delve into the history of scholastic debates about the unity of science and identify the principles behind the interpretation of Descartes's method offered in this book.

Further support from the University of Notre Dame's Institute for Scholarship in the Liberal Arts (ISLA) enabled me to travel to Paris for an international conference on the Cambridge manuscript organized by Daniel Garber at the Institut d'études avancées de Paris in 2018. I am grateful to these institutions for their support.

At Johns Hopkins, I would like to thank Hent de Vries (now at NYU), whose mentorship and support over a period now spanning more than fifteen years I will never be able to repay. I would also like to thank Eckart Förster, Michael Fried, Ruth Leys, Paola Marrati, Yitzhak Melamed, Marva Philip, Michael Williams, and Meredith Williams. The education I received at Johns Hopkins remains second to none.

At Michigan, I would like to thank Tad Schmaltz, who met with me every other week for three years, read and helped me improve my papers, and introduced me to the wider world of scholars in early modern philosophy. I could not have hoped for a better mentor. I would also like to thank the organizers and participants of the University of Michigan's Premodern Colloquium, especially Tom Willette and Helmut Puff, for inviting me to present my research and receive valuable feedback. Finally, I would like to thank my colleagues and friends whose support made an

enormous difference to both my life and my career, including Elizabeth Anderson, Alina Clej, Ed Curley, S.E. Kile, Louis Loeb, Monu Lahiri, Don Lopez, Tomoko Masuzawa, Keith Mitnick, Yasmin Moll, Jennifer Nelson, Yopie Prins, Laura Reutsche, Mireille Roddier, Scott Selberg, Anton Shammas, Jamie Tappenden, Linda Turner, Silke Maria-Weineck, and Damon Young.

At Notre Dame, I would like to thank my colleagues and friends in the Program of Liberal Studies, the Program in History and Philosophy of Science, and the Department of Philosophy, including Francesca Bordogna, Katie Bugyis, Eric Bugyis, Chris Chowrimootoo, Therese Corey, Steve Fallon, Anna Geltzer, Robert Goulding, Debbie Kabzinski and Becky Badger, Katharina Kraus, Julia Marvin, Felicitas Munzel, Jenny Martin, Sam Newlands, Emma Planinc, Clark Power, Andrew Radde-Gallwitz, Gretchen Reydams-Schils, Denis Robichaud, Joseph Rosenberg, Even Ragland, Fred Rush, Phil Sloan, Tom Stapleford, Henry Weinfield, and Steve Watson. I would also like to thank Hussein Abdulsater, Corey Garibaldi, David Hooker, Leonardo Francalari, Chante Mouton Kinyan, Sara Marcus, Ebrahim Moosa, Azareen Van der Vliet Oloomi, Viveca Pattison Robichaud, Roy Scranton, Yasmin Solomonescu, and Aldo Tagliabue. I wrote the bulk of this manuscript while at Notre Dame, and without the support of my friends and colleagues, I would not have been able to get through it.

At Toronto, I would like to thank my colleagues in the Department of Philosophy, including Donald Ainslie, Rebecca Comay, Robert Gibbs, Mark Kingwell, Willi Goetschel, William Paris, Martin Pickavé, Michael Rosenthal, Marleen Rozemond, Nick Stang, Trevor Teitel, and Owen Ware.

In Paris, I owe a special thanks to Denis Kambouchner, whom I met in Summer 2014 and who has been a mentor and very close friend ever since. This book bears the mark of Denis's influence in more ways than I can enumerate here. I would also like to thank Frédéric de Buzon for his incredibly helpful comments on Chapter 10, David Rabouin for helping me navigate my way through the complexities of Descartes's mathematics, and Jean-Luc Marion and Vincent Carraud for inviting me to participate in the events organized by the Centre d'études cartésiennes.

In Australia, I would like to thank John Schuster, who read and criticized previous drafts of Chapter 10, but not without invaluable suggestions on how the chapter might be improved, despite his disagreements.

In Cambridge, I would like to thank Richard Serjeanston, who met with me in October 2017, walked me over to the library, showed me a copy of the Cambridge manuscript, and even provided me with a guided philological tour of the manuscript. Very few people had seen the manuscript in the flesh at this point, and I hope that my publications since have proven that nothing can replace the painstaking labor Richard and Michael Edwards have devoted to the manuscript.

I would like to thank Peter Dear, Robert Goulding, Denis Kambouchner, Sam Newlands, and Tad Schmaltz for their participation in a book manuscript workshop held at the University of Notre Dame on February 18, 2021. The book is in much better shape due in no small part to their comments and criticisms. Many thanks to Tom Stapleford for encouraging me to organize the workshop and to the Program of Liberal Studies for funding it.

I presented parts of this book at various conferences over the years. I would like to thank Patrick Brissey, Gideon Manning, Jack Stetter, and Stephen I. Wagner for their comments at the APA (Pacific and Central Division). John Carriero provided very helpful comments on an early draft of Chapter 8. The *Atelier Franco-Américain de philosophie moderne* (organized by Jean-Pascal Anfray, Frédéric de Buzon, Daniel Garber, Denis Kambouchner, Steve Nadler, Sophie Roux, Tad Schmaltz, and held annually in either Ann Arbor, Madison, Paris, or Princeton) provided me with many opportunities to present work in progress, receive comments from some of the finest historians of early modern philosophy, and become a part of an international scholarly community. I hope to return the favor one day by hosting the *Atelier* here in Toronto.

The two anonymous referees for Oxford University Press provided detailed and critical feedback that greatly improved the book as a whole.

Among those whom I have not yet mentioned, I would like to thank Arash Abazari, Vlad Beronja, Johannes Birke, Andrew Bush, Martijn Bujis, Marco Coco, Sara Elamin, Mohammed Elghoul, Elena Fabietti, Jane Fletcher, Jeroen Gerrits, Elham Hatef, Alexander James, Waleed Jammal, Amy Kraus, Mike Leonard, Greg Piton Saint Martin, Samuel Monsalve, Larry McGrath, Rebecca Pekron, Bican Polat, Avreimi Rot, Aviva Rot, Johannes Schade, Martin Shuster, Nils Schott, Ian Singleton, Natalya Sukhonos, and Joshua Thompson.

I would also like to thank my editor at Oxford, Peter Momtchiloff, with whom I intend to continue working in future, and the entire team at Oxford, who transformed this manuscript from a digitally stored file on my computer into a book available throughout the world.

Parts of Chapters 1 and 2 derive from three articles, “Method, Practice, and the Unity of *Scientia* in Descartes’s *Regulae*,” “Extrinsic Denomination and the Origins of Early Modern Metaphysics: The Scholastic Context of Descartes’s *Regulae*,” and (with Denis Kambouchner) “Descartes’s Method” (forthcoming in *The Cartesian Mind*, ed. Jorge Secada and Cecilia Wee). I would like to thank the *Journal of Early Modern Studies*, Springer, and Routledge for permission to draw on material from these three articles. Parts of Chapter 3 derive from my entry, “Descartes’ Method,” and appear here with the generous permission of the *Stanford Encyclopedia of Philosophy*. Chapter 8 is drawn from my article, “The Origins of Cartesian Dualism” (*Journal of the American Philosophical Association*) and is here reprinted with the generous permission of Cambridge University Press. Chapter 10 is drawn from “Descartes’ Deduction of the Law of Refraction and the Shape of the Anaclastic Lens in Rule 8” (*HOPOS: The Journal of the International Society for the History of Philosophy of Science*), and is reprinted with the generous permission of Chicago University Press.

Finally, I would like to thank my family: my wife, Constance de Font-Réaulx, who endured my worst anxieties about this book and whose unwavering support continues to get me through my days; our baby daughter, Saba, who is a miracle; my parents, Amira and Rifaat Dika, who sacrificed everything for us; and my sisters, Sahar and Hala Dika, whom I love dearly.

Toronto
September 2022

Preface

In this book, I provide a systematic interpretation of Descartes's method in his first, posthumously published treatise, *Rules for the Direction of the Mind* (*Regulae ad directionem ingenii*) (henceforth *Rules*), most likely composed during the 1620s, but not published until 1684 (in Dutch translation) and 1701 (in Latin), many years after Descartes's death in 1650.[1] By "systematic" I simply mean that the interpretation is informed by a principle: Descartes's method is a problem-solving cognitive disposition or *habitus* that can be actualized in a variety of well-defined ways, depending always on the parameters of the problem at hand. I have aimed to provide an interpretation that, in addition to being systematic, is also as comprehensive as possible: I cover *Rules* in its entirety as well as a number of important developments in Descartes's method after *Rules* (whether I have succeeded is another matter, and is in any case up to the reader to decide).

I have deliberately chosen not to organize the book according to the order of the text, but rather according to the order of problems that must be solved in order to learn the method. Unlike Descartes's *Meditations*, where the order of the text reflects the order of problems that must be solved in order to discover the primary notions or principles of metaphysics, in *Rules* the order of the text does not consistently reflect the order of problems that must be solved in order to learn the method. Descartes's conception of the treatise matured as he wrote it. The first problem that must be solved by the method—the problem of the limits of knowledge—does not appear until Rule 8, and Descartes discusses it *after* having discussed a problem that can only be solved once the problem of the limits of knowledge has been solved: the problem of the law of refraction and the shape of the anaclastic lens. The problem of the law of refraction and the shape of the anaclastic lens, moreover, is what Descartes terms an "imperfectly understood problem," or a problem in which the conditions relevant to the solution are

not known in advance, but must be found. These problems typically arise in natural philosophy, and according to Descartes's division of the treatise, they belong to the third part (Rules 25–36), which Descartes never completed (*Rules* ends prematurely at Rule 21). To solve imperfectly understood problems, one must first become proficient in solving what Descartes terms "perfectly understood problems," or problems in which the conditions relevant to the solution are known in advance. These problems typically arise in mathematics, and according to Descartes's division of the treatise they belong to the second part (Rules 13–24).

Were I to have organized the book according to the order of the text, I would have had to violate the order of problems that must be solved in order to learn the method, which struck me as inappropriate. Thus, I have organized the book according to Descartes's division of the treatise into three parts (a division he himself does not consistently respect): operations and simple propositions (Chapters 2–3); the problem of the limits of knowledge (the first problem that must be solved by the method) (Chapters 5–8); perfectly understood problems (Chapter 9); and imperfectly understood problems (Chapter 10). This order, I believe, best exhibits how practice in the method develops in degrees from the simplest problems to more complex problems, up to the point where problems in mathematics and natural philosophy can be solved.

The organization of this book reflects not only an understanding of how Descartes intended his method to be acquired and applied but also an understanding of *Rules* as a treatise that, despite its imperfections, exhibits underlying *unity*. My emphasis on the unity of the treatise contrasts sharply with other interpretations, above all that of J.-P. Weber in *La constitution du texte des* Regulae. Weber's insistence on dating, not only the treatise as a whole or even individual rules but even parts of rules (by both year and month) over a period of nine years has contributed to the perception that Descartes's *Rules* is a patchwork, which Descartes failed to "rigorously compose," and which contains a "mass of contradictions […] within a rule […] no less than between rules." Indeed, Weber continues, "*Rules* does not reveal one method, but rather many, which succeed one another, perfect one another, or mutually annul one another." In short, *Rules* is "allusive, excessively composite and, consequently, by and large contrived."[2] Even when Descartes's readers disagree with the details of Weber's chronology, they nevertheless retain his thesis that *Rules* should be divided, not

internally into three parts, according to its own criteria of unity, but rather externally, according to discrete "stages of composition" that are nevertheless incredibly difficult to individuate based on the available evidence.[3] When one delves into the details of such interpretations, one invariably finds that the division of *Rules* into stages of composition is based, as it must be, on the author's own interpretation of the text. As is well known, Descartes never dated *Rules*, and none of the manuscript copies indicate when the original manuscript was written. Any dating, proposed by anyone (including myself), is based entirely on interpretations of textual and circumstantial evidence.

There is no doubt that Descartes wrote *Rules* over a period of many years (how many years is less clear),[4] and that, like many other texts in the history of philosophy, real or apparent gaps and discontinuities pose serious challenges for the interpreter. Everything is further complicated by the fact that Descartes never completed *Rules*. Throughout this book, I take into consideration, whenever appropriate, gaps, discontinues, and developments in Descartes's position, as exhibited in any one manuscript, in differences between the relevant manuscripts, or between *Rules* and Descartes's subsequent texts. However, I do not *base* my interpretation of *Rules* on a chronological division of the treatise into stages of composition.[5] Instead, I provide an interpretation of *Rules* based on Descartes's own principal of unity, which divides the treatise into three parts of twelve rules each, and which is itself based on the order of problems that need to be solved in order to learn the method. This does not mean that I simply *ignore* chronological considerations. Far from it. It only means my interpretation does not fundamentally depend on any division of the treatise into chronological stages.

1 See Descartes 1684 and 1701. The consensus (see, e.g., Weber 1964, 109–45, 184–94; Gaukroger 1995, 181, and Schuster 2013, 307–49) is that Descartes abandoned *Rules* by 1628, but new evidence from the Cambridge manuscript suggests that Descartes may have continued composing *Rules* after 1628. See Serjeanston and Edwards in Descartes forthcoming. On the manuscripts, title(s), and composition of *Rules*, see Appendix.

2 See Weber 1964, 1–2.

3 Weber's enduring influence on the scholarship on Descartes's *Rules* is evident in Garber 1992, Gaukroger 1995; Schuster 1977 and 2013. Gaukroger, for instance, describes himself as "[f]ollowing the general thrust of Weber's account (but not the details, which are often too fine-grained to bear the evidence) […]" (Gaukroger 1995, 111). Schuster endorses Weber's thesis that "the *Regulae*, in fact,

were composed in stages between 1619–1628 and that different strata in the text correspond to different stages in the development and reformulation of Descartes's methodological ideas," but he also points out that he makes "drastic revisions" whenever necessary (Schuster 2013, 227). Garber 1992, 317, n. 1 acknowledges that Weber 1964 is the "standard account of the composition of *Rules*," but while he finds "the basic chronology correct," he is "a bit skeptical that one can make such fine distinctions." He relies principally on Weber's chronology and Schuster's revisions in Schuster 1980 and 1986.

4 For my own view, see the Appendix.

5 Indeed, recent evidence suggests, *pace* Gaukroger and Schuster, that *mathesis universalis* forms no part of the earliest stage of composition (as they claim; see references in n. 3 above), since it is nowhere to be found in the Cambridge manuscript, which seems to be copied from an earlier draft of *Rules*. For more discussion, see Chapter 4 and the Appendix. This is only one example of the risks involved in basing one's interpretation of *Rules* on a conjectural chronology.

Abbreviations

Descartes

CSM/CSMK — *The Philosophical Writings of Descartes* (1985–1991). Cited by volume number and page number. "CSM 1:7" = "*The Philosophical Writings of Descartes*, volume 1, page 7." "CSMK" is employed for volume 3.

AT — *Oeuvres de Descartes* (1996). Cited by volume number and page number. "AT 10:3" = "*Oeuvres de Descartes*, volume 10, page 3."

Aquinas

ST — *Summa theologiae* in *Opera omnia* (1882– and English translation in Aquinas 1945). Cited by year, volume number, and page, followed by part number, question number, and article number. "Aquinas 1891, 6:344 (*ST* I–II, q. 64, art. 4)" = "*Opera omnia*, volume 6, page 344 in *Summa theologiae*, question 64, article 4."

Scotus

QM — *Quaestiones…super libros Metaphysicorum Aristotelis* in *Opera omnia* (1891–1895 and English translation in 1998). Cited by year, volume number, and page number, followed by book number and question number. "Scotus 1891, 7:303–8 (*QM*, lib. 6, q. 1)" = "*Opera omnia*, volume 7, pages 303–8 in *Quaestiones…super libros Metaphysicorum Aristotelis*, book 6, question 1."

Ord. — *Ordinatio* in *Opera omnia* (1950–2013). Cited by year, volume number, and page number, followed by book number, distinction number, and question number or, in the case of the prologue, prologue and question number. "Scotus 1950–2013, 1:96 (*Ord.*, Prol., q. 3)" = "*Opera omnia*, volume 1, page 96 in *Ordinatio*, Prologue, question 3."

Rep. — *Reportatio* in *Opera omnia* (1891–1895). Cited by year, volume number, and page number, followed by question number and article number. "Scotus 1894, 22:9 (*Rep.*, q. 1, art. 2)" = "*Opera omnia*, volume 22, page 9 in *Reportatio*, question 1, article 2."

Ockham

Ord. — *Ordinatio* in *Opera theologica* (1967–1988 and in partial English translation in Ockham 2021). Cited by year, volume number, and page number, followed by book number, distinction number, question number, and article number or, in the case of the prologue, book number, prologue, question number, and article number. "Ockham 1967–1988, 1:111 (*Ord.* 1, Prol., q. 2, art. 3)" = "*Opera theologica*, volume 1, page 111 in *Ordinatio*, book 1, prologue, question 2, article 3."

Expos. Phys. — *Expositio in libros Physicorum Aristotelis* in *Opera philosophica* (1974–1988 and partial English translation in 1990). Cited by year, volume number, and page number, followed by book number or, in the case of the prologue, prologue and section number. "Ockham 1974–1988, 4:7 (*Expos. Phys.*, Prol., §3)" = "*Opera philosophica*, volume 4, page 7 in *Expositio in libros Physicorum Aristotelis*, Prologue, section 3."

SL — *Summa logicae* in *Opera philosophica* (1974–1988 and English translation in Ockham 1974 and 1980 (Parts I–II, respectively) and 2007 (Part III-II)). Cited by year, volume number, and page number, followed by part number and chapter number. "Ockham 1974–1988, 1:540 (*SL* III-II, cap. 21)" = "*Opera philosophica*, volume 1, page 540 in *Summa logicae*, Part III-2, chapter 21."

Suárez

DM — *Disputationes metaphysicae* in *Opera omnia* (1856–1878 and in various English translations). Cited by year, volume number, and page number, followed by disputation number, section number, and paragraph number separated from the section number by a period. "Suárez 1856, 26:695 (*DM*, disp. 44, sec 11.3)" = "*Opera omnia*, volume 26, page 695 in *Disputationes metaphyiscae*, disputation 44, section 11, paragraph 3."

Conimbricenses

In De anima — *In tres libros De anima Aristotelis Stagiritae* in [1598] 1604. Cited by year and page, followed by book number, chapter number, question number, and article number. "Conimbricenses [1598] 1604, 505 (*In De Anima*, lib. 3, cap. 8, q.7, art. 2)" = "*In tres libros De Anima Aristotelis Stagiritae*, page 505, in book 3, chapter 8, question 7, article 2."

When comparing the edition of Descartes's *Rules for the Direction of the Mind* in AT to other manuscripts of *Rules*, I cite as follows:

Rules$_{\text{AT}}$ — *Rules* in AT.

Rules$_{\text{CM}}$ — The contents of the Cambridge manuscript of *Rules*.

CM — The document of the Cambridge manuscript of *Rules* in Descartes 1626/1627?.

When appropriate, individual rules are cited as follows: "Rule 12$_{\text{CM}}$" = "Rule 12 in the Cambridge manuscript," and "Rule 12$_{\text{AT}}$" = "Rule 12 in AT." Reference to other manuscripts of *Rules*, such as the Hanover manuscript, are not abbreviated. When citing directly from CM, I cite by folio number and indicate recto or verso by "r" or "v" in superscript. "CM fo. 16$^{\text{r}}$" = "Cambridge manuscript, folio 16, recto."

English translations

Citations of available English translations of Latin texts are by year and page number, are preceded by the abbreviation “trans.,” and follow citations of the original Latin texts after a semi-colon.

Introduction: Descartes's Method

Universality without Uniformity

From at least the mid-nineteenth century up to the 1970s, most historians of philosophy regarded Descartes's method as the undisputed source of Cartesian science (including metaphysics). In 1826, Victor Cousin, who first translated *Rules* into French, described it as a treatise in which "one sees even more Descartes's fundamental aim and the spirit of the revolution that created modern philosophy."[1] Cousin was not alone in his enthusiasm for Descartes's method. "The Cartesian philosophy," the French philosopher and Descartes scholar Louis Liard wrote in 1880, "is effectively the product, the commentary, and the justification of the method."[2] Eight years later, A. Boyce-Gibson, who first introduced *Rules* to Anglophone historians via a series of important articles, would similarly argue that "the *Regulae* undoubtedly remain the best exposition we have of that natural logic, under the guidance of which Descartes's whole thought lived and moved."[3] Finally, in 1952, J. L. Beck, author of what remains one of the most extensive English-language commentaries on *Rules* to date, summed up his estimation of Descartes's method in one sentence: "The Cartesian 'revolution' is primarily a revolution in method."[4] For these philosophers and scholars, there was simply no question: Descartes's method is the source of Cartesian science. To understand the latter, one must understand the former.[5]

While historians of philosophy emphasized the importance of Descartes's method, philosophers and historians of science raised serious questions about its relation to Cartesian science. As early as 1934, the French philosopher of science Gaston Bachelard described Descartes's method as a "purely mechanical method, which requires no effort […] and cannot be truly fecund."[6] In 1956, the Russo-French historian of science

Alexandre Koyré denied that science "has ever started with a *tractatus de methodo* and progressed by the application of such an abstractly derived method […]. Descartes's *Discourse on Method* […] was written not before, but after the scientific 'essays' to which it forms the preface. […] Thus, even Cartesian science was no more an outcome of a methodological revolution than that of Galileo[…]."[7] Koyré would soon be followed by Thomas Kuhn and Paul Feyerabend in denying that the methods devised by Bacon, Descartes, Newton, and others played any substantive role in the historical constitution of modern science.[8] Consequently, many historians of science came to regard Descartes's method—and, indeed, all scientific methods—as little more than a rhetorical device for the presentation of scientific discoveries produced by other means. Methods persuade others to accept discoveries that could not have been made by means of these methods themselves. Early modern scientists who exalted the fruits of their method systematically distorted their own practices, seduced by the force of their own "mythic discourse."[9]

Perhaps due to the doubts raised by philosophers and historians such as Koyré, Kuhn, and Feyerabend, Descartes's method has had a cooler reception among historians of philosophy since the 1980s. Desmond Clarke has argued that Descartes's method in *Rules* is "too vague or general to be of use in explaining the method to be used in physical science."[10] Peter Dear has referred to the "unjustified assumption that Descartes's method was an instrumentally efficacious technique by which he arrived at his results."[11] Even when it is conceded that Descartes may actually have employed his method in science, it has also been argued that he abandoned it as early as 1630. Daniel Garber has argued that Descartes only applied his method to one paradigmatic case (the deduction of the rainbow in *Meteorology* VIII), and that Descartes himself failed to appreciate the extent to which he had abandoned the method, even as he continued to advertise its importance in the 1630s and 40s.[12]

Thus, when one surveys the literature on Descartes's method written by historians of science and historians of philosophy over the past four decades, one is left with the distinct impression that Descartes at best only applied his method to one, maybe two paradigmatic cases. At worst, he never applied it at all, and could not have.

Indeed, one need not survey recent literature to find critical evaluations of the efficacy of Descartes's method; Descartes's contemporaries themselves complained that he had failed to provide them with clear examples of his method even in his own scientific writings,[13] and Leibniz famously derided Descartes's method as wholly vacuous. The rules of the method, he wrote, are "like the precepts of some chemist: take what you need, do what you should, and you will get what you want [*Sume quod debebes et operare ut debes, et habebis quod optas*]."[14] The fault is in part Descartes's own. The only published exposé of the method Descartes ever provided during his lifetime can be found in what effectively amounts to a single paragraph in *Discourse* II (see AT 6:18–19, CSM 1:120)! Descartes never published (or even intended to publish) anything more on his method. In correspondence, he repeatedly insisted that he never intended "to teach the method but only to discuss it" (to Mersenne, February 27, 1637, AT 1:349, CSMK 3:53); that he is "not actually teaching the method, but trying rather to give some demonstration of it in the three consecutive treatises appended to the *Discourse*" (to an unknown recipient, end of May 1637, AT 1:370, CSMK 3:58); that in "the *Dioptrics* and *Meteorology* I merely tried to show that my method is better than the usual one; in my *Geometry*, however, I claim to have demonstrated this" (AT 1:478, CSMK 3:77–8). In a letter to Vatier, Descartes openly admits that "I could not demonstrate the use of this method in the three treatises which I gave, because it prescribes an order of research which is quite different from the one I thought proper to exposition" (to Vatier, February 22, 1638, AT 1:559, CSMK 3:85).[15] With one exception (his deduction of the rainbow in *Meteorology* VIII), nowhere in the *Essays* (beyond the *Geometry*) does Descartes demonstrate the application of his method (hence Garber's argument described above). When seen in this context, the reaction of Descartes's contemporaries to his bold claims about the efficacy of his method is not in the least bit surprising.[16]

Furthermore, there appears to have been serious confusion about what Descartes's method *is* among his contemporaries and immediate successors. The German theologian and philosopher Johannes Clauberg (1622–1665) regarded *Discourse* and *Meditations* as most representative of Descartes's method (Gassendi also regarded *Meditations* as most representative of Descartes's method), while Descartes's first biographer, Daniel Lipstorp (1631–1684), regarded the method employed in *Geometry* as most

representative.[17] After the publication of *Meditations*, many came to regard Descartes's method as equivalent to the "method of doubt," which led philosophers and theologians in England and France to argue that the method leads only to skepticism, atheism, and a host of other depravities deemed injurious to religion (including homosexuality).[18] By the time a Latin edition of *Rules* had been published in the *Opuscula posthuma* (1701), debates about Descartes's method had already crystalized around these other texts. To be sure, manuscripts of *Rules* did enjoy an underground existence in the seventeenth century. Baillet had a copy (see AT 10:351–7), as did Nicolas Joseph-Poisson,[19] Leibniz, and Arnauld and Nicole, who cite it in *Logic, or the Art of Thinking*, the so-called *Port-Royal Logic*.[20] And as we now know, an early draft of *Rules* also made its way to England.[21] But access to the manuscript remained restricted and in any case did not necessarily translate into understanding; both those with and those without access to *Rules* in the United Provinces and France took liberties in their reconstructions of Descartes's method in order to blunt the force of Descartes's rejection of Aristotelian logic.[22] To my knowledge, *Rules* never played a substantial role in major debates about Descartes's method until the mid-nineteenth century.

When *Rules* finally did become part of these debates,[23] many commentators agreed on one basic thesis: *the unity and universality of Descartes's method depends on its uniform application to all problems, be they problems in metaphysics, natural philosophy, geometry, or morals*.[24] The "Uniformity Thesis" certainly explains why philosophers and historians of science have come to regard Descartes's method as immaterial to the practice of Cartesian science. How can a method that must be uniformly applied be "truly fecund," as Bachelard put it? The Uniformity Thesis has determined even the most divergent interpretations of Descartes's method over the past century by both historians of philosophy as well as philosophers and historians of science. Among historians of philosophy, the difference is that whereas earlier commentators oftentimes (but not always) regarded Descartes as having succeeded in uniformly applying his method to all problems, more recent commentators do not believe that he succeeded, such that what I will argue are *variations in the application of one method* have been consistently interpreted as *different methods*. For example, Descartes's method in *Rules* and *Discourse* II is

often distinguished from the "method of doubt" employed in *Discourse* IV and *Meditations*[25] as well as the "hypothetico-deductive" method in *Discourse* VI, the *Essays*, and *Principles* III–IV.[26] Even within *Rules*, it has been argued that Descartes must, but fails to, apply the method in the same way across the board.[27] In both earlier and more recent scholarship, the underlying assumption is that in principle Descartes remained committed to the Uniformity Thesis. Evaluations of the success or failure of Descartes's method are based on the Uniformity Thesis as a norm.

My interpretation of Descartes's method traces a different route. A careful examination of the relevant texts—above all *Rules*—reveals that Descartes never endorsed the Uniformity Thesis. On the contrary, he rejected it. For Descartes, sustained practice in the method produces a cognitive disposition or "*habitus*" that exists in the human mind (*ingenium*) and enables it to solve problems in the sciences.[28] The actualization of this *habitus* may, within limits, differ from case to case, depending always on the problem at hand. *The parameters of the problem dictate how the method must be applied, such that different problems require (or may require) different modes of application*. Descartes's method not only tolerates more than one mode of application; it *requires* it as a *principle*.[29] *While universal in scope, the method is not uniform in application*. The tendency to conflate universality, on the one hand, with uniformity, on the other, has constituted a major source of confusion in the historical reception of Descartes's method. While every application of the method bears the hallmark of certain operations—principally, intuition, deduction, and enumeration—each application of the method nevertheless remains singular, and no one application can be regarded as paradigmatic, at least not without serious qualification. The difficulty involved in discerning Descartes's method in his texts (and the tendency to distinguish between many methods in his texts) is due not to Descartes's failure to employ his method, but rather to the unfounded expectation (on the part of his readers) that he will always—must always—employ it in the same way.

I have described Descartes's method as an acquired cognitive disposition or "*habitus*." What is a "*habitus*," and in what sense can Descartes's method be described as one? As I show in more detail in Chapter 1, the term "*habitus*" is a Latinization of Aristotle's *hexis* (ἕξις), canonically defined in *Categories* 8 as a quality that is stable and difficult to change.[30]

In Aristotelian psychologies, *habitus* are caused in the soul by acts, and they facilitate the production of such acts in future. Once acquired, intelligible species (or concepts), sciences, productive arts and skills, and virtues (both intellectual and ethical) exist as *habitus* in the relevant part of the soul and empower the soul to act in determinate ways. Unless they are somehow hindered, these *habitus* can be actualized whenever necessary.[31] Geometry is a clear example of a scientific *habitus*. Once acquired, the ability to demonstrate conclusions about continuous magnitudes (e.g., the Pythagorean theorem) remains durably in the intellect as an ability or quality that is stable and difficult to change even when it is not actualized (e.g., when I am not doing mathematics). Some *habitus* are virtues, while others are vices. Those that *perfect* the operation of the subject in which they inhere are virtues, and those that *degrade* the operation of the subject in which they inhere are vices. In *Nicomachean Ethics* VI, Aristotle famously distinguishes between the highest theoretical and practical virtues. The highest *habitus* in the intellect is theoretical wisdom (*σοφία* or, in Latin, *sapientia*). Theoretical wisdom combines two other intellectual *habitus*: understanding (*νοῦς* or, in Latin, *intellectus*), or the *habitus* whereby one understands indemonstrable first principles, and science (*ἐπιστήμη* or, in Latin, *scientia*), or the *habitus* whereby one demonstrates conclusions from these principles.[32] The highest *habitus* in the practical intellect is practical wisdom (*φρόνησις* or, in Latin, *prudentia*), which is the *habitus* whereby one deliberates and acts in a manner most conducive to the good. Deploying virtue in practical wisdom notably requires considering what is appropriate to the occasion in each case. As Aristotle puts it in *Nicomachean Ethics* II. 2 1104ᵃ7–10: "[The] agents themselves must in each case consider what is appropriate to the occasion, as happens also in the art of medicine or of navigation."[33]

In what sense, then, can Descartes's method be described as a *habitus*? The method can only be learned by practice (acts), which cause a durable cognitive disposition in the mind, one that is indeed "stable and difficult to change." That is the very definition of *habitus*. Furthermore, the cognitive disposition produced by practice in the method *perfects the operation of the mind (ingenium*) in science. The method is not only a *habitus*; it is a determinate species of *habitus*: a *virtue*. Thirdly, the cognitive disposition acquired by practice in the method yields science based on the intuition of first principles and the deduction of conclusions from these principles (in a

manner no doubt very different from Aristotelian demonstrations, as I show in Chapters 2–3). The method is not only a virtue, but is (or produces) the highest intellectual virtue: *theoretical wisdom* (*sapientia*). Finally, and perhaps most importantly, the application of the method *depends on the ability of the agent to assess whatever is "appropriate to the occasion"* (i.e., the problem), "as happens also in the art of medicine or of navigation." The method is an *art*, and it incorporates the locally responsive rationality that constitutes *practical wisdom*. In all of these respects, Descartes's method is a *habitus*, one that even draws on or integrates forms of rationality that Aristotle regarded as irreducibly heterogeneous: theoretical and practical wisdom.

Before it achieves discursive articulation in a corpus of propositions, Cartesian method and science exist as an acquired cognitive disposition that literally empowers minds to *solve problems* and thereby *produce* the propositional order of science. As Descartes puts in an important letter to Hogelande: "By 'science' I mean the skill [*peritiam*] to solve every problem, and thus to discover by one's own efforts [*propria industria*] everything capable of being discovered in that science by means of our native human intelligence [*humano ingenio*]" (February 8, 1640, AT 3:722–3, CSMK 3:144).[34] And as Descartes puts it in Rule 3, one becomes a scientist only by acquiring the "intellectual aptitude to solve any given problem [*ingenio apti ad quaecumque problemata resolvenda*]" (AT 10:367, CSM 1:13). As these passages indicate, Descartes's method is, ontologically, a *habitus*—a skill or, better yet, an art, as I indicated above. As I will argue throughout this book, practice in the method produces cognitive virtues in the mind, so that the practitioner of the method may solve problems by assessing their degree of complexity and defining an order of operations whose execution yields a solution to the problem (and, therefore, yields "science" or "certain and evident cognition").

An "habitual" interpretation of Descartes's method along the lines sketched above is both textually informed and more philosophically satisfying than standard interpretations of Descartes's method, according to which the method is reducible to a system of purely discursive rules and the relevant conception of "rule-following" is one in which sensitivity to context (i.e., sensitivity to problems) plays little or no role. Like any ethical virtue, the Cartesian scientific *habitus* is flexible, since it may (indeed, must) be *responsive to the unique demands different problems place on the*

human mind. On an habitual interpretation, Descartes's method is finally freed from the rigid constraints imposed by the Uniformity Thesis, which does indeed make it impossible to see how one and the same method can be applied to different problems in different ways.

I develop my habitual interpretation of Descartes's method in five parts and eleven chapters.

In Part I (Chapters 1–2), I argue that Descartes's conception of method and science as *habitus* emerges in *Rules* as he develops his thesis about the unity of the sciences as a whole via a critique of the presuppositions behind the scholastic debate about the unity of science. Chapter 1 reconstructs the scholastic debate from Aquinas (1225–1274) to Suárez (1548–1617). The scholastic debate about the unity of science turned on the *ontology* of scientific *habitus*. Does the unity of a science consist in one, ontologically simple (indivisible) intellectual *habitus*, which retains its simple unity even as it extends to previously unknown principles and conclusions (Aquinas, Henry of Ghent, Cajetan), or does the unity of a science rather consist in a complex and coordinated unity of many really distinct intellectual *habitus*, one corresponding to each principle and conclusion in a science (Scotus, Ockham, Suárez, and the Conimbricenses)? I conclude Chapter 1 by arguing that despite important differences between the participants in the scholastic debate, the debate remained centered on the unity of individual sciences, not on the unity of the sciences as a whole. The scholastics individuated sciences or scientific *habitus* by their respective objects (e.g., geometry by continuous magnitude, arithmetic by discrete magnitude, etc.), and Aristotle's ban on genus-crossing in the sciences (i.e., employing the principles of one science to demonstrate conclusions in another science) aggressively policed the boundaries between different scientific *habitus*, effectively rendering the Cartesian thesis that the sciences are an interconnected whole—the unity of science as it has been understood since the seventeenth century—inconceivable.

This sets the stage for Chapter 2, where I argue that Descartes takes a decisive step beyond his scholastic predecessors and contemporaries, not by denying that science is a *habitus*, but rather by suspending Aristotle's ban on genus-crossing and denying that scientific *habitus* should be distinguished by their objects.[35] The "Cartesian scientific *habitus*" extends to all sciences because it extends to all objects and problems in the sciences without exception. This thesis underlies Descartes's conception of the unity

of science in *Rules*, and I develop the mechanics behind the thesis in considerable detail. I also argue that the Cartesian scientific *habitus* differs from scholastic theories of scientific *habitus* in three important respects: in addition to coordinating different sciences in order to solve problems, the Cartesian scientific *habitus* is not reducible to any one scientific principle or demonstration and it is not restricted to reproducing the acts whereby it is acquired. The Cartesian scientific *habitus* is fundamentally *productive*, not merely *retentive*. Most scholastics argued that scientific *habitus* only retain what the mind has already learned and do *not* extend beyond the acts by which they are acquired.

In Part II (Chapters 3–4), I discuss the operations of the method (intuition, deduction, and enumeration) (Chapter 3). While these operations are natural and so are not *acquired* by acts, they can and must be *perfected* by acts. Otherwise, there can be no science. I argue that the *sustained perfection* of these operations engenders the Cartesian scientific *habitus* in the human mind by degrees according to a definite order, beginning with practice in solving problems in the simplest sciences. By learning how to solve a problem in the simplest sciences, I acquire the ability to solve problems in the more advanced sciences. This leads into Chapter 4, where I argue that *mathesis universalis*, far from being identical to Descartes's method or even to Descartes's mathematics in Rules 13–21 or *Geometry*, is the science in which one first learns how to apply the operations of the method to problems that are so easy and simple as to seem "almost childish," such as finding mean proportionals (e.g., 2, 4, *x*, 32, etc.) (AT 10:384, CSM 1:23).[36] Practice in *mathesis universalis* produces the first degree of the Cartesian scientific *habitus*, which empowers the mind to define different types of problems as well as the order of operations needed to solve them. *Mathesis universalis* is what one scholar has aptly described as a "logic of problems."[37] The *habitus* one acquires via practice in *mathesis universalis* may then be transferred to the "more advanced sciences," as Descartes puts it in Rule 4 (AT 10:404, CSM 1:35).

Before one can solve problems in the more advanced sciences, however, one must first determine the class of problems the human mind has the capacity to solve in general. Descartes describes the problem of the limits of knowledge in Rule 8 as the "first problem of all that should be examined by means of the Rules described above [Rules 1–7]" (AT 10:397–8, CSM 1:31). In Part III (Chapters 5–8), I show how Descartes employs the method

in his solution to the problem of the limits of knowledge (Chapter 5),[38] and I reconstruct each part of the solution (Chapters 6–8). Descartes's solution to the problem of the limits of knowledge includes a theory of the human cognitive faculties and a theory of the objects of knowledge (the "simple natures"). Whereas most interpretations of Descartes's theory of the faculties in *Rules* focus exclusively on Descartes's mechanization of Aristotelian faculty psychology,[39] I argue that Descartes's theory of the faculties in *Rules* has two components: a rudimentary mechanical description of the physiology and psychology of cognition (from the affection of the sensory organs to intentional consciousness in *vis cognoscens*), and a normative description of how the faculties *should* be employed under different conditions. The normative description of how the faculties should be employed is the immediate foundation of the Cartesian scientific *habitus*, and its ontological foundations rest on Descartes's dualism in *Rules* (Chapter 8). Only once Descartes's theory of the faculties has become fully embodied in cognitive practice as a *habitus* is the operator of the method in a position to solve particular problems in the sciences.

Accordingly, in Part IV (Chapters 9–10), I turn to particular problems in the sciences. In Chapter 9, I reconstruct Descartes's method in mathematics in Rules 13–21, and in Chapter 10, I carry out Descartes's proposed deduction of the law of refraction and the shape of the anaclastic lens in Rule 8.[40] Finally, in Part V (Chapter 11), I explain why Descartes abandoned *Rules* as a treatise, but not the method described therein. I then discuss why he decided to pursue metaphysics and systematic natural philosophy as early as 1629, and I also discuss Descartes's method after *Rules*. I do not reconstruct the application of the method to problems in metaphysics, mathematics, and natural philosophy after *Rules* (which would require a book of its own), but rather limit myself to the more modest task of identifying transformations in Descartes's method after *Rules*, including the relation between method and system in and after 1629, Descartes's revision of the theory of simple natures in *Principles* and related texts and correspondence, and the relation between the theory of simple natures and Descartes's ontology of substance, attribute, and mode, also in *Principles* and related texts and correspondence.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0001

[1] Descartes 1824–1826, 11:i, cited in Marion 1975, 18.

[2] Liard 1880, 573.

[3] Boyce-Gibson 1898a, 149. See also Boyce-Gibson 1888b. The only secondary sources Boyce-Gibson cites are French (Millet 1867 and his own Boyce-Gibson 1896, written in French), never English. With one or two minor exceptions, Smith 1902 and Keeling [1934] 1968 also cite French sources exclusively. I have been unable to find any scholarly articles on *Rules* written in English prior to Boyce-Gibson's articles.

[4] Beck 1952, 8.

[5] For other positive appraisals of Descartes's method over the last century, see Natorp 1882, 1–26; Cassirer 1902, 1937; Sirven 1928, 171, 222–5, 442; Laporte 1945, 19; Röd 1971, 13; Marion 1975; Hintikka 1978, 87; Heidegger 1962. Not everyone agreed. For example, Serrus 1933, 77–125 argued that Descartes's method could only be successfully applied to problems in mathematics and natural philosophy, but not to problems in metaphysics. By contemporary standards, even Serrus's argument seems rather presumptuous. More recent developments in the historiography of science have rendered the thesis that Descartes applied his method at all rather controversial. See, e.g., Schuster 1977, 2013.

[6] See Bachelard 1934, 165–6. See also Bachelard 1972, 35–44 and the discussion in Lecourt 1968, 72.

[7] Koyré 1956, 15. It is true that Descartes wrote *Discourse* after he had already composed the *Essays*, but the method described in *Discourse* is a highly abbreviated summary of *Rules* (Gilson in Descartes 1987a, 196–214 draws out the parallels in considerable detail), which Descartes composed long before the *Essays*. See Chapter 11, Section 11.2.

[8] See Kuhn 1977, 137, where Kuhn laments the "widespread conviction" amongst historians of science "that scientists discover truth by the quasi-mechanical (and perhaps not very interesting) application of scientific method." Like Bachelard (see n. 6 above), Kuhn describes the application of method as "mechanical." See also Kuhn ibid., 150, where he describes what he dubs the "myth of method." Like Bachelard and Kuhn, Feyerabend 1975, 14 writes: "The idea of a method that contains firm, unchanging, and absolutely binding principles for conducting the business of science meets considerable difficulty when confronted with the results of historical research." Feyerabend too regards the application of method as mechanical ("firm, unchanging, and absolutely binding").

[9] Schuster 2013, 270–3. See also Schuster and Yeo 1986.

[10] Clarke 1982, 180.

[11] Dear 2000, 159.

[12] See Garber 1992, 46–50 and 2001, 47–51.

[13] See, e.g., letter to Vatier, February 22, 1638, AT 1:559–560, CSM 1:85.

[14] Leibniz 1875–1889, 4:329.

[15] See also *Conversation with Burman*, AT 5:153, CSMK 3:338: "…for the method and order of discovery is one thing, and that of exposition another." I discuss Descartes's important distinction between the "order of research" and the "order of exposition" in Chapter 10, Section 10.9.

[16] Garber 1988 accurately describes the immediate reception of Cartesian science by his contemporaries as "the revolution that did not happen." One could perhaps describe the immediate reception of Descartes's method by his contemporaries in the same way.

[17] See Lipstorp 1653, 8; Gassendi 1658, 1:65–6; Clauberg 1652, 1–7. See also the discussion in Sirven 1928, 16 and 222 and Gilson in Descartes 1987a, 79.

[18] See Voetius 1648–1669, 1:177; Schook 1643; Descartes's letter to Voetius, May 1643, AT 8B:1–199, CSMK 3:220–4 (abridged) and Descartes and Schook 1988; Verbeek 1992; Schmaltz 2017, 15–63.

[19] See Poisson 1670.

[20] See Arnauld and Nicole 1664, 391ff and AT 10:471–2, CSM 1:77–8.

[21] See Serjeanston and Edwards in Descartes forthcoming.

[22] See Ariew 2014, 157–65 and Schmaltz 2017, 64–121.

[23] See Liard 1880; Natorp 1882; Cassirer 1902, 1937; Boyce-Gibson 1898a, 1898b; Smith 1902; Hamelin 1921; Gilson in Descartes 1987a; Sirven 1928; Serrus 1933; Merrylees 1934; Keeling [1934] 1968, 1937; Laporte 1945; Alquié 1950; Beck 1952; Joachim 1957; Mahnke 1967; Buchdahl 1969; Röd 1971; Marion 1975; Schouls 1980, 2000; Clarke 1982; Dear 2000; Garber 1992, 2001; Rabouin 2009; Schuster 1977, 2013; Dubouclez 2013; Nelson and Rogers 2015; Nelson 2017.

[24] See, e.g., Boyce-Gibson 1898b, 348: "With Descartes there is no question of a method that shall adapt itself to the material it is destined to organize; his method is consequently both uniform and universal." Boyce-Gibson [1932] 1967, 186 also refers to Descartes's "insistence on a uniform method" in Rule 1. See also Liard 1880, 574; Keeling [1934] 1968, 66: "…So methodological precepts must be quite general; they must be applicable indifferently to any subject-matter and to any type of problem." More recently, see Schouls 1980, 63: "[When] we speak of a universal method, no diversity is introduced when this method is applied to different kinds of subject-matter."

[25] Curley 1978, 35–45 regards the method employed in *Discourse* IV and *Meditations* as distinct from the method described in *Rules*, as does Broughton 2002, 2–7. For both of these commentators, the method of doubt is not explicitly prescribed in *Rules*, and so the method of doubt cannot be regarded as an application of the method of *Rules*.

[26] McMullin 1968; Smith 1966, 51, 96–7; Duhem 1954, 43–6; Dambska 1957; Roth 1937; Segond 1932; Stock 1931; Milhaud 1921; Tannery 1896 maintain in one way or another that Descartes's method becomes hypothetico-deductive in *Discourse* VI and the *Essays*. Dellsén 2017; Garber 2001; Hatfield 1988; Sabra 1981; Allard 1963; Beck 1952 argue that Descartes's method becomes hypothetico-deductive in *Principles* III–IV. Clarke 1982 maintains that Descartes's method is consistently hypothetico-deductive from *Rules* on.

[27] For example, Garber 1992, 42–4 argues that Descartes did not apply his method to the problem of the limits of knowledge in Rules 8 and 12 because his solution to the problem of the limits of knowledge does not resemble his solution to the problem of the anaclastic lens in Rule 8 or his deduction of the rainbow in *Meteorology* VIII. He assumes that the method must be uniformly applied to all of these problems, and infers that the method is not applied to the first problem because it does not resemble the application of the method to the other two (which do resemble one another). For more detailed discussion, see Chapters 5–7, 10, and 11.

[28] I discuss the concept of *habitus* in more detail in Chapters 1–2 and Descartes's concept of *ingenium* in more detail in Chapter 2, Section 2.2.

[29] See also Dika and Kambouchner forthcoming, Kambouchner 2019, and Dika 2015.

[30] Aristotle 1984, 1:14 (*Categories* 8 $8^{b}27$–8).

[31] On the history of medieval theories of *habitus*, see Faucher and Roques 2018.

[32] See Aristotle 1984, 2:1801–2 (*Nicomachean Ethics* VI. 7) and Aquinas 1882–, 6:355–66 (*ST* I–II, q. 57, art. 2); trans. Aquinas 1945, 2:431.

[33] See Aristotle 1984, 2:1744.

[34] In this letter, Descartes employs “science” in a manner sufficiently inclusive to be synonymous with “method.” In other contexts, however, he employs “science” in a narrower sense to refer, among other things, to the *results* of the application of the method to a determinate problem. Which sense he has in mind is typically clear from the context.

[35] Beck 1952 and Marion 1975 argue that Descartes rejects the Aristotelian definition of science as a *habitus* in *Rules*. I discuss their position in more detail in Chapter 2, Section 2.1.

[36] Many scholars have argued that *mathesis universalis* is identical to the method as such, which embraces both the mathematical and the non-mathematical sciences. See Marion 1975 and van de Pitte 1979. See also Heimsoeth 1912 and Röd 1971. Others have argued that *mathesis universalis* embraces a more restricted terrain: Cartesian mathematics, which they interpret in a variety of ways. Sasaki 2003, 189–205, esp. 195, 197, and 200; Liard 1880, 1911; Boutroux 1900; and Klein [1934–1936] 1992 identify *mathesis universalis* and the algebraic geometry developed in Rules 13–21 and *Geometry*. Schuster 1980 and 2013, 230–5 argues that *mathesis universalis* is identical to what he terms “physico-mathematics.”

[37] See Rabouin 2009, 278–85.

[38] Garber 1992, 42–4 denies that Descartes applies the method to the problem of the limits of knowledge because it does not resemble his proposed solution to the problem of the law of refraction and the shape of the anaclastic lens in Rule 8.

[39] See, e.g., Beck 1952, 15–30; Marion 1975, 113–31; Clarke 2003, 16–45, 46–50, 81–9; Gaukroger 1995, 152–86; Schuster 2013, 214–360.

[40] Schuster 1977, 2013 denies that Descartes employed or could have employed the method in his solution to the problem of the law of refraction and the shape of the anaclastic lens in Rule 8.

PART I

THE HABITUAL UNITY OF SCIENCE

Aquinas to Descartes

1

The Habitual Unity of Individual Sciences

Aquinas to Suárez

1.1 *Hexis/Habitus*

In Chapter 1, I reconstruct the scholastic debate about the unity of scientific *habitus* from Aquinas (1225–1274) to Suárez (1548–1617). In Section 1.1, I reconstruct the general role played by the concept of *habitus* in Aristotelian psychologies. In Section 1.2, I discuss Aristotle's theory of demonstration in *Posterior Analytics*, above all his ban on genus-crossing (*μετάβασις*) in the sciences (i.e., employing the principles of one science in order to demonstrate conclusions in another science). I argue that Aristotle's ban on genus-crossing restricted the scholastic debate about the unity of science to a debate about the unity of individual sciences. In Sections 1.3–1.5, I turn to Aquinas, Scotus, Ockham, and Suárez, whom I have chosen due both to their historical importance and to the fact they will help isolate Descartes's distinctive position in Chapter 2. It is not my intention to defend any one theory, but rather only to describe how a succession of different theories respond to the same problem: how can a science have many principles, demonstrations, and objects, and yet be "one" science? I begin Section 1.3 by explaining why Aquinas solves this problem by arguing that the unity of a science consists in a *habitus* that disposes the intellect to consider everything in the science under one formality (*ratio*). Aquinas's thesis requires explanation because Aristotle does not establish the unity of any science by reference to a *habitus* in the intellect, but rather by reference to a genus of being. "A science is one if it is of one genus," writes Aristotle in *Posterior Analytics* I.28.[1] I argue that Aquinas establishes the unity of a

science by reference to a *habitus* in the intellect because the science that most interests Aquinas is sacred doctrine (*sacra doctrina*), and sacred doctrine has more than one genus as its object (God, angels, human beings, etc.). Aquinas establishes the unity of sacred doctrine by arguing that the objects of sacred doctrine, however ontologically diverse, are nevertheless considered by the intellect under one formality (*ratio*): the formality of revelation, which is the first object of the *habitus* of faith. Aquinas's solution to the problem of the unity of sacred doctrine framed how the scholastic debate about the unity of science would be conducted well into the seventeenth century. In Section 1.4, I reconstruct the scholastic debate about the unity of science in and after Aquinas. As I indicated in the Introduction, participants in the debate divide into two camps: those who maintain that a scientific *habitus* is a simple quality, which retains its simplicity (indivisibility) even as it extends to previously unknown principles and demonstrations (Aquinas, Henry of Ghent, Cajetan), and those who maintain that sciences are composed of many really distinct *habitus*, one *habitus* corresponding to each principle and demonstration in the science (Scotus, Ockham, Suárez, and the Conimbricenses, each of whom offer different solutions to the problem of how these many *habitus* nevertheless constitute one science). I conclude Chapter 1 by arguing that the scholastic debate about the unity of science ends in *aporia*. As Suárez recognizes toward the end of his discussion of the problem in *Disputationes metaphysicae* (disp. 44, sec. 11), different sciences have irreducibly different degrees of unity, and it is simply not possible to impose any one criterion of unity on all of them.

An appropriate place to turn in order to introduce the concept of *habitus* more fully than I did in the Introduction is Aquinas's *Treatise on* Habitus (*ST* I–II, q. 49–54), which divides into six questions: the essence of *habitus*; the subject of *habitus*; the formation of *habitus*; the increase of *habitus*; the corruption of *habitus*; and the distinction of *habitus*. I neither can nor need to reconstruct Aquinas's entire theory of *habitus* here, but I will rely on Aquinas's division of the treatise in order to briefly reconstruct the general role played by the concept of *habitus* in Aristotelian psychologies.

(1) *The Essence of* Habitus. As indicated in the Introduction, the term "*habitus*" is a Latinization of Aristotle's *hexis* (*ἕξις*), defined in *Categories* 8 as a quality that is stable and difficult to change.[2] Unlike other, more

ephemeral qualities, such as blushing or sweetness, *habitus* are durable possessions:

> Such are the branches of knowledge and the virtues. For knowledge seems to be something stable and difficult to change if one has even a moderate grasp of a branch of knowledge, unless a great change is brought about by illness or some other such thing. So also virtue; justice, temperance, and the rest seem to be not easily changed.[3]

Once acquired, intelligible species (or concepts), theoretical sciences, productive arts, and virtues (both ethical and intellectual) all exist as *habitus* in the relevant part of the soul. They need not be activated at all times, but when they are not activated, they do not thereby cease to exist; they continue to exist in the soul habitually as second potentialities/first actualities (over and above the faculties of the soul, which are first potentialities).[4] These *habitus* can be reactivated whenever necessary because they are under rational control.[5] As Augustine puts it in a definition frequently cited by Aquinas: "*Habitus* is that whereby something is done when necessary." A "*habitus* is that whereby we act when we will," as Ibn Rushd (Averröes) puts it in another definition also frequently cited by Aquinas.[6]

(2) *The subject of* habitus. *Habitus* (including those that determine the body to some operation) reside principally in the soul.[7] Regarding those *habitus* that determine the soul to some operation, they are not restricted to any one activity in the soul. On the contrary, they determine every rational activity in the soul (i.e., every activity that can be determined by reason).[8] *Habitus* are necessary because the rational faculties (the intellect and the will) are "capable of determination in several ways and to various things."[9] Fire cannot acquire a *habitus*, because by its nature fire can only act in one way.[10] Animals, insofar as they are wholly determined by natural instinct, also cannot acquire *habitus*.[11] The human intellect and the will, by contrast, are "capable of determination in several ways," and so they need a *habitus* in order to determine them to one act as opposed to another. *Habitus*, Aquinas explains, are "midway between pure potentiality and complete act [*medius inter puram potentiam et actum perfectum*],"[12] and "it belongs to every *habitus* to have relation to an act."[13] Thus, *habitus* mediate between faculties and their acts.

(3) *The formation, increase, and corruption of* habitus. *Habitus* are caused and increased by acts (they can also be corrupted by contrary acts, e.g., an ethical virtue can be corrupted by contrary acts of vice). These acts leave traces in the soul, and with repetition these traces become durable possessions. Indeed, *habitus* are caused by acts and acts are caused by *habitus* in a virtuous circle. “Like acts cause like *habitus*.”[14] Just as I acquire an ethical *habitus* by ethical actions, so too do I acquire arts and sciences by the relevant practical and theoretical actions (e.g., playing an instrument; demonstrating mathematical propositions). In *Nicomachean Ethics* II.1 1103^{a}31–1103^{b}1, Aristotle describes how ethical *habitus* are acquired “by first exercising them, as also happens in the case of the arts as well. For the things we have to learn before we can do, we learn by doing, e.g., men become builders by building and lyre-players by playing the lyre; so too we become just by doing just acts, temperate by doing temperate acts, brave by doing brave acts.”[15] In *De Anima* II.5 417^{a}31–32, Aristotle describes how scientific *habitus* are acquired by “repeated transitions from one state to its opposite under instruction” (e.g., instruction in mathematics, which effects repeated transitions from ignorance to knowledge).[16]

(4) *The distinction of* habitus. As mediators between faculties and their acts, *habitus* can dispose their subjects in one of two ways: well or badly.[17] Good *habitus* dispose their subjects well, and are virtues, while bad *habitus* dispose their subjects badly, and are vices. Whether a *habitus* disposes its subject well or badly depends on whether it disposes it to perform acts that are conducive to its *natural end*. Ethical vices dispose the soul to acts no less than ethical virtues do, but they are not conducive to the end of the practical intellect and the rationally determined will: the good. Ethical virtues, by contrast, dispose the will well. Sciences also dispose their subjects well because they are conducive to the end of the speculative intellect: the true.[18] Any *habitus*—intellectual or ethical—that disposes its subject to acts conducive to its end are virtues.[19] “Virtue is that which makes its possessor good, and his work good likewise.”[20]

In *Nicomachean Ethics* VI—Aristotle’s famous classification of the intellectual virtues—he distinguishes between three such virtues: understanding (*νοῦς*), the *habitus* of first principles; science (*ἐπιστήμη*), the *habitus* of demonstration; and wisdom (*σοφία*), the *habitus* that combines both understanding and science.[21] Based on Aristotle’s classification of the

intellectual virtues, Aquinas distinguishes between two senses of *habitus* in Aristotle's treatises. In *Nicomachean Ethics* VI, Aquinas argues, *habitus* refers *inter alia* to the intellectual virtues. But in *Categories* 8, *habitus* refers to individual sciences, such as geometry or arithmetic:

> In the *Ethics*, the Philosopher considers the intellectual *habitus* insofar as they are intellectual virtues. Now they are called virtues because they perfect the intellect in its operation; for "virtue makes its possessor good and renders his work good." So he distinguishes between virtues of this sort in as much as [they are] speculative *habitus* [that] perfect the intellect [*animae speculativa*] in different ways. In one way the speculative part of the soul is perfected by understanding [*intellectum*], which is the *habitus* of principles, through which some things become known of themselves. In another way it is perfected by a *habitus* through which conclusions demonstrated from these principles are known, whether the demonstration proceeds from inferior causes, as in science [*scientia*], or from the highest causes, as in wisdom [*sapientia*]. But when sciences are differentiated insofar as they are *habitus*, they must be distinguished according to their objects [*obiecta*], that is, according to the things of which the sciences treat. And it is in this way that both here and in the *Metaphysics* speculative philosophy is distinguished into three parts.[22]

Individual scientific *habitus* are distinguished by their respective objects. As Aquinas puts it in *ST* I–II, q. 54, art. 2: "Acts differ in species according to the diversity of their objects […]. But *habitus* are dispositions to acts. Therefore *habitus* are also distinguished according to the diversity of their objects."[23] Geometry is distinguished by its object (e.g., continuous magnitude), under which everything in the science is considered. The intellectual virtues, by contrast, are not distinguished by *object*, but rather by the manner in which they perfect the intellect—i.e., by *function*. While both senses of *habitus* are no doubt relevant to the scholastic debate about the unity of science, this debate is not principally a debate about the intellectual virtues; it is a debate about the unity of individual sciences.

1.2 Aristotle's Ban on Genus-Crossing (*μετάβασις*) in *Posterior Analytics* I.7

Modern readers may well wonder why the scholastic debate about the unity of science focused on the unity of individual sciences, and not on the unity of the sciences as an interconnected totality. As I indicated above and will show in more detail presently, Aristotle's ban on genus-crossing in the sciences (i.e., employing the principles of one science in order to

demonstrate conclusions in another science) restricted the scholastic debate about the unity of science to a debate about the unity of individual sciences. In *Posterior Analytics* I.7, Aristotle argues that every science has its own principles and demonstrations. These principles and demonstrations cannot be transferred from one science to another *unless* both sciences deal with the same genus (either *simpliciter* or in some respect). Sciences that deal with different genera are irreducibly heterogeneous. For example, arithmetic and geometry deal with quantity, but arithmetic deals with discrete quantity (number), and geometry deals with continuous quantity (lines and figures). The *per se* properties of number (e.g., even, odd, etc.) and the *per se* properties of magnitude (e.g., straight, curved, etc.) are different.[24] There are no "curved" numbers or "odd" lines. The *ontological differences* between the genera these (or any) two sciences deal with determines *logical differences* between their respective principles, since discrete quantity has properties that continuous quantity does not have and vice versa, and principles can only express relations between genera and their *per se* properties. For example:

All numbers are either even or odd.

All odd numbers have remainders when divided by two.

The number 7 has a remainder when divided by two.

Therefore, the number 7 is odd.

In this demonstration, as in any scientific demonstration, the genus serves as the middle term between the two extremes. The genus is "number," "even" and "odd" are *per se* properties of number, and it is demonstrated that the number "7" has the property "odd." This example illustrates that no conclusion in arithmetic can ever extend beyond number. Arithmetical demonstrations must be about number because their principles can only express the *per se* properties of number. Geometrical demonstrations must be about magnitudes because their principles can only express the *per se* properties of magnitudes. In general, then, any syllogism whose principles are about one genus, but whose conclusions are about another, has crossed both an ontological and a logical boundary, and so is not scientific. Genus-crossing and Aristotelian science are incompatible:

> One cannot […] prove anything by crossing from another genus – e.g., something geometrical by arithmetic. […] [Of] things whose genus is different – as arithmetic and geometry, one cannot apply arithmetical demonstrations to the accidentals of magnitudes, unless magnitudes are numbers […]. Arithmetical demonstrations always include the genus about which the demonstration is, and so do the others; hence it is necessary for the genus to be the same, either *simpliciter* or in some respect, if the demonstration is going to cross. That it is impossible otherwise is clear; for it is necessary for the extreme and the middle terms to come from the same genus.[25]

For a demonstration to cross from one science to another, "it is necessary for the genus to be the same, either *simpliciter* or in some respect." For example, geometrical principles and demonstrations can serve as principles and demonstrations in optics, because optics deals with continuous quantity in nature (light rays), and so it has the same genus as geometry does, not "*simpliciter*," but rather "in some respect." Similarly, arithmetical principles and demonstrations can serve as principles and demonstrations in harmonics, because harmonics deals with numerical ratios between notes.[26] Aristotelian scholastics described these cases as cases of "subalternation" between sciences. One science subalternates another when its principles and demonstrations cross over into another science. Only in cases like these—i.e., subalternation cases that are not cross-generic—does Aristotle authorize the transference of principles and demonstrations from one science to another.[27] In all other cases, principles and demonstrations do not cross over.

While Aristotle does not mention scientific *habitus* in *Posterior Analytics* I.7, based on his definition of the sciences as *habitus* in Categories 8 Aristotelian scholastics inferred that the ban on genus-crossing in the sciences is a ban on combining different scientific *habitus*. To assert that sciences are distinguished by their genera is to assert that scientific *habitus* are also so distinguished, and that these *habitus* do not interact unless the one science subalternates the other, such that *the maximum possible degree of scientific unity is the unity of an individual scientific* habitus. The irreducible diversity of the sciences, on the one hand, and the irreducible diversity of scientific *habitus*, on the other, are but two sides of the same coin. The unification of the sciences in one *habitus*, therefore, has no place in Aristotelian science.

Aristotle's ban on genus-crossing in the sciences had a decisive impact on scholastic theories of science from the thirteenth century to the late sixteenth century.[28] In different ways, Aquinas, Scotus, Ockham, and

Suárez all accepted Aristotle's ban on genus-crossing, and although they frequently disagreed about how best to define the unity of a science, they only debated about the unity of individual sciences. The unity of science in the Cartesian sense—a unity that includes all sciences—remained inconceivable. Any such possibility would have been regarded as absurd, since everybody agreed that being divides into different genera and that differences among the sciences are rooted in differences among genera. The basic principles of Aristotelian first philosophy entail the ban on genus-crossing in the theory and practice of science, and rule out the possibility of the unity of science as Descartes would later describe it in Rule 1 of *Rules*.

1.3 Aquinas's Habitual Interpretation of the Unity of Science

In Section 1.2, I showed how Aristotle's ban on genus-crossing restricted the scholastic debate about the unity of science to a debate about the unity of individual sciences. I also argued that the diversification of sciences and the diversification of scientific *habitus* are but two sides of the same coin, such that the unification of the sciences in one *habitus* has no place in Aristotelian science. But Aristotle's theory of science does not explain why the concept of *habitus* becomes so central in scholastic debates about the unity of science. As we have seen in Section 1.2, according to Aristotle the unity of a science is based on its genus, not on a *habitus* in the soul. To my knowledge, Aristotle never explicitly employs the concept of *habitus* in order to establish the unity of *any* science. He only defines the sciences as *habitus* and leaves it at that. For Aquinas, by contrast, the unity of a science consists in a *habitus* that disposes the intellect to consider everything in the science under one formality (*ratio*). Both Aristotle and Aquinas define the sciences as *habitus*, but only Aquinas systematically insists that the *unity* of a science *consists* in a *habitus*. Why does Aquinas establish the unity of a science by reference to a *habitus* in the soul, and not simply by reference to a genus beyond the soul?

My argument in this section is as follows. Since Aristotle argues that sciences are individuated by their genus,[29] he creates two interrelated problems Aquinas must address about sacred doctrine: "Whether sacred doctrine is a science?" and "Whether it is one science or many?"[30]

According to Aquinas, sacred doctrine is a science, and it deals with more than one genus (God, angels, human beings, etc.). Clearly, the unity of sacred doctrine cannot be based on a genus because there is no one genus it deals with. This leads Aquinas to locate the unity of sacred doctrine, not in a genus, but rather in a *habitus* that disposes the intellect to consider everything in sacred doctrine, however ontologically diverse, under one formality: the formality of revelation, which is the first object of the *habitus* of faith. In the remainder of this section, I reconstruct Aquinas's answer to the two problems about sacred doctrine enumerated above as well as the historical context in which these problems arose.

The Latin translation of Aristotle's *Posterior Analytics* in the second quarter of the twelfth century[31] immediately gave rise to questions about whether theology is a demonstrative science in the Aristotelian sense.[32] Alexander of Hales (c. 1185–1245) was the first to pose the question in his *Universae theologiae summa*,[33] and debates over the question continued at both the University of Paris and the University of Oxford.[34] Hales himself did not regard theology as a demonstrative science. Indeed, prior to Aquinas, *all* participants in the debate about whether theology is a demonstrative science in the Aristotelian sense *deny* that theology is a science: in Paris, Odo Rigaldus (d. 1275), William of Meliton (d. 1257), and Albert the Great (c. 1200–1280) (Aquinas's teacher); in Oxford, Richard Fischacre (c. 1208–1248) and Robert Kilwardby (d. 1279) all deny that theology is an Aristotelian demonstrative science.[35] To be sure, they regarded theology as a science in a more inclusive sense (as opposed to belief or opinion), but not as a demonstrative science. Whenever they employ the term "*scientia*" to describe theology, it is science in this more inclusive sense that they have in mind. Prior to his own intervention, Aquinas occupied a minority position in the debate.

To demonstrate that sacred doctrine is an Aristotelian demonstrative science, and that it is *one* science, Aquinas had to address three objections in circulation since Alexander of Hales: (1) For Aristotle, every demonstration is based on self-evident principles. The articles of faith, however, are not self-evident, since "their truth is not admitted by all."[36] Therefore, sacred doctrine is not a science. (2) Sacred doctrine deals with the actions of individuals, such as Abraham, Isaac, and Jacob. A science of individuals, however, is not possible, according to Aristotle; there is no

science of Abraham, Isaac, or Jacob. Therefore, sacred doctrine is not a science.[37] (3) Finally, even if sacred doctrine is a science, it is not *one* science, but rather many sciences. For Aristotle, "a science is one if it is of one genus,"[38] but sacred doctrine deals with more than one genus (God, angels, human beings, etc.),[39] which belong to "separate philosophical sciences."[40] Sacred doctrine violates Aristotle's ban on genus-crossing. Therefore, it is not a science. Sacred doctrine, it seems, violates nearly every requirement of Aristotelian demonstrative science.

As I will show in more detail below, when Aquinas argues that sacred doctrine is a science, he synthesizes and extends prior developments in the history of the debate. The relevant parts of Aquinas's argument may be summed up in five basic theses: (1) Just as the principles of any science are indemonstrable (on pain of regress or circularity), so too are the principles of sacred doctrine.[41] (2) Just as the principles of any science are known by the *habitus* of first principles (*intellectus*), which is a natural *habitus*, so too the principles of sacred doctrine are known by the *habitus* of faith (*habitus fidei*), which is a supernaturally infused *habitus*. (3) Just as the principles of optics or harmonics are self-evident, not in themselves, but rather in relation to a higher science (geometry and optics, respectively), so too are the principles of sacred doctrine (i.e., the articles of faith) self-evident, not in themselves, but rather in relation to a higher science: the science of God and the blessed (*scientia divina*). Sacred doctrine is subalternate to the science of God and the blessed no less than optics is subalternate to geometry or harmonics to arithmetic. (4) Sacred doctrine does not deal with individual actions qua individual, but rather as examples.[42] Beyond his argument that sacred doctrine is a science, Aquinas must also establish that sacred doctrine is *one* science, not many. He argues (5) that the unity of sacred doctrine consists, not in its end, as Albert the Great had claimed,[43] but rather in a *habitus*, which disposes the intellect to consider everything in sacred doctrine, however ontologically diverse, under the formality of revelation, which is the first object of the *habitus* of faith.

I will now develop Aquinas's arguments in more detail.

To the objection that the principles of sacred doctrine are not self-evident, Aquinas responds by (1) drawing an analogy between the articles of the Christian faith and the indemonstrable first principles of an Aristotelian demonstrative science, and (2) introducing an analogy between

the *habitus* whereby the articles of faith are known (*habitus fidei*) and the *habitus* whereby first principles are known (*intellectus*). These two analogies can already be found in the prologue to Aquinas's commentary on Peter Lombard's *Sentences* (c. 1252–1257):

> This science [sacred doctrine] has the articles of faith for its first principles which through the infused light of faith are known *per se* by having faith [*qui per lumen fidei infusum per se noti sunt habenti fidem*], much as principles are naturally instilled in us by the light of the agent intellect.[44]

> It should be said that whereas the *habitus* of first principles is not acquired by the other sciences but is had by nature, the *habitus* of conclusions deduced from the first principles is acquired. So too in this doctrine the *habitus* of faith [*habitus fidei*], which is like the *habitus* of principles, is not acquired, but the *habitus* of those things which are deduced from them and which contribute to its defense [is].[45]

Just as the first principles of a science are indemonstrable, so too are the revealed articles of the Christian faith. Just as the first principles of a science are known by *intellectus*, a natural *habitus*, so too are the articles of faith known by the *habitus fidei*, a supernaturally infused *habitus* of faith. Neither of these *habitus* is acquired; both are given, one by nature, and the other by God.

Aquinas responds to the objection that no science can have dubitable first principles by (2) drawing a third analogy in *Summa theologiae* (c. 1265–1273). His response exploits Aristotle's theory of subalternation in *Posterior Analytics*. He first points out that Aristotle distinguishes between sciences "which proceed from principles known by the natural light of the intellect, such as arithmetic and geometry and the like," and sciences "which proceed from principles known by the light of a higher science: thus optics proceeds from principles established by geometry, and music from principles established by arithmetic." "So it is," he concludes, "that sacred doctrine is a science because it proceeds from principles made known by the light of a higher science, namely, the science of God and the blessed," for "just as music accepts on authority the principles taught by the arithmetician, so sacred science accepts the principles revealed by God."[46] For God, these principles are not dubitable. The science of God and the blessed is to sacred science as geometry is to optics. The principles of sacred doctrine are, therefore, "reducible to the knowledge of a higher science,"[47] in which they are not dubitable. Sacred science need not demonstrate these principles any more than optics need demonstrate the

principles of geometry. Since subalternation is permitted in Aristotelian science, the fact that sacred doctrine is a subalternate science in no way undermines its scientific credentials. On the contrary, it bolsters them.

As for the objection that there is no science of the individual, Aquinas easily disposes of it by (3) arguing that "individual facts are not treated in sacred doctrine because it is concerned with them principally," but rather because they serve as "examples to be followed in our lives (as in the moral sciences)" and because they "establish the authority of these men through whom divine revelation, on which the sacred scripture or doctrine is based, has come down to us."[48]

Based on these arguments, Aquinas effectively completed the transformation of theology in the thirteenth century from a discipline (*disciplina*) based almost exclusively on scriptural hermeneutics into a *bona fide* Aristotelian demonstrative science.[49] But does he have everything he needs to show that sacred doctrine is *one* science, and not many? Hardly. To establish the *unity* of sacred doctrine, Aquinas must introduce additional arguments.

As we have seen, according to Aristotle, "a science is one if it is of one genus."[50] Sacred doctrine, however, deals with more than one genus.[51] It deals with God, angels, human beings, corporeal creatures, and human morality, which belong to "separate philosophical sciences."[52] In short, sacred doctrine cannot be one science because it violates Aristotle's ban on genus-crossing. Aquinas responds to this objection by introducing a fourth analogy between the unity of a faculty and the unity of a *habitus* and by distinguishing between the formal object of a science and its many material objects:

> Sacred doctrine is one science. The unity of a power or *habitus* is to be gauged by its object, not indeed, in its material aspect [*materialiter*], but as regards the formality under which it is an object [*secundum rationem formalem obiecti*]. For example, man, ass, stone, agree in the one formality of being colored; and color is the formal object of sight. Therefore, because Sacred Scripture (as we have said) considers some things under the formality of being divinely revealed [*secundum quod sunt divinitus revelata*], all things which have been divinely revealed have in common the formality of the object of this science [*communicant in una ratione formali obiecti*]. Hence, they are included under sacred doctrine as under one science.[53]

Aquinas's first premise is that the unity of a *habitus* does not differ from the unity of a faculty. In both cases, unity is based on the object of the faculty

or *habitus*, not "in its material aspect, but as regards the formality under which it is an object." Aquinas illustrates this premise by arguing that the unity of the faculty of sight consists in the fact that its objects, however materially different they may be from one another, nevertheless share the "formality of being colored." These materially different objects are different in genus and species, but so long as they share in the formality of color, they are objects of sight. Since sacred doctrine is a *habitus*, Aquinas must now establish that the objects of sacred doctrine, however materially different they may be from one another, also share a formality. He argues that they share the "formality of being divinely revealed." While he does not mention it here, this formality is the first object of the supernaturally infused *habitus* of faith discussed above.[54]

To sum up. Aquinas's argument that sacred doctrine is a science is based on four analogies: two logical and two psychological. (1) He employs a logical analogy between the indemonstrability of the first principles of an Aristotelian demonstrative science and the articles of the Christian faith; (2) he exploits Aristotle's mechanism of subalternation in order to establish a second logical analogy, in which sacred doctrine is subalternate to the science of God and the blessed as optics and harmonics are subalternate to geometry and arithmetic, respectively; (3) he employs a psychological analogy between the natural *habitus* of first principles (*intellectus*), and supernaturally infused *habitus* of faith (*habitus fidei*); (4) he employs a second psychological analogy between the unity of a faculty and the unity of scientific *habitus* (see Figure 1.1).[55]

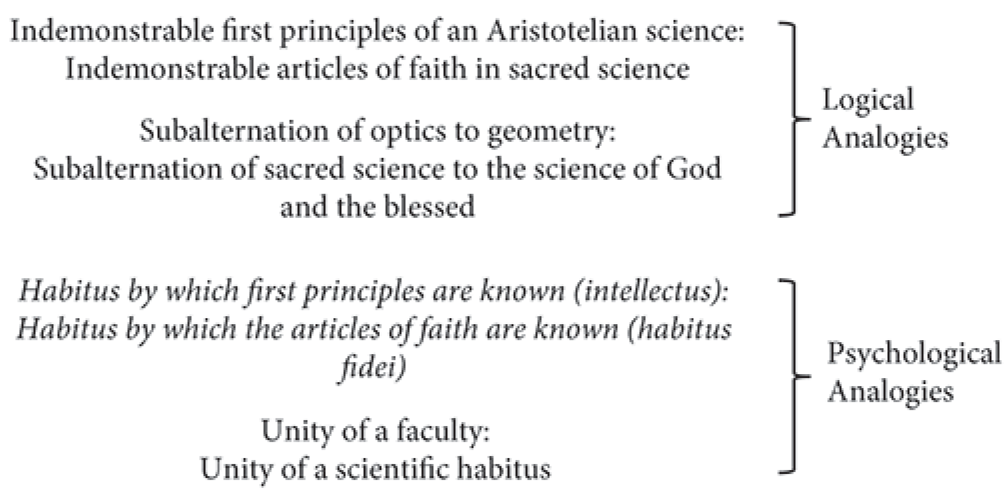


Figure 1.1 The Structure of Sacred Doctrine in Aquinas

It would be difficult to overestimate the impact Aquinas's strategy for establishing that sacred doctrine is a science would have on the subsequent history of the theory of science in the Latin West. For those who defended him no less than those who proposed alternatives, Aquinas established the basic terms in which debates about the unity of science would be conducted well into the seventeenth century. By the time Suárez entered the fray in *Disputationes metaphysicae* (1597), the scholastics had long been embroiled in debates about the unity of scientific *habitus*.

1.4 The Ontology of Scientific *Habitus*: Simple Quality or Complex Order?

1.4.1 Aquinas's Gradient Ontology of Scientific *Habitus*

Thus far, I have focused on how Aquinas exploits Aristotle's theory of science in order to establish that sacred doctrine is a science. However, Aquinas's arguments about the unity of sacred doctrine in *Summa theologiae* I, q. 1 rest on a deeper ontology of *habitus* developed in other parts of the treatise. According to Aquinas, *habitus* are simple (indivisible) qualities; they have no parts, and so cannot be divided into parts. This creates a problem, since *habitus* can also increase. Scientific *habitus* increase as the intellect learns more principles and demonstrations over time by many different acts. How can a *habitus* be both simple *and* acquired over time by many different acts? Is it not rather the case that each act engenders a really distinct *habitus* whose existence does not depend on the *habitus* engendered by previous acts? Aquinas does not address this problem in *Summa theologiae* I, q. 1, but it is important, since the debate about the unity of science after Aquinas really turns on *how best to ontologically explain the increase of a scientific* habitus. In different ways, Scotus, Ockham, and Suárez argue against Aquinas that distinct principles and demonstrations are learned by distinct acts, and that these acts produce really distinct scientific *habitus*, not one. In this section, I reconstruct each of their positions, beginning with Aquinas.

In *Summa theologiae* I–II, q. 54, art. 4, Aquinas argues that all *habitus*—intellectual and ethical—are ontologically simple (indivisible). They have no parts, and so cannot be divided into parts. True to the literary form of the *Summa*, Aquinas begins by enumerating three objections to his argument.[56] The first objection runs as follows:

> It would seem that one *habitus* is made up of many *habitus*. For whatever is engendered, not at once, but little by little, seems to be made up of several parts. But a *habitus* is engendered, not at once, but little by little out of several acts […]. Therefore one *habitus* is made up of several.[57]

This objection hinges on an inference from the major premise that "whatever is engendered, not at once, but little by little, seems to be made up of several parts" and the minor premise that *habitus* are produced "not at once, but little by little," to the conclusion that they are "made up of several" *habitus*. The objection is based on *Summa theologiae* I–II, q. 51, art. 3, where Aquinas argues that, with some exceptions, *habitus* are caused, not at once, but by several repeated acts that are identical in species.[58] In *Summa theologiae* I–II, q. 54, art. 4, Aquinas inserts his earlier argument into the objection and adds the major premise that whatever is produced little by little seems to be made up of several parts. Since, according to the objection, these parts are themselves *habitus*, a *habitus* is a complex unity of many *habitus*, not a simple quality.

The second objection builds on the first by citing Cicero's claim that there are "many parts of fortitude, temperance, and other virtues."[59] Presumably, each part of these virtues disposes the soul to perform specifically different acts. This would certainly support the claim that *habitus* are made up of several parts.

The third objection focuses specifically on the case of scientific *habitus*: "Further, one conclusion suffices both for an act and for a *habitus* of scientific knowledge. But many conclusions belong to but one science, to geometry, for instance, or to arithmetic. Therefore one *habitus* is made up of many."[60] The third objection concludes as the first does, but from the premise that one need learn only one conclusion (e.g., the Pythagorean theorem) in order to acquire the relevant *habitus*. Once one has learned the relevant conclusion, one has acquired a *habitus* that is stable and difficult to destroy. Since one already has one *habitus* from one conclusion alone, any other conclusions learned (e.g., that similar triangles are proportional to one

another) must be a distinct *habitus*. In *Summa theologiae* I–II, q. 51, Aquinas argues that even one conclusion produces a scientific *habitus* in the soul. The third objection builds on his argument by adding that in learning many conclusions, one acquires many *habitus*, not only one.

Aquinas's response to these three objections rests principally on Aristotle's definition of *habitus* as a quality in *Categories* 8. His major premise is that all *habitus* are qualities, and his minor premise is that all qualities are ontologically simple: "A *habitus*, since it is a quality, is a simple form. But nothing simple is made up of many [parts]. Therefore one *habitus* is not made up of many." He then reintroduces the analogy between faculties and *habitus* employed in *Summa theologiae* I, q. 1, where he employed it in order to establish the unity of sacred doctrine (see Section 1.3 above):

> Hence, just as a power, while it is one, extends to many things insofar as they have something in common (i.e., some general objective aspect [*ratione obiecti*]), so also a *habitus* extends to many things insofar as they are related to one thing, for instance, to some specific objective aspect, or to one nature, or to one principle, as was stated above. If, then, we consider a *habitus* from the standpoint of the things to which it extends, we shall find a certain multiplicity in it. But since this multiplicity is directed to something one, on which the *habitus* is chiefly intent, hence it is that a *habitus* is a simple quality, not composed of several *habitus*, even though it extends to many things. For a *habitus* does not extend to many things save in relation to something one, whence it derives its unity.[61]

Just as one faculty extends to many things without compromising its simplicity, so too one *habitus* extends to many things without compromising its simplicity. The simplicity of a *habitus* is not in the least bit affected by the fact that it extends to many things, so long as it extends to them under one formality.

After reintroducing his analogy between faculties and *habitus*, Aquinas responds to each objection. Each response deploys what I term Aquinas's "gradient ontology of *habitus*," which distinguishes between lower and higher degrees or grades of one and the same simple *habitus* and is based on the ontological difference between *degrees*, on the one hand, and *parts*, on the other. *Habitus* are qualities, and as Aristotle argues in *Categories* 8, all qualities "admit of a more and a less; for one thing is called more pale or less pale than another, and more just than another. Moreover, it itself sustains increase (for what is pale can still become paler) […]."[62] Deploying his gradient ontology of *habitus*, Aquinas responds to the first

objection by arguing that the increase of a *habitus* never introduces parts into the *habitus*: *habitus* are engendered little by little, not because they have parts, but rather because they have degrees. Degrees are not parts. "It [the *habitus*] begins by being imperfectly in the subject, and is gradually perfected."[63] The perfection of a *habitus* does not render it complex. In his response to the second objection, Aquinas simply denies that there are many parts of virtue, and instead argues that, in the case of prudence, "memory, understanding and foresight, as also caution and docility" are "secondary virtues to a principal virtue," i.e., are distinct virtues subordinate to prudence, not parts of prudence.[64] To the third objection, Aquinas responds that the ability to demonstrate even one conclusion in a science is indeed a scientific *habitus*, but it is not the perfect *habitus* of the relevant science. It is imperfect, and as it extends to more conclusions, "the *habitus* which was in him previously is made more perfect, insofar as it extends to more things."[65] However many conclusions a *habitus* may extend to, it is not fractured into parts, but only perfected in degree. Aquinas's gradient ontology of *habitus* enables him to demonstrate that there is no necessary inference from the premises that *habitus* are produced little by little or that they can be extended to new objects to the conclusion that *habitus* are complex. Where a mereological ontology of *habitus* sees only the addition of parts, Aquinas's gradient ontology sees the intensification of degrees.

Given Aquinas's insistence that *habitus* are simple qualities, it is surprising that he sometimes distinguishes between *two* senses of *habitus*: (1) that whereby something is held, which is the "proper and essential" sense, and (2) that which is held by a *habitus*, which is the "secondary" sense.[66] The intellectual virtue or *habitus* whereby principles are known (*intellectus*) is a *habitus* in the primary and essential sense, but the principles held by this *habitus*—which are numerically many—are themselves *habitus* in the secondary sense. There are two main reasons for why Aquinas distinguishes between these two senses of *habitus*. (1) Principles and demonstrations "are sometimes considered by reason actually, while sometimes they are in reason only habitually."[67] Aquinas introduces the secondary sense of *habitus*, not in order to establish the unity of a science, but rather in order to describe the being that principles and demonstrations have when they exist non-occurrently in the intellect. Nevertheless, the secondary sense of *habitus* does have an important

implication. Since in any science there are many principles and demonstrations, and since these principles and demonstrations are not always activated in the intellect, in the secondary sense of *habitus* there are equally as many scientific *habitus* as there are intelligible species that compose the principles and demonstrations of a science; a science is the "ordered aggregate [*ordinata aggregatio*]" of these *habitus*.[68] But why are these *habitus* only *habitus* in the secondary sense, and not in the primary and essential sense? Because (2) Aquinas denies any identity between the *habitus* whereby something is held and that which is held by a *habitus*. As he puts it in *Summa theologiae* I–II, q. 94, art. 1: "Now that which a man does is not the same as that whereby he does it, for he makes a becoming speech by the *habitus* of grammar."[69] The *habitus* of grammar is that whereby speech acts are produced, but it is not identical to any of these speech acts, nor does it seem to resemble them. Similarly, he continues, "in speculative matters, the indemonstrable principles are not the *habitus* itself whereby we hold these principles; they are rather the principles of which we possess the *habitus*."[70] The *habitus* whereby principles are known is an ability to recognize principles once their terms have been appropriately cognized, not the principles themselves. I mention Aquinas's distinction between these two senses of *habitus* because, as we will see in Sections 1.4.2–1.4.4, after Aquinas the "secondary" sense of scientific *habitus* as an ordered aggregate of propositions progressively replaces its "proper and essential" sense as a simple quality.[71] Aquinas's gradient ontology of *habitus* is replaced by other ontologies in which sciences are complex accidental unities composed of many really distinct *habitus*. Since Scotus, Ockham, and Suárez argue that sciences are made up of many really distinct *habitus*, they are obliged—as Aquinas is not—to explain how it can be that many really distinct scientific *habitus* nevertheless constitute *one* science.

1.4.2 Scotus's Virtual Containment Ontology of Scientific *Habitus*

I concluded Section 1.4.1 by noting that after Aquinas, the "secondary" sense of *habitus* as an ordered aggregate of propositions progressively replaces its "proper and essential" sense as a simple quality in the soul. Subsequent scholastics defined science as a complex accidental unity of

many really distinct scientific *habitus*. Why? Because they came to regard Aquinas's gradient ontology of scientific *habitus* as untenable. This is especially clear in Duns Scotus (1265–1308). In *Questions on the Metaphysics of Aristotle*, lib. 6, q. 1, Scotus aims to completely overturn Aquinas's gradient ontology of scientific *habitus* via a debate with Henry of Ghent (c. 1217–1293). While Henry of Ghent is no Thomist, he does subscribe to a gradient ontology of scientific *habitus*. Henry maintains that scientific *habitus* are simple qualities in the soul. He also maintains that the unity of a science consists in the formality under which everything in the science is considered by the intellect (a *modus considerandi*). When the intellect learns more principles and demonstrations, the scientific *habitus* it already possesses acquires no additional parts; it only becomes more perfect.[72] Scotus mobilizes a number of arguments in order to demonstrate that the gradient ontology of scientific *habitus* is simply untenable. This leads him to articulate an alternative ontology of scientific *habitus*, which I term his "virtual containment ontology of scientific *habitus*," according to which the unity of a science is based on its primary object. The primary object of a science is a real object, not a formality, and it virtually contains (i.e., has the virtue or power to cause in the intellect) the many really distinct *habitus* that compose the science.[73] Scotus can have his cake and eat it too: the unity of a science can be secured directly in the primary object of the science, such that the many really distinct scientific *habitus* this object causes in the intellect in no way undermine the unity of the science.

Scotus produces many arguments against the gradient ontology of scientific *habitus* shared by Aquinas and Henry of Ghent:[74]

(1) *Specifically distinct acts produce specifically distinct* habitus. Scotus makes this argument via an analogy between ethical and scientific *habitus*. The analogy is permitted because there is no ontological difference between ethical and scientific *habitus* qua *habitus*. Individual ethical *habitus* can only be produced by specifically identical acts, not by specifically distinct acts. (These acts may differ numerically, since one act of temperance may be performed today, another act of temperance tomorrow, and so on. But these acts do not differ specifically, since they are all acts of temperance.) Only temperate acts produce the *habitus* of temperance; they do not produce other ethical *habitus* (e.g., justice, liberality, etc.). Just as

specifically distinct acts cannot produce one ethical *habitus*, Scotus argues, so too specifically distinct acts cannot produce one scientific *habitus*. The principles and demonstrations of a science are learned by many specifically distinct acts. These acts differ from one another because the principles and demonstrations themselves differ from one another. "Therefore to the extent [that] there is a difference of acts, to that degree there is a difference of *habitus*."[75] Specifically distinct acts "generate specifically diverse *habitus* […] because according to their specifically proper notions they impress natural likenesses on potencies that differ in a similar way as they differ […]."[76] On the gradient ontology of scientific *habitus*, any number of acts, however specifically distinct they may be from one another, never produce more than one simple scientific *habitus*, so long as they are directed to the same formal object. But if this were so, Scotus argues, "someone could claim that there is only one moral virtue which is generated by any act whatsoever and then is augmented, not, however, by similar acts but by different sorts of acts as well, just as here it is claimed that this is so in regard to the *habitus* of science."[77] If many specifically distinct acts can produce one scientific *habitus*, then many specifically distinct acts can produce one ethical *habitus*. An ethical *habitus*, however, can only be produced by specifically identical acts. Given the analogy between ethical and scientific *habitus*, it follows that specifically distinct acts produce distinct scientific *habitus*, not one simple scientific *habitus*. In Aquinas and Henry of Ghent, the extension of a scientific *habitus* in no way compromises its simplicity, while in Scotus, learning a new principle or demonstration requires the production of a specifically and really distinct scientific *habitus* in the intellect. Extension requires real addition.

(2) *One and the same scientific* habitus *cannot extend to both known and unknown propositions.* Suppose I believe that a proposition is true, when it is not. False beliefs, according to Scotus, are *un*scientific *habitus*, which he terms "dispositional ignorance" or error, as opposed to merely "negative ignorance" (the mere absence of knowledge about whether a proposition is true or false).[78] As Scotus understands it, since the gradient ontology of scientific *habitus* is committed to the existence of one simple scientific *habitus*, this *habitus* must extend to every principle and demonstration in a science. This *habitus* even extends to those principles and demonstrations not yet known by the intellect, since it remains one and the same *habitus* once the intellect comes to learn them. On the supposition that I am

dispositionally ignorant or in error about some proposition, then on the gradient ontology I must have two opposed *habitus* vis-à-vis one and the same proposition, and that is impossible. It is "impossible […] to have coexisting in the same intellect opposite *habitus* with respect to the same proposition. […] Therefore, with respect to an unknown conclusion, we do not have a *habitus* which is its science."[79] Any scientific *habitus* I already have cannot extend to any proposition I am dispositionally ignorant of. This is confirmed by the fact that "when an unknown conclusion is learned, the *habitus* opposed to the ignorance that preceded it is induced."[80] Further confirmation comes from the fact that the scientific *habitus* I already have only facilitates the performance of so many acts and no more. "From that *habitus* no act – no matter how imperfect – could be had with respect to" an unknown conclusion.[81] The only way to learn an unknown conclusion is by means of a specifically distinct act, which, in turn, produces a specifically and really distinct *habitus* in the intellect, according to argument (1) discussed above.

(3) *In learning a new conclusion, one can forget something that was known before*. *No simple* habitus, *however, can be simultaneously perfected and corrupted.* Aquinas argues that frequency of consideration perfects a scientific *habitus*, but he is vulnerable to Scotus's argument that, by frequently considering one conclusion instead of another, I may perfect my knowledge of one conclusion and forget my knowledge of another. "It is possible that in learning new conclusions, one either forgets something that was known before or knows it less perfectly than before the new conclusions were known, when what was previously known was more frequently considered."[82] This is a very common experience, "for one who is learning the second book of geometry, while he is occupied with what is there, he neglects to consider the conclusions of the first book."[83] This, Scotus concludes, "is impossible if it is the same *habitus*,"[84] since no one simple *habitus* can be simultaneously perfected and corrupted. Simple *habitus* have no parts, so it is not possible that one part is perfected, while another part is corrupted. It is either the whole *habitus* that is perfected, or the whole *habitus* that is corrupted, but one and the same simple *habitus* cannot be both perfected and corrupted at the same time. Therefore, sciences are not simple *habitus*.

(4) *The more perfect cannot be caused by the less perfect*. Aquinas and Henry of Ghent regard the knowledge of a principle as a first, minimal degree of a scientific *habitus*, which acquires more perfection when a conclusion is deduced from the principle. However, Scotus argues, the conclusion is formally caused by the principle, and the effect can never exceed the cause in perfection. Consequently, a less perfect degree of a *habitus* cannot cause a more perfect degree. As we will see in more detail below, according to Scotus, the principle virtually contains the conclusion, and so, contra Aquinas and Henry of Ghent, the principle must be more perfect than the conclusion, which does not virtually contain the principle.[85]

(5) *There is no analogy between the intensification of a simple quality and the perfection of a scientific* habitus. As we have seen in Section 1.4.1, Aristotle's definition of *habitus* as a species of quality motivates Aquinas's and Henry's gradient ontology of scientific *habitus*, in which scientific *habitus* are analogized to simple qualities such as color. For Scotus, the analogy between simple qualities like color and scientific *habitus* conceals profound ontological differences: "[In] whatever quality there are grades of difference, according to more and less," and these grades of difference "have an essential order, such that A includes B virtually, and C, A and so on to the highest degree, as is evident in the case of whiteness."[86] However, no such ordered grades "are present with respect to the diverse knowables." I can learn one conclusion and another conclusion in geometry, and "each of them can be known indifferently before the others."[87] Since these conclusions do not virtually contain or have the power to cause one another, they do not have the same relation to one another as the grades of a simple quality do. When it comes to diverse knowables, there is no gradient order analogous to the order found in simple qualities. Thus, Scotus concludes, "those things that regard diverse knowables are not degrees of the same *habitus*, but are rather different *habitus*."[88]

If one accepts Scotus's criticisms of Aquinas and Henry of Ghent, then the gradient ontology of scientific *habitus* collapses entirely. As I indicated above, the problems associated with Aquinas's and Henry of Ghent's ontology of scientific *habitus* lead Scotus to articulate an alternative ontology of scientific *habitus*, which I have termed his "virtual containment ontology of scientific *habitus*," according to which the unity of a science is based on its primary object. In Scotus's words, the primary object of a science "contains virtually in itself all the truths of the *habitus* of the

science."[89] What does this mean? It means that the primary object of a science has the virtue or power to cause every principle and demonstration of the science in the human intellect. How does it do that? For Scotus, the primary object of a science is and must be a *real* object, not a *formal* object, and as such it has *causal power*. For example, in the case of what Scotus terms "theology" (*theologia*),[90] the primary object is God. When the intellect cognizes God, it acquires an intelligible species of God, which is a simple *habitus*. This intelligible species is the logical subject of the science (the entity, God, is the object of the science). Scotus terms this simple *habitus* the "common" scientific *habitus*, and he argues that since subjects contain their *per se* properties, the subject of a science contains the principles of the science. Furthermore, the principles contain the conclusions and, therefore, the entire science. Consequently, the subject has the power to cause in the human intellect the many really distinct scientific *habitus* (principles and conclusions) that ultimately compose the science. He terms the latter the "proper" scientific *habitus*, and he describes the causal process as follows:

> [T]here can be one *habitus* with respect to many propositions [*potest esse unus habitus respectu multorum complexorum*]. For since conclusions are virtually contained in principles, and the principles are virtually contained in their subject – for the subject includes the predicate in the first principles […] – it follows that when the simple notion of the subject [is] known quidditatively, the principles and conclusions are virtually included in such a subject, and thus the complete knowledge which is apt by nature to be known of such a subject is had.[91]

> [F]ollowing this order [from conclusion to principle, and from principle to subject] one eventually arrives at some simple subject, which is the subject of a principle or principles. From the notion of this subject all that pertains to the essence is known and the notion of the subject is not known from anything else. This therefore is rightly called the first or primary subject; it contains virtually in itself [*primo continet in se virtualiter*] the knowledge that pertains to the science.[92]

> [There] can be a twofold *habitus*. One is proper, which formally inclines to thinking about it [i.e., the proposition] as a natural likeness left behind by its consideration. The other is the common [*habitus*] which virtually inclines one to think about it [i.e., the proposition] by inclining one formally to think about another, in which such a proposition is virtually contained.[93]

Scotus's strategy here is ingenious. The argument proceeds from the uncontroversial premise that the conclusions of a demonstration can only be inferred from the principles of the demonstration. Because they are immediate, the principles are not inferred from any prior principles, but

rather are known by their terms alone (*cognoscuntur ex terminis*), which are either subjects or predicates (*per se* properties). The predicate in a first principle is known by its subject (*ex ratione subjecti*). The subject, however, is not known by any other subject. The "simple notion of the subject" is a simple *habitus*, and it virtually contains all "the knowledge that pertains to the science." In short, the conclusions are virtually contained in the principles, and the principles are virtually contained in the subject, such that the simple scientific *habitus* that disposes the intellect to consider the subject (common scientific *habitus*) virtually contains every principle and demonstration (really distinct proper scientific *habitus*) in the science. In theology, God virtually contains "the immediate propositions [i.e., the principles], because the subject of those propositions [i.e., God] contains the predicate, and thus it [the subject, God] contains the evidence for the whole proposition."[94] God is the subject of the principles of theology, and he ontologically contains whatever properties are predicated of him in these principles. The knowledge the human intellect has of God's essence is a common scientific *habitus* and contains the proper scientific *habitus* (principles and conclusions) that compose theology as a science.

Thus, to demonstrate that God has property *x* requires demonstrating that property *x* is caused by God's essence. Since God's essence is simple, Scotus famously argues that these properties are not really distinct or separable from one another in God, but rather that they are only formally distinct, i.e., distinct in definition prior to any operation of the intellect without being separable either from one another or from God.[95] Scotus further argues that these formally distinct properties *bear definite relations of priority and posteriority to one another*. For example, there is a definite ontological and epistemic order between God's essence and his attributes: God's essence is ontologically prior to God's intellect, and God's intellect is ontologically prior to God's knowledge, since the intellect is ontologically prior to any one of its acts. This "essential order" makes theology possible as a science; God's attributes can be demonstrated by reference to his essence according to this order.[96] Since, moreover, the ontological order that obtains between formally distinct properties in a substance is not only identical to, but also *produces* the epistemic order in which the intellect appropriately cognizes the substance, knowledge of God's essence produces the common scientific *habitus* in which all proper scientific *habitus* are

virtually contained. "And thus with respect to all of these [complexes] there is one virtual *habitus*."[97]

Why does Scotus insist that the primary object of a science must be *real*, and not merely formal, as Aquinas and Henry maintained? For Aquinas and Henry, the object of theology is a *formal* object: the formality of revelation. By considering all things under the formality of revelation, I ensure that, however ontologically diverse the many material objects of sacred doctrine may be, whatever predicates these objects have are predicated of them qua revealed (and not, say, qua in motion or qua being, which are the formal objects of physics and metaphysics, respectively). For Scotus, common predication is insufficient: there must be a *causal* relation between the primary object of a science and its principles and demonstrations;[98] what is common to the objects of theology must be common "according to virtuality, […] not by reason of common predication."[99] Scotus interprets Aristotle's second mode of *per se* predication in *Posterior Analytics* as requiring precisely such a causal relation.[100] In the second mode of *per se* predication, the subject appears in the definition of the predicate, as "straight belongs to line and so does curved." "Straight" and "curved" are *per se* predicates of "line," since it appears in their definition.[101] The predicates are virtually contained in the subject because they are (formally) *caused* by the subject. For Scotus, mere Thomist formalities fail to secure the unity of a science because *there can be no causal relation between such a formality and the principles and conclusions of a science*. Only "some one concept on which the resolution of all the principles and conclusions ultimately rests" virtually contains the science. "It is clear," Scotus continues, "that what is simply the first subject of the science is such [an object] that contains virtually [all that is knowable in the science] […] and not something common to all things considered in that science."[102]

Thus, Scotus does *not* dispense with the thesis that there is one simple scientific *habitus* per science. Insofar as sciences are composed of many really distinct proper scientific *habitus*, there are as many sciences as there are principles and conclusions: it "can be conceded that there are as many sciences as there are knowables [*quot scibilia, tot scientiae*]."[103] However, insofar as there is one common scientific *habitus* that is caused by the primary object of a science and that virtually contains the many really distinct proper scientific *habitus* that compose the science, there is only one,

simple scientific *habitus* per science. A science is one because the many really distinct proper scientific *habitus* are virtually contained in a simple common scientific *habitus,* which is itself virtually contained in the primary object of the science. The simple common scientific *habitus*, moreover, does not contain the many really distinct proper scientific *habitus* as parts, but only in the sense that it has the virtue or power to cause them in the intellect when the intellect activates it. Whatever their number, these proper scientific *habitus* in no way undermine the unity of the science. Thus, between those who maintain that the unity of a science consists in one simple scientific *habitus* alone (Aquinas; Henry of Ghent; Cajetan) and those who maintain that the unity of a science consists in many really distinct scientific *habitus* alone (Gonsalvus Hispanus?),[104] Scotus occupies a comfortable and rather attractive intermediate position.

Having overturned Aquinas's gradient ontology of scientific *habitus* and relocated the source of scientific unity in the primary object of a science, Scotus has all the materials he needs to strike at what is arguably the heart of Aquinas's strategy for establishing the unity of a science: the analogy between the unity of a faculty and the unity of a scientific *habitus* asserted in *Summa theologiae* I, q. 1 (see Sections 1.3 and 1.4.1 above). Scotus argues that the primary object of a faculty (e.g., sight), which is a formality (e.g., the formality of being colored), only *moves* the faculty, but does not "contain virtually in itself the perfection of that power […]," while the primary object of a science *does* contain the perfection of the *habitus* it produces in the intellect. Aquinas's analogy becomes a disanalogy: while the formal object of a faculty may move the faculty, the formal object does not contain the perfection of the faculty, while the primary object of a science does contain the perfection of the *habitus* it produces in the intellect. Aquinas's gradient ontology of scientific *habitus* is left with very little to stand on.

To sum up. Scotus effectively overturns Aquinas's strategy for establishing the unity of a science. Whereas Aquinas argues that the unity of a science consists in a simple scientific *habitus* that may be extended over time, Scotus argues that there are in any science as many proper scientific *habitus* as there are principles and demonstrations. These proper scientific *habitus* are nevertheless virtually contained in one simple, common scientific *habitus*, which is caused by and virtually contained in the primary object of the science. Scotus's concept of (proper) scientific

habitus more closely corresponds to Aquinas's secondary sense of *habitus* (see Section 1.4.1 above): that which is held by the *habitus* (the intelligible species or proposition). For Scotus, every intelligible species, principle, and conclusion individuates a distinct *habitus* in the intellect. Between Aquinas and Scotus, the criterion whereby scientific *habitus* are individuated changed. For Aquinas, the extension of a scientific *habitus* to more principles and conclusions does not compromise its simplicity and does not require the production of yet another *habitus*. For Scotus, by contrast, when the intellect learns another principle or demonstration, this produces a distinct (proper) scientific *habitus* in the intellect. Scotus's criterion, according to which specifically distinct acts terminate in specifically distinct *habitus*, directly contradicts the principle underlying Aquinas's gradient ontology of scientific *habitus*, and becomes even more radical in Ockham.

1.4.3 Ockham's Aggregate Ontology of Scientific *Habitus*

As we have seen, Scotus regards the relation between the primary object of a science and scientific *habitus* as a causal relation. The primary object of a science virtually contains and, therefore, has the virtue or power to cause the *habitus* (both common and proper) that compose the relevant science in the intellect. Ockham accepts Scotus's thesis that sciences are composed of many *habitus*, but he denies that there is one primary object or subject of a science that virtually contains all the *habitus* of the science. The basic principle behind Ockham's rejection of Scotus's virtual containment ontology of scientific *habitus* is that subjects virtually contain neither their properties nor knowledge of their properties.[105] Scotus argues that God virtually contains his formally distinct properties and has the virtue or power to cause knowledge of these properties in the human intellect according to a definite order. For Ockham, however, substances do not contain any formally distinct properties. The reasons behind Ockham's rejection of the formal distinction are complex, but his basic argument is that the formal distinction leaves no room for real distinctions between things. If *x* is F and *y* is not F, it will always be possible to argue that *x* and *y* are only formally distinct (or distinct in definition alone), and not really

distinct.[106] In Ockham's ontology, there are only individuals, and the only ontological distinctions there are in individuals are real distinctions between substances and their properties. One consequence of Ockham's rejection of the formal distinction is that where Scotus could argue that the formally distinct properties of a substance bear definite relations of priority and posteriority to one another, there is no such ontological order between formally distinct properties in Ockham's ontology because there simply are no such properties in his ontology. Since, moreover, Scotus maintains that the ontological order that obtains between formally distinct properties in a substance is not only identical to, but also produces the epistemic order in which the intellect appropriately cognizes the substance and its properties, Ockham's denial of the former, ontological order entails his denial of the latter, epistemic order. This is why Ockham *denies that theology is a demonstrative science*: what Scotus takes to be God's formally distinct properties cannot be demonstrated on the basis of God's essence, since there are no distinctions whatever in God's essence, which is absolutely simple.[107] *All theological propositions are self-evident.*

In the *Ordinatio*, Ockham produces many arguments against Scotus's virtual containment ontology of scientific *habitus*. These arguments separately establish two principal theses: "That it does not belong to a subject to virtually contain knowledge of its properties [*passiones*]," and that "it does not belong to a subject to virtually contain its properties."[108] The first thesis is epistemic, and does not depend on denying that subjects virtually contain their properties.

On behalf of the first thesis, Ockham makes the following arguments:

(1) *Ontological (virtual) containment does not entail epistemic containment.* Ockham argues that "an efficient cause does not contain its effect less virtually or less perfectly than a subject contains its property."[109] The analogy between cause and effect, on the one hand, and subject and property, on the other, is established via the thesis that causes do not contain their effects less perfectly than subjects contain their properties. The analogy permits Ockham to draw inferences about the epistemic containment of a property in its subject from inferences about the epistemic containment of an effect in its cause. Thus, because knowledge of a cause does not contain or have the power to produce knowledge of its effects, knowledge of a subject does not contain or have the power to produce

knowledge of its properties. Causes and effects have different quiddities, and Ockham argues that even Scotus acknowledges that "knowledge of distinct quiddities requires distinct grounds of knowing [*rationes cognoscendi*]; therefore, distinct knowledge of one quiddity without a ground for knowing another is not sufficient to produce knowledge of another."[110] Ockham concludes that knowledge of the subject of a science does not virtually contain or have the power to produce knowledge of its properties. This may seem question-begging. For Scotus, the formally distinct properties of a substance do *not* require distinct grounds of knowing, since, as we have seen in Section 1.4.2, they have an essential order among themselves, such that knowledge of the subject of these properties yields knowledge of the properties according to this order. Ockham, however, denies the formal distinction. Any ontological relation between a substance and its properties must, therefore, be a real distinction, in which case Ockham's argument against Scotus succeeds.

Ockham addresses two objections to his argument. According to the first objection, "distinct quiddities are required when one [quiddity] is not contained in the other."[111] Ockham responds that if this were so, "for knowing all the inferior quiddities, a distinct knowledge of the sun itself would suffice, because the sun contains virtually all these quiddities."[112] The sun, Ockham argues à la Scotus, virtually contains its effects, and yet here too, distinct knowledge of the sun does not by itself contain knowledge of these effects. If the objector retorts that the sun does not contain knowledge of these quiddities because the sun is only their partial, not total, cause, Ockham responds that in this case yet other causes are required to produce the knowledge of the relevant effects. He then reintroduces the analogy between causes and subjects: "Since, therefore, a subject is, in the same way, not always the total cause of the property, a distinct knowledge of the subject does not suffice for having a distinct knowledge of the property."[113]

The second objection challenges Ockham's analogy between causes and effects, on the one hand, and subjects and properties, on the other, and rests on what he regards as a false premise: viz., that the dependence of an effect on its cause is less perfect than the dependence of a subject on its properties. "Sometimes," he responds, "a property does not depend on its subject save as an accident depends on its subject; but that is more

imperfect than the dependence of an effect on its efficient cause."[114] Efficient causes *produce* their effects, while subjects do not always produce their accidents. The objection fails.

(2) *Knowledge of the subject never contains even the most imperfect knowledge of its properties.* According to Scotus, "what contains perfect knowledge of something, contains imperfect knowledge of it."[115] But, Ockham argues, knowledge of a subject does not contain even the most imperfect knowledge of its properties, "because it does not contain the nominal definition [*non continet quid nominis ipsius*],"[116] let alone the real definition (*quid rei*). Therefore, he concludes, knowledge of the subject does not virtually contain knowledge of its properties.

(3) *If anything virtually contains the conclusion, it is the middle term of a demonstration, not the subject.* This is clear because Aristotle regards the middle term and the cause of the conclusion as one and the same, and virtual containment is a species of cause.[117] Subjects, however, are not middle terms. Therefore, the subject does not virtually contain the conclusion. This argument may seem contrived, since Scotus very clearly insists that the *definition of the subject is the middle term in any demonstration*. For example:

God has the most perfect nature.

The most perfect nature has the most perfect intellect.

Therefore, God has the most perfect intellect.[118]

Why, then, does Ockham make this argument? Because the middle term is not always contained in the definition of its subject. It is possible to have knowledge of the subject and yet not know its properties due to lack of knowledge of the *extrinsic causes of these properties*. For example:

An eclipse occurs when the moon is interposed between the earth and the sun.

The moon is interposed between the earth and the sun.

Therefore, there is an eclipse.

I can have knowledge of the moon without knowing that it is the cause of the eclipse because knowledge of the moon does not virtually contain

knowledge that the moon is the cause of the eclipse.[119]

(4) *Knowledge of relative properties depends on more than knowledge of the subject of these properties.* According to Scotus, Ockham writes, “nothing leads to a knowledge of something else save because it contains it virtually or essentially.”[120] However, subjects have many relative properties that can “in no way be known save when something is pre-known that is not contained in the subject either virtually or essentially.”[121] For example, beatitude, createdness, and charitability are properties that created rational beings have in relation to God, such that knowledge of these properties is neither essentially nor virtually contained in knowledge of their subjects.[122]

Furthermore, these properties import something more perfect into their subjects. Consequently, they cannot be known from their subjects alone, since knowledge of the more perfect cannot be contained in knowledge of the less perfect.[123] Ockham recognizes that Scotus may object here that these properties are not proper properties (*passiones proprie loquendo*), since subjects must always be more perfect than their properties, and subjects must cause their properties. Ockham’s reply is that these properties are no less proper than any other, since the proposition, “Rational creatures are beatifiable” is necessary, and as such it is necessary either according to the first or second mode of *per se* predication. Since beatitude is not contained in the definition of rational creatures, but rational creature is contained in the definition of beatitude, the proposition is necessary according to the second mode of *per se* predication. Consequently, “beatifiable” is no less a property than any other property, and so it is simply not the case that subjects must always be more perfect than their properties or that they must cause their properties.[124]

(5) *Scotus proves too much.* If knowledge of subjects virtually contains knowledge of their properties, any property of being would be self-evidently known from knowledge of being alone, since it would be virtually contained in knowledge of being. This is clearly false.[125] This may seem unfair, since Scotus only requires that knowledge of a property is virtually contained in *distinct* knowledge of its subject. Ockham replies that in the case of being *there can only be distinct knowledge*, since according to Scotus being is *absolutely simple* (*simpliciter simplex*), and the distinct knowledge of a subject contains knowledge of its properties, such that any proposition about being must be self-evident. Furthermore, either a

proposition that has being as its subject is self-evident or it is not. If it is, then every such proposition is self-evident. If not, then *pace* Scotus it is not the case that knowledge of a subject virtually contains knowledge of its properties.

On behalf of the second thesis—that it is not in the nature of a subject to virtually contain its properties—Ockham makes one argument:

(1) *Properties are not virtually contained in or caused by their subjects.* Since virtually containing something means having the virtue or power to cause it, and since causes are really distinct from their effects, Ockham recognizes only two ways in which a subject can virtually contain a property. Either the subject causes a really distinct property, or it has the power to cause the concept of such a property in the intellect. No subject has the power to cause the concept of any other thing unless it can cause knowledge of that thing. It has already been shown, however, that no subject causes knowledge of any other thing. Furthermore, not all properties are really distinct from their subjects; properties such as createdness, truth, and goodness are properties of all beings, but they are not really distinct from them.[126] Therefore, they are not virtually contained in their subjects.

On the basis of these arguments, all of which in one way or another stem from his rejection of the formal distinction, Ockham effectively eliminates the last remaining scientific *habitus* that could have secured the unity of a science: the common scientific *habitus*, which is caused by the primary object of a science, and which virtually contains all of the principles and conclusions (proper scientific *habitus*) of the science. As Ockham puts it in *Expositio in libros Physicorum Aristotelis*: "[I]t is no part of the concept of 'a subject' that it should 'virtually' contain the whole knowledge of the conclusions, or be something which comes first, and to which everything else is referred."[127] Ockham also denies Aquinas's thesis that a science is numerically one in the sense in which physical qualities and individuals are one: "[N]either metaphysics nor philosophy of nature nor mathematics is numerically one science [*non est una scientia secundum numerum*] in the same way as this whiteness and this heat and this man and this donkey are numerically one."[128]

How, then, does Ockham explain the unity of a science?

For Ockham, science in the proper sense (*scientia proprie dicta*) pertains only to knowledge of an entire demonstration, not the principles of a

demonstration alone.[129] Only the conclusion of a demonstration extends knowledge of a subject beyond its essential properties, which are themselves immediately known *per se primo modo,* and so are indemonstrable.[130] Ockham's definition of science in the proper sense reflects Aristotle's own distinction in *Nicomachean Ethics* VI between the intellectual virtues of science (*scientia*), the *habitus* of demonstration; understanding (*intellectus*), the *habitus* of principles; and wisdom (*sapientia*), the *habitus* that combines both science and understanding (see Section 1.1 above).[131] Since knowledge of the conclusion of a demonstration alone constitutes science in the proper sense, and since sciences have many conclusions, but no primary object or subject, Ockham argues that *there are as many scientific* habitus *in any science as there are conclusions.* Sciences have neither one subject nor one object; they have *as many subjects as there are subjects of conclusions, and as many objects as there are conclusions*.[132] Whereas Scotus only conditionally endorses the principle that "there are as many sciences as there are knowables [*quot scibilia, tot scientiae*],"[133] Ockham, having dispensed with primary objects and subjects, endorses the principle unconditionally. A science is not a simple *habitus*, but rather a collection of many *habitus*. The unity of the many *habitus* that compose a science is best understood not by analogy with the simplicity of a physical quality (Aquinas's gradient ontology), nor by reference to the primary object or subject of a science (Scotus's virtual containment ontology), but rather by analogy to the *unity of a political body*. Every demonstration is a science, and the unity of a science such as metaphysics or physics consists in its *hierarchical logical order alone*:

> Hence we have to say that metaphysics is not numerically one science. The same is true of the philosophy of nature, which is a collection of many *habitus* [*collectio multorum habituum*] […]. It is one in the same sense that a city, or a nation, or an army, which includes men and horses and other necessary things, or a kingdom, or a university, or the world, is said to be one.[134]

> Science […] is not one in number [*non est una numero*], but contains many *habitus* distinct not only in species but frequently also in genus. But they are mutually ordered [*ordinem tamen aliquem inter se habentes*], and owing to this special order, which other objects of science or knowledge [*scibilia vel cognoscibilia*] do not have, they can be called, and are called, in common usage [*usum loquentium*], one science [*una scientia*].[135]

The unity of a science consists in the hierarchical logical order of many really distinct scientific *habitus*. This order is characterized by a descending

scale of generality in the extension of the terms that compose the propositions of the science. These propositions can be ordered in three ways: according to the extension of their predicates; according to the extension of their subjects; or according to the extension of both their subjects and predicates.[136] Here is an example of the first type of order.[137] Any number of demonstrated conclusions, "S is P_1," "S is P_2," "S is P_3," "S is P_n" constitute one science if, and only if, P_1 has a broader extension than P_2, P_2 has a broader extension than P_3, P_3 has a broader extension than P_n, etc.[138]

A major consequence of Ockham's theory of science is that no proposition intrinsically belongs to metaphysics or natural philosophy or any other science. Which science a proposition belongs to depends on whether it has a place in some logical order, and insofar as one and the same proposition may belong to different logical orders (either as a principle or a conclusion), it may belong to many sciences.[139] This can happen in at least three ways. (1) One part of one science can be subalternated to another science: "[It] is not impossible that some science should be subalternated in one part to another science, but not in another part. For instance, perspective is in one part subordinated to geometry, but not in another part."[140] Many, but not all, propositions in perspective are subordinated to geometry. Other examples include metaphysics and theology, which share many propositions, since any conclusion about being in general can serve as a principle in a demonstration about God, who is a being.[141] Similarly, logical and metaphysical principles serve as principles in demonstrations in other sciences, such as natural philosophy and mathematics.[142] (2) One part of one science can be subalternated to two or more sciences at once: it is "possible that one science should be subalternated to different sciences in the same part, namely when one science knows one principle for a conclusion and another science knows another principle for the same conclusion." (3) One part of one science may be subalternate to one science, while another part may be subalternate to another science: "[One] part of a science can be subalternated to one science, and another to another, as, for instance, one part of the science of nature can be subalternated to geometry and another to arithmetic."

One and the same scientific *habitus* can, therefore, belong to different logical orders and, therefore, different sciences. This is enormously

important because it considerably relaxes Aristotle's ban on genus-crossing in the sciences. Subalternation is a relation, not between entire sciences, but rather between parts of sciences, which can relate to one another in a variety of ways, and which are themselves sciences in no less a sense than any of the sciences they are parts of. In Ockham, "subalternation becomes the norm rather than the exception […]."[143] By denying that sciences have one primary subject or object and insisting that the unity of a science consists in its logical order alone, Ockham boldly embraces what is arguably the most radical position on the unity of science in the scholastic debate about scientific *habitus*. As we will see below, Suárez is not entirely comfortable with Ockham's denial that sciences have primary objects; sometimes sciences are composed of many diverse logical series that do not exhibit the logical order Ockham requires, but that nevertheless are about the same object.

1.4.4 Suárez's Pluralist Ontology of Scientific *Habitus*

In *Disputationes metaphysicae*, disp. 44, sec. 11.18, Suárez introduces two principles that determine his ontology of scientific *habitus*. First, against Aquinas and Henry of Ghent, but with Scotus and Ockham, Suárez argues that "the real increase of a *habitus* [*augmentum reale habituum*], especially its extensive increase, is not possible without a real addition [*non posse fieri sine additione reali*]." The second principle is that "what is added with the extensive increase of a *habitus* cannot have a proper and per se union with the part of the *habitus* that is presupposed."[144] When I learn a demonstration, I acquire an ability that is not contained in (cannot be actualized by) any scientific *habitus* I already have. There must, therefore, be some real addition to the scientific *habitus* I already have.[145] This leaves open two ontological possibilities: either the *habitus* I already have acquires a part (an accidental form or quality) that inheres in it as in a subject (in which case the selfsame *habitus* only becomes more mereologically complex), or I acquire another, really distinct scientific *habitus*. Suárez endorses the second possibility. His main reason is that the *habitus* I acquire when I learn a demonstration does not have a per se union with any scientific *habitus* I already have. For Suárez, a per se union only obtains

between "partial entities" that need one another in order to constitute one thing. For example, matter and form need one another in order to constitute one substance (e.g., a human being). Form inheres in matter, and empowers matter to act. No part of the same quality, however, inheres in any other part or empowers it to act. Scientific *habitus* can act *independently of one another*, and so *they do not ontologically depend on one another*.[146] These *habitus* are not, therefore, parts of one *habitus.* When I learn a demonstration, I acquire a really distinct scientific *habitus*.[147] Consequently, a science is an accidental unity of many scientific *habitus* in one faculty (the intellect), much like the parts of a house are an accidental unity (since these parts also do not ontologically depend on one another). Like Ockham and Scotus before him, Suárez clearly repudiates Aquinas's gradient ontology of scientific *habitus* and endorses the thesis that the unity of a science is an accidental unity of many scientific *habitus*. As we will see, however, he also repudiates both Scotus's virtual containment ontology of scientific *habitus* and Ockham's aggregate ontology of scientific *habitus*.

How does Suárez explain the unity of a science?

Suárez distinguishes between two complementary sources of scientific unity: (1) the "effective subordination [*subordinatio effectiva*]" of conclusions when one can be derived from another, and (2) the "reference or attribution to the same total object [*attributio seu respectus ad idem objectum totale*]."[148] By "effective subordination," Suárez means one logical series in which all conclusions are continuously deduced from previous conclusions and principles. However, Suárez argues that the effective subordination of scientific *habitus* cannot alone secure the unity of a science, because there are many sciences in which conclusions are not effectively subordinated, but rather constitute diverse logical "series" or "lines." For example, in the science of man (*scientia de homine*) and physics, some conclusions may indeed be deduced from others, but many conclusions cannot be deduced in this way, since they are derived from the essence of the relevant object.[149] Suárez introduces reference or attribution to the same total object in order to explain how these diverse logical series can nevertheless constitute one science. This constitutes a clear repudiation of Ockham's theory of the unity of scientific *habitus*. It also burdens Suárez's theory with explaining exactly how the object of a science can secure its unity.

Suárez develops his own theory by considering the theories held by his predecessors. He begins by considering the thesis "held by many" that the object of a science must be one in species, real, simple, and virtually contain the entire science.[150] Suárez does not mention Scotus by name here, but he is clearly referring to Scotus's virtual containment ontology of scientific *habitus* (see Section 1.4.2 above). Suárez zeroes in on Scotus's thesis that the primary object of a science must be one in species. As we have seen in Section 1.4.2, according to Scotus, the unity of a science must be anchored in an object that is real, not merely common to many, as Aquinas and Ghent had argued (mistakenly, according to Scotus). This object cannot be one in genus alone; genera are common to many. The object of a science must be real, since only a real object has causal power and as such can virtually contain the entire science. Since, however, there is no science of individuals, this real object must be one *in species*. Thus, even though theology "considers many specifically distinct things," such as God, angels, and creation, "nevertheless [its] first object […] is one according to species and […] virtually contains all," and this object is God qua deity.[151] Indeed, according to Scotus, *all* sciences have objects that are one in species. There are, therefore, *as many sciences as there are species of being*. In the *Reportatio*, Scotus explicitly addresses the objection that "generally in the sciences something is assigned as a first object that is [generically] common to those things to which the science extends, such as 'being' in metaphysics, 'body' or 'mobile being' in physics, or 'magnitude' in geometry, and so in all the other sciences."[152] This objection is based on Aristotle's thesis in *Posterior Analytics* that a science is one if its genus is one. It is an objection because, as we have seen, according to Scotus the object of theology is one, not in genus, but rather in species: God qua deity. Scotus responds to this objection by arguing that "a science generically one has one first generic object. But from many specifically different objects, of which each is a first object of a science specifically one, it is possible to abstract one common object, and thus all those sciences have a generic unity, and therefore they pertain to one generic science."[153] All objects that are generically one are abstracted from objects that are specifically one: "[Where] a common subject is assigned […] it is the first subject of one generic science and therefore is abstracted from the special objects of many sciences, each of which is one according to species […]."[154] Scotus's theory is rather interesting. A science that is specifically one has the *highest*

possible degree of scientific unity, since there is no science of individuals, and all objects that are generically one are abstracted from objects that are specifically one. Furthermore, as we have seen in Section 1.4.2, a science whose object virtually contains the entire science can be deduced in a wholly a priori manner.

Attractive though it may be, Suárez argues that science is rarely as a priori as Scotus requires. Sciences, as human beings can acquire them, can neither always be individuated by species alone nor deduced in a wholly a priori manner from the subject of the science alone. Suárez concludes that Scotus's theory does not successfully explain how the object of a science can secure its unity.[155]

But Suárez also rejects the most common opinion: Aquinas's thesis that the objects of the theoretical sciences are established by abstraction. For Aquinas, the object of mathematics is established by abstracting quantity from the other accidents of substance; the object of physics is established by abstracting the form of material things from their material individuation; and the object of metaphysics is wholly abstracted from matter, because it does "not depend upon matter" and "can exist without matter."[156] Whereas Scotus seems to establish *too many maximally unified sciences* (one per species), Aquinas seems to establish *too few minimally unified sciences* (one per genus). A genus, Suárez argues, is too broad, since it includes many different species, and these species have differences that are essential and yet can neither be derived from the genus nor from one another. Furthermore, these species have no necessary connection among themselves.[157]

Having rejected Ockham's, Scotus's, and Aquinas's theories of the unity of scientific *habitus*, Suárez ends *Disputationes metaphysicae*, disp. 44, sec. 11.69 by arguing that so long as the objects of a science can be known under *some* formality (*ratio*), then *some* type of unity can be secured for a science.[158] He distinguishes between three grades of unity that the formal object of a science may have: specific, generic, and trans-generic. The science of man has a species as its object, and so it has specific unity. Physics has the genus of natural being as its object, and so it has generic unity. Metaphysics has the trans-generic formality of real being as its object, and so it has trans-generic unity. Each of these grades, Suárez writes, is "inexact, imperfect, and quasi-artificial [*hanc unitatem scientiae non esse*

exactam et perfectam, sed quasi artificiale],"[159] because these sciences are ultimately composed of many scientific *habitus* that constitute diverse logical series unified only by a common formality—the last remaining pillar of scientific unity in what would otherwise reduce to a mere heap of scientific *habitus*. Suárez concludes that there is no "certain and general rule" by which to determine the degree of unity any object must have in order to secure the unity of a science. For Suárez, a one-size-fits-all theory of scientific unity is simply not possible. Sciences have irreducibly different degrees of unity; there is no use in insisting on only one. It is the objects of the sciences themselves that resist any one theory of the unity of science.

1.5 Genus-Crossing and Subalternation

As we have seen, by the end of the sixteenth century, the scholastic debate about the unity of science remained as open as ever. At one extreme, Aquinas's and Henry of Ghent's gradient ontology of scientific *habitus* quickly lost traction. At the other extreme, Ockham's aggregate ontology of scientific *habitus* seemed to undermine the unity of science altogether. In Suárez, the object of a science came to bear the burden of securing the unity of the science, but he solemnly concludes that there is no "certain and general rule" by which to determine the degree of unity any object must have in order to secure the unity of a science.

In Section 1.2 I argued that Aristotle's ban on genus-crossing in the sciences restricted the scholastic debate about the unity of science to a debate about the unity of individual sciences. However, the scholastics did not all interpret Aristotle's ban on genus-crossing in the same way. On the contrary, they interpreted it in a broad variety of ways, and in some cases relaxed the ban substantially.[160] As we have seen, Ockham, who relaxed Aristotle's ban on genus-crossing most radically, argued that subalternation need not be an all-or-nothing affair: partial sciences can subalternate one another in different ways (see Section 1.4.3 above). Nevertheless, however complex the mechanisms of subalternation became, even Ockham recognized that subalternation requires that the subalternating and subalternated sciences remain *distinct* sciences:

> [That] can be called a subalternating science, either in itself or in some part of it, whenever an entire science cognizes a universal principle of some conclusion or property, and another science cognizes the conclusion, *in such a way that these sciences still do not make up a single science* [*ita tamen quod istae scientiae non constituunt unam totalem scientiam*].[161]

In subalternation, "the subalternating and subalternate sciences do not go together to make up a single science. They cannot be merged into a single axiomatic system."[162] Subalternation is a relation between two or more sciences. Otherwise, there is only one science, which cannot subalternate or be subalternated to itself. The Cartesian thesis that the totality of the sciences form a unity should not, therefore, be regarded as a consequence of even the most radical Aristotelian theory of subalternation.[163]

In *Rules*, Descartes completely suspends Aristotle's ban on genus-crossing in the sciences. He explicitly denies that sciences should be distinguished by "differences in their objects," or that "each should be studied separately, without regard to any of the others." "For the sciences," he writes, "as a whole are nothing but human wisdom [*humana sapientia*], which always remains one and the same, however different the subjects to which it is applied, it being no more altered by them than sunlight is by the variety of things it shines on" (AT 10:360, CSM 1:9). It "must be acknowledged that the sciences are so closely interconnected that it is much easier to learn them all together than to separate one from the other [*Credendumque est, ita omnes inter se esse connexas, ut longe facilius sit cunctas simul addiscere, quam unicam ab alijs separare*]" (AT 10:361, CSM 1:10). Nevertheless, despite the scholarly consensus that Descartes's theory of the unity of science in *Rules* is fundamentally at odds with scholastic theories of scientific *habitus*,[164] in Chapter 2 I argue that the unity of science in *Rules* depends, not on a rejection of scholastic theories of scientific *habitus*, but rather on a radical transformation of them.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0002

[1] Aristotle 1984, 1:143. See also Livesey 1990, 278 and 1989, 55–6.

[2] Aristotle 1984, 1:14 (*Categories* 8 8b27–28).

[3] Aristotle 1984, 1:14 (*Categories* 8 8b29–35; translation slightly modified). See also Aquinas 1882–, 6:310–12 (*ST* I–II, q. 49, art. 2); trans. Aquinas 1945, 2:368–71.

[4] See Aristotle 1984, 1:682–3: “When thought has become each thing in the way in which a man who actually knows is said to do so (this happens when he is now able to exercise the power on his own initiative), its condition is still one of potentiality, but in a different sense from the potentiality which preceded the acquisition of knowledge by learning or discovery; and thought is then able to think of itself” (*De Anima* III.4 429[b]6–9). See also Aquinas 1882–, 6:321 (*ST* I–II, q. 50, art. 4); trans. Aquinas 1945, 2:381.

[5] In modern English, “habits” are dispositions, but they are not typically or always regarded as under rational control (i.e., under the control of the intellect and the will). The English word “habit” translates, not *habitus*, but rather *consuetudo*. See Faucher and Roque 2018, 4.

[6] See, e.g., Aquinas 1882–, 6:312 (*ST* I–II, q. 49, art. 3); trans. Aquinas 1945, 2:371–2. Aquinas cites Augustine 1844–1864, 40:390 and Averröes 1550–1552, 6:169v.

[7] See Aquinas 1882–, 6:317–18 (*ST* I–II, q. 50, art. 1); trans. Aquinas 1945, 2:375–6.

[8] This includes the sensitive faculties “according as they act at the command of reason.” See Aquinas 1882–, 6:319 (*ST* I–II, q. 50, art. 3); trans. Aquinas 1945, 2:379–80 and *Nicomachean Ethics* I.13 1102[b]13–1103[a]3 in Aristotle 1984, 2:1741–2.

[9] Aquinas 1882–, 6:315 (*ST* I–II, q. 49, art. 4); trans. Aquinas 1945, 2:373.

[10] See Aquinas 1882–, 6:327 (*ST* I–II, q. 51, art. 2) trans. Aquinas 1945, 2:389. See also *Nicomachean Ethics* II.1 1103[a]1–25 in Aristotle 1984, 2:1742–3.

[11] Aquinas 1882–, 6:319–20 (*ST* I–II, q. 50, art. 3); trans. Aquinas 1945, 2:379–80.

[12] Aquinas 1882–, 6:321 (*ST* I–II, q. 50, art. 4); trans. Aquinas 1945, 2:382.

[13] Aquinas 1882–, 6:312 (*ST* I–II, q. 49, art. 3); trans. Aquinas 1945, 2:372.

[14] Aristotle 1984, 2:1743 (*Nicomachean Ethics* II.1 1103[b]21), cited in Aquinas 1882–, 6:336 (*ST* I–II, q. 52, art. 3); trans. Aquinas 1945, 2:399.

[15] See Aristotle 1984, 2:1743.

[16] See Aristotle 1984, 1:664.

[17] Aristotle 1984, 2:1614 (*Metaphysics* V.20 1022[b]10–12), where Aristotle defines *hexis* as a “disposition according to which that which is disposed is either well or ill disposed, either in itself or with reference to something else.” See also Aquinas 1882–, 6:310–12 (*ST* I–II, q. 49, art. 2); trans. Aquinas 1945, 2:368–71.

[18] On the ends of the speculative and practical intellect, see, e.g., Aquinas 1882–, 5:278–9 (*ST* I. q. 79, art. 11); trans. Aquinas 1945, 1:764.

[19] Aquinas 1882–, 6:349 (*ST* I–II, q. 55, art. 1); trans. Aquinas 1945, 2:413: “Virtue denotes a certain perfection of a power. Now a thing’s perfection is considered chiefly in relation to its end. But the end of power is act. Therefore power is said to be perfect according as it is determined to act. […] [The] rational powers, which are proper to man, are not determined to one particular action, but are inclined indifferently to many; but they are determined to acts by means of *habitus* […]. Therefore human virtues are *habitus*.”

[20] Aristotle 1984, 2:1747 (*Nicomachean Ethics* II.6 1106[a]15–17), cited in Aquinas 1882–, 6:352 (*ST* I–II, q. 55, art. 3); trans. Aquinas 1945, 2:415.

[21] See Aristotle 1984, 2:1801–2 (*Nicomachean Ethics* VI.6–7).

[22] Aquinas 1965, 166–7 (*Expositio super librum Boethii De trinitate*, q. 5, art. 1); trans. Aquinas 1986, 15–16.

23 Aquinas 1882–, 6:342; trans. Aquinas 1945, 2:408.

24 In *Posterior Analytics* I.4, Aristotle argues that a *per se* property is (1) any property that is contained in the definition of the subject (and is, therefore, part of its essence) (e.g., "line to triangle and point to line"); (2) any property whose definition contains the subject (e.g., "as straight belongs to line and so does curved"). Aristotle distinguishes between two other types of *per se* property, but they are not relevant here. See Aristotle 1984, 1:118. For discussion, see Hintikka 1972; McKirahan 1992; Ferejohn 2013.

25 Aristotle 1984, 1:122 (*Posterior Analytics* I.7 75^{a}38–75^{b}11). On Aristotle's ban on genus-crossing in scholastic Aristotelianism up to Ockham, see Livesey 1982, 1990. On Descartes's rejection of Aristotle's ban on genus-crossing in *Rules*, see Dika 2018; Ariew 1990; and Chapter 2.

26 See Aristotle 1984, 1:122 (*Posterior Analytics* I.7 75^{b}16–17).

27 See McKirahan 1978 and 1992, 50–68.

28 See Livesey 1982.

29 Aristotle 1984, 1:143 (*Posterior Analytics* I.28 81^{a}38).

30 Aquinas distinguishes between "the theology [*theologia*] which is part of philosophy," which depends on natural reason alone, and the "theology included in sacred science [*sacram doctrinam*]," which depends on both natural reason and revelation. See Aquinas 1882–, 4:7 (*ST* I, q. 1, art. 1); trans. Aquinas 1945, 1:6. *ST* is devoted to the theology included in *sacra doctrina*, not the *theologia* included in philosophy. On Aquinas's concept of sacred doctrine as a science, see Jenkins 1997, 11–50.

31 On the translation and reception of Aristotle's *Posterior Analytics* in the Latin West, see Pasnau 2014 and Longeway 2011, 1062–6.

32 See Krebs 1912; Chenu 1957, 1967; Grabmann 1948; Dumont 1961, 1962; Köpf 1974; Torrell 2003.

33 Alexander of Hales 1622, 1:1 (*Universae theologiae summa*, Prologue, q. 1, m. 1, "*An doctrina sacra scripturae, vel Theolgiae sit scientia*?").

34 For a reconstruction of the debate, see Chenu 1957, 1967.

35 For details and references, see Chenu 1967, 41–52.

36 Aquinas 1882–, 4:8 (*ST* I, q. 1, art. 2); trans. Aquinas 1945, 1:6–7.

37 Aquinas 1882–, 4:8 (*ST* I. q. 1, art. 2); trans. Aquinas 1945, 1:7. See *Metaphysics* VI.2 and *Nicomachean Ethics* VI.8 1142^{a}25–30 in Aristotle 1984, 2:1620–2 and 1803, and *Posterior Analytics* I.28 87^{a}38 in Aristotle 1984, 1:132.

38 Aristotle 1984, 1:143 (*Posterior Analytics* I.28 87^{a}38).

39 Aquinas 1882–, 4:11 (*ST* I, q. 1, art. 3); trans. Aquinas 1945, 1:7.

40 Aquinas 1882–, 4:11 (*ST* I, q. 1, art. 3); trans. Aquinas 1945, 1:7.

41 See William of Auxerre, *Summa aurea super IV libros Sententiarum*, lib. 3, tr. 12, c. 1, cited in Chenu 1967, 59.

42 See also Alexander of Hales 1622, 1:2 (*Universae theologiae summa*, Prologue, q. 1, m. 1); Albert the Great 1890–1899, 25:17–18 (*Commentarii in I Sententiarum*, lib. 1, d. 1, art. 3).

43 Albert the Great 1890–1899, 25:17 (ibid.).

44 Aquinas 1929–1956, 1:14 (*Scriptum super libros Sententiarum*, Prologue, q. 1, art. 3); trans. Aquinas 1998, 62; translation slightly modified.

45 Ibid.

46 Aquinas 1882–, 4:9 (*ST* I, q. 1, art. 2); trans. Aquinas 1945, 1:7.

47 Ibid.

48 Ibid.

49 See Chenu 1957, 1967.

50 Aristotle 1984, 1:143 (*Posterior Analytics* I.28 87[a]38).

51 Aquinas 1882–, 4:11 (*ST* I, q. 1, art. 3); trans. Aquinas 1945, 1:7.

52 Ibid.

53 Aquinas 1882–, 4:12 (*ST* I, q. 1, art. 3); trans. Aquinas 1945, 1:8.

54 See Aquinas 1882–, 11:165 (*ST* III, q. 11, art. 6); trans. Aquinas 2007, 4:2088: "The divinely infused light is the common formality for understanding what is divinely revealed, as the light of the intellect is with regard to what is naturally known."

55 One can find traces of Aquinas's analogy between the unity of a faculty and the unity of a science in Aristotle in Aristotle 1984, 2:1585 (*Metaphysics* IV.2 1003[b]19–23). See also Livesey 1989, 55–6.

56 As we will see, Scotus and Ockham raise similar objections. See Sections 1.4.2–1.4.3 below.

57 Aquinas 1882–, 6:344 (*ST* I–II, q. 54, art. 4); trans. Aquinas 1945, 2:410–11.

58 The exceptions are *intellectus*, the *habitus* of first principles, which is completely engendered once the possible intellect receives a principle, and supernaturally caused *habitus*, such as miraculous health or the science of the scriptures given to the apostles.

59 Aquinas 1882–, 6:344 (*ST* I–II, q. 54, art. 4); trans. Aquinas 1945, 2:411. See Cicero 1949, 331 (*De inventione* II.54).

60 Aquinas, ibid.

61 Ibid.

62 Aristotle 1984, 1:17.

63 Aquinas 1882–, 6:344 (*ST* I–II, q. 54, art. 4); trans. Aquinas 1945, 2:411.

64 Aquinas 1882–, 6:371 (*ST* I–II, q. 57, art. 6); trans. Aquinas 1945, 2:439.

65 Aquinas 1882–, 6:344 (*ST* I–II, q. 54, art. 4); trans. Aquinas 1945, 2:411.

66 Aquinas 1882–, 7:168–169 (*ST* I–II, q. 94, art. 1); trans. Aquinas 1945, 2:772–773.

67 Aquinas 1882–, 7:168 (*ST* I–II, q. 94, art. 1); trans. Aquinas 1945, 2:773.

68 Aquinas 1882–, 13:162 (*Summa contra gentiles*, lib. 1, cap. 56). For more discussion, see Maurer 1990. As Maurer also points out (ibid., 75–6), on the basis of the distinction between the primary and essential sense of *habitus* and its secondary sense, many Thomists, such as Capreolus [1409–1432] 1899–1908, 1:34 and Soncinas 1505, VI, q. 9, fol. 59v., wrongly took Aquinas to be arguing that sciences are an aggregate of many *habitus*, not a simple *habitus*. Maurer cites Cajetan in Aquinas 1882–, 6:345, n. 2–5 as someone who criticizes other Thomists for this misunderstanding. Like Aquinas, Cajetan maintains that a science is a simple *habitus* in the intellect.

69 Aquinas 1882–, 7:168 (*ST* I–II, q. 94, art. 1); trans. Aquinas 1945, 2:772.

70 Aquinas 1882–, 7:168 (*ST* I–II, q. 94, art. 1); trans. Aquinas 1945, 2:773.

71 See Maurer 1990, 75.

[72] See Henry of Ghent 1979–2008, 13:88–99 (*Quodlibet* IX, q. 4). For discussion, see Bermon 2018, 306–7.

[73] Scotus 1950–2013, 1:96 (*Ord.*, Prologue, q. 3); Scotus 2012, 69. On Scotus's concept of the unity of science, see O'Connor 1968, 3–50; Demange 2004, 2009a, 2009b; Bermon 2018.

[74] See Scotus 1891–1895, 7:303–8 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:8–13.

[75] Scotus 1891–1895, 7:305 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:8.

[76] Ibid.

[77] Ibid.

[78] The Aristotelian source of the distinction between dispositional ignorance and negative ignorance seems to be *Posterior Analytics* I.16 79[b]23–28 in Aristotle 1984, 1:130. See also Aquinas 1970, 86.

[79] Scotus 1891–1895, 7:305 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:9.

[80] Ibid.

[81] Ibid.

[82] Scotus 1891–1895, 7:306 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:9–10.

[83] Scotus 1891–1895, 7:306 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:10.

[84] Ibid.

[85] Scotus 1891–1895, 7:306 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:11.

[86] Scotus 1891–1895, 7:307 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:11.

[87] Ibid.

[88] Ibid.

[89] Scotus 1950–2013, 1:96 (*Ord.*, Prologue, q. 3); trans. Scotus 2012, 69.

[90] On the differences between Aquinas's concept of sacred doctrine and Scotus's concept of theology, see Gilson 1952, 44–84 and Cross 1999, 3–15.

[91] Scotus 1891–1895, 7:308 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:14.

[92] Scotus 1891–1895, 22:9 (*Rep.*, Prologue, q. 1, art. 2); trans. Scotus 2004–2008, 1:5.

[93] Scotus 1891–1895, 7:308 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:13. See also Bermon 2018.

[94] Scotus 1950–2013, 1:96 (*Ord.*, Prologue, q. 3); trans. Scotus 2012, 69.

[95] See Scotus 1950–2013, 6:192 (*Ord.*, 1.8.1.4, n.192): "There is therefore there [among the divine attributes] a distinction that is in every way prior to the [operation of] the intellect, and it is this: that wisdom actually exists naturally, and goodness actually exists naturally, and actual wisdom is formally not actual goodness," cited in Cross 1999, 43. On Scotus's theory of distinctions, see Blander 2020; Adams 2008, 1976; Cross 2004, 2002; Dumont 2005; Edwards 1977; Grajewski 1944; Wengert 1965; Wolter 1965.

[96] On Scotus's concept of "essential order," see *De primo rerum omnium principio* in Scotus 1891–1895, 4; trans. Scotus 1966. See also Flores 2000; Demange 2009a.

[97] Scotus 1891–1895, 7:308 (*QM*, lib. 6, q. 1, sc. 3); trans. Scotus 1998, 2:14.

[98] See Demange 2004, 2009a, 2009b. Aquinas regards God as the subject-matter of sacred doctrine, but not as the source of its unity. See Aquinas 1882–, 4:19–21 (*ST* I, q. 1, art. 7); trans. Aquinas 1945, 1:12. See also Livesey 1989, 58.

99 Scotus 1891–1895, 22:11 (*Rep.*, Prologue, q. 1, art. 2); trans. Scotus 2004–2008, 1:10.

100 See *Posterior Analytics* I.4 in Aristotle 1984, 1:118. See also McKirahan 1992, 226–30; Ferejohn 2013; and the discussion in Section 1.2 above.

101 Cf. Demange 2004: 95–101, esp. 99. On the second mode of *per se* predication in Aristotle, see McKirahan 1992, 87–93 and Ferejohn 2013, 90–5.

102 Scotus 1891–1895, 22:11 (*Rep.*, Prologue, q. 1, art. 2); trans. Scotus 2004–2008, 1:10.

103 Scotus 1891–1895, 7:308 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:14; translation modified.

104 Scotus does not cite an author who defends the thesis that the unity of a science consists in many really distinct scientific *habitus* alone, but the editors cite the Spanish Franciscan Gonsalvus Hispanus (1255–1313) as a possible source. See Gonsalvus Hispanus 1935, 152 (*Quaestiones disputate*, q. 9), cited by Etzkorn and Wolter in Scotus 1998, 2:5, n. 10.

105 See Ockham 1967–1988, 1:229–240 (*Ord.* 1, Prologue, q. 9); Ockham 2021, 117–123.

106 See references in n. 95 above.

107 See Ockham 1967–1988, 1:111 (*Ord.* 1, Prologue, q. 2, art. 3); trans. Ockham 2007, 235: "[N]othing intrinsic to God can be demonstrated of the divine essence in such a way that the divine essence is made subject in itself and something that is really the divine essence is predicated of itself. […] For if any proposition is known per se, it will surely be one in which what is really the same is predicated of what is really the same. But the divine essence is really the same as whatever is really God. Therefore no such proposition can be doubted, nor, consequently, can it be demonstrated." As Dumont 1992, 424 has argued, "after Ockham, many theologians would cease treating the question of whether theology was a science as a topic at all."

108 See Ockham 1967–1988, 1:229 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117; translation modified. For a summary of Ockham's arguments in *Ord.* 1, q. 9 against Scotus on scientific *habitus*, see Leff 1975, 320–9.

109 Ockham 1967–1988, 1:229 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117.

110 See Ockham 1967–1988, 1:229 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117; translation modified. Ockham cites Scotus 1891–1895, 12:255 (*Ord.* 2, d. 3, q. 10, n. 15).

111 Ockham 1967–1988, 1:229 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117.

112 Ockham 1967–1988, 1:229–230 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117.

113 Ockham 1967–1988, 1:230 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117.

114 Ockham 1967–1988, 1:230 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 117–118.

115 Ockham 1967–1988, 1:232 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 118.

116 Ibid.

117 Ibid. Ockham cites Aristotle 1984, 1:148 (*Posterior Analytics* II.2 90a6–7): "For the middle term is the cause [*αἴτιον*]…" See Section 1.2 above.

118 See Webering 1953, 134.

119 Ockham 1967–1988, 1:232 (*Ord.*, Prologue, q. 9); trans. Ockham 2021, 119.

120 Ockham 1967–1988, 1:233 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 119.

121 Ibid.

122 Ibid.

123 Ockham 1967–1988, 1:234 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 120.

[124] Ockham 1967–1988, 1:234 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 119–120.

[125] Ockham 1967–1988, 1: 236–237 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 121.

[126] Ockham 1967–1988, 1: 239–240 (*Ord.* 1, Prologue, q. 9); trans. Ockham 2021, 122.

[127] Ockham 1974–1988, 4:9 (*Expos. Phys.*, Prologue, §3); trans. Ockham 1990, 9.

[128] Ockham 1974–1988, 4:6–7 (*Expos. Phys.*, Prologue, §3); trans. Ockham 1990, 6.

[129] Knowledge of the principles of a demonstration is also science in a broader sense, but not science in the proper sense. See Pelletier 2013, 21–6.

[130] See Pelletier 2013, 23.

[131] See Ockham 1974–1988, 4:6 (*Expos. Phys.*, Prologue, §2); trans. Ockham 1990, 5, cited in Pelletier 2013, 23, n. 34.

[132] "We must also realize that there is a difference between the object and the subject of knowledge. For the object of knowledge is the whole proposition [i.e., the conclusion] that is known; the subject, however, is only a part of this proposition, namely the subject-term. For instance, the object of my knowledge that every man is educable is the entire proposition; its subject, however, is the term 'man.'" Ockham 1974–1988, 4:9 (*Expos. Phys.*, Prologue, §3); trans. Ockham 1990, 9.

[133] Scotus 1891–1895, 7:308 (*QM*, lib. 6, q. 1); trans. Scotus 1998, 2:14.

[134] Ockham 1974–1988, 4:7 (*Expos. Phys.*, Prologue, §3); trans. Ockham 1990, 7; translation modified.

[135] Ockham 1967–1988, 1:9–10 (*Ord.*, Prologue, q. 1), translated in Maurer 1990, 81.

[136] Ockham 1967–1988, 1:219 (*Ord.*, Prologue, q. 8), cited in Pelletier 2013, 31, n. 56.

[137] For more detailed discussion, see Pelletier 2013, 31–34.

[138] See Pelletier 2013, 32.

[139] See Pelletier 2013, 33; Livesey 1989; Livesey 1982, 330–88.

[140] Ockham 1974–1988, 1:540 (*SL* III-II, cap. 21), cited in Pelletier 2013, 37, n. 68; trans. Ockham 2007, 181.

[141] See Ockham 1967–1988, 1:10 (*Ord.* 1, Prologue, q. 1); trans. Ockham 2021, 10.

[142] Logical principles are not part of the content of the demonstrations of natural philosophy and mathematics, but are rather exhibited in the form of these demonstrations. See Longeway in Ockham 2007, 379, n. 102.

[143] Pelletier 2013, 37.

[144] Suárez 1856–1878, 26:699. Suárez traces these principles back to Scotus, whom he cites as an authority. He also cites Peter of Auriol, Gregory of Rimini, Ockham, and Gabriel Biel. On Auriol's theory of the unity of science, see Spade 1972. For a summary of the differences between Auriol, Ockham, and Rimini on scientific *habitus*, see Bermon 2018, 301–21. On the broader ontology of *habitus* in Suárez, see Perler 2018, 365–84.

[145] Suárez 1856–1878, 26:695 (*DM*, disp. 44, sec. 11.3).

[146] See Suárez 1856–1878, 26:695 (*DM*, disp. 44, sec. 11.5). Suárez defines *per se* unity in Suárez 1856–1878, 25:127–9 (*DM*, disp. 4, sec. 3.6–8).

[147] Suárez 1856–1878, 26:696 (*DM*, disp. 44, sec. 11.8).

[148] Suárez 1856–1878, 26:713 (*DM*, disp. 44, sec. 11.63).

[149] Ibid.

[150] Suárez 1856–1878, 26:714 (*DM*, disp. 44, sec. 11.65).

[151] Scotus 1891–1895, 22:11 (*Rep.*, Prologue, q. 1); trans. Scotus 2004–2008, 1:10.

[152] Scotus 1891–1895, 22:10 (*Rep.*, Prologue, q. 1); trans. Scotus 2004–2008, 1:6.

[153] Scotus 1891–1895, 22:11 (*Rep.*, Prologue, q. 1); trans. Scotus 2004–2008, 1:10.

[154] Scotus 1891–1895, 22:12 (*Rep.*, Prologue, q. 1); trans. Scotus 2004–2008, 1:1:11.

[155] Suárez 1856–1878, 26:715 (*DM*, disp. 44, sec. 11.67).

[156] Suárez 1856–1878, 26:715 (*DM*, disp. 44, sec. 11.68). The citations are from Aquinas 1965, 165–6 (*Expositio super librum Boethii De trinitate*, q. 5, art. 1); trans. Aquinas 1986, 14. Suárez does not directly address sacred doctrine here, but only the manner in which the objects of the theoretical sciences are established according to Aquinas.

[157] Suárez 1856–1878, 26:715 (*DM*, disp. 44, sec. 11.68). For more discussion, see Doyle 1991, 328–31.

[158] Suárez 1856–1878, 26:715 (*DM*, disp. 44, sec. 11.69). This is also the position defended by the Conimbricenses. See Conimbricenses [1606] 1611, 677 (*In primum librum Posteriorum Aristotelis*, lib. 1, cap. 23, q. 1, art. 2). The Conimbricenses defend the two theses that “*scientiam totaelm non esse simplicem qualitatem, sed aggregationem multorum habituum efficientium unam scientiam per ordinem ad unam abstractionem*” and “*habitum tendere in obiectum sub eadem ratione formali […]*.”

[159] Suárez 1856–1878, 26:715 (*DM*, disp. 44, sec.11.69).

[160] See Livesey 1982 and 1989. See also Laird 1983 and Gagné 1969.

[161] Ockham 1974–1988, 1:541 (*Summa logicae*, III-2, cap. 21), trans. Ockham 2007, 181–2; my emphasis.

[162] Longeway in Ockham 2007, 379, n. 102.

[163] For a similar argument, see Ariew 1990, 299.

[164] See, e.g., Beck 1952, 14–25 and Marion 1975, 25–30.

2

The Habitual Unity of Science

Descartes

2.1 The Cartesian Scientific *Habitus*: Basic Properties

In Chapter 1, I reconstructed the scholastic debate about the unity of scientific *habitus* from Aquinas to Suárez. I argued that, due to Aristotle's ban on genus-crossing, the debate focused exclusively on the unity of individual sciences, not the unity of the sciences as an interconnected totality. I concluded that the debate ended in *aporia*. Suárez came to see that there is no "certain and general rule" by which to determine the degree of unity any object must have in order to secure the unity of a science. In this chapter, I argue that in *Rules* Descartes transforms the scholastic debate by showing how there is one *habitus* that extends to all possible objects of science and that unifies the sciences themselves. This *habitus* Descartes terms "*sapientia*" in Rule 1, and one can acquire *sapientia* by practice in the method, which is the source of *scientia* and which coordinates the sciences as needed in solutions to particular problems.

In this section, I introduce the Cartesian scientific *habitus*. I argue that Descartes models the unity of science (*sapientia*), not on Aristotelian *theoretical* wisdom, which is identical to the highest science (*πρώτη φιλοσοφία*, *proté philosophia*, "first philosophy" or "metaphysics") and does not include or unify the other sciences, but rather on Aristotelian *practical* wisdom (*φρόνησις*, *phronesis* or *prudentia*, in Latin), which unifies the ethical virtues in deliberations about what to do in particular cases. After addressing a prominent objection to an "habitual" interpretation of Descartes's method, I enumerate the principal properties that distinguish

the Cartesian scientific *habitus* from the scholastic theories of scientific *habitus* examined in Chapter 1. In Section 2.2, I discuss the subject of the Cartesian scientific *habitus*, the formation of the Cartesian scientific *habitus*, and the ontology of the Cartesian scientific *habitus*. In Section 2.3, I argue that the unity of science in *Rules* depends on an explicit rejection of Aristotle's ban on genus-crossing, and in Sections 2.4–2.5 I show how Descartes concretely overcomes the ban via his theory of the object of science (the "simple natures"), which establishes that one scientific *habitus* can indeed extend to all possible objects of science. Finally, in Section 2.6, I explain salient differences between Rule 1_{AT} and Rule 1_{CM}.

For Aristotle, *phronesis* is the *habitus* whereby the intellect deliberates about the means to an end set by virtue. For example, the generous person already has generosity as an end. However, the end is indeterminate; having generosity as an end is one thing, but determining what being generous requires in any particular case is quite another. The generous person must determine how to be generous in relation to particular individual(s), such that rendering the end determinate varies between cases depending on the relevant individual(s). (This is true in the arts too. The doctor already has health as an end, but they must determine how to produce health in particular individuals, such that rendering the end determinate varies between cases depending on the relevant individual.) Importantly, *phronesis* is not restricted to deliberation about one virtue in isolation from all other virtues; it deliberates about the means to one virtuous end "in a way that also situates that end within the ends of all the other virtues as well."[1] This is because being virtuous requires understanding the human good in general.[2] For example, to give money to someone when I owe the money to someone else is *not* an act of generosity because it is not based on an understanding of the good in general, which *also* requires that I act *justly*, *courageously*, *temperately*, etc. Thus, *phronesis* requires *all* the virtues, not only one; the virtues are themselves interconnected to one another in *phronesis*, such that one virtue without another is no virtue at all (e.g., generosity without justice, etc.). Conversely, the virtues also require *phronesis*, since being virtuous requires a reliable deliberative disposition or *habitus* that issues in virtuous actions (otherwise, virtue is totally accidental and not properly rooted in the soul). In short, since "every virtue requires *phronesis*, which is one, and *phronesis* requires all the virtues, then every virtue requires all the virtues."[3] As Aristotle puts it: "For with the presence

of the one quality, practical wisdom, will be given all the excellences."[4] *Phronesis* must coordinate *all* virtues in every case, and the actualization of any virtue requires *phronesis*; otherwise, there is neither *phronesis* nor, therefore, virtue.

The Cartesian scientific *habitus* is in many respects structurally analogous to Aristotelian *phronesis*. In the title of Rule 1, Descartes defines the *end* of science: "The end of our studies should be to direct the mind with a view to forming true and sound judgments about whatever comes before it [*Studiorum finis esse debet ingenij directio ad solida et vera, de ijs omnibus quae occurrunt, proferenda judicia*]" (AT 10:359, CSM 1:9; translation modified). Just as *phronesis* is the *habitus* whereby the intellect deliberates about the means to an end set by virtue, so too the method is the *habitus* whereby the intellect deliberates about the means to an end set by science. Furthermore, as I will show in more detail in Chapter 3, like virtuous ends, which must be rendered determinate here and now, so too must the end of science be rendered determinate here and now. As we have seen in the case of *phronesis*, rendering the end determinate and devising the means varies between cases depending on the relevant individual(s). The same is true of the method. In each problem, the agent must prescribe an order of operations whose execution yields a solution to a problem (and, therefore, yields "science"), and the order prescribed varies between cases depending on the relevant problem.[5] In Rule 4, Descartes describes the method as the *means*: "We need a method if we are to investigate the truth of things [*Necessaria est methodus ad rerum veritatem investigandam*]" (AT 10:371, CSM 1:15). The method is the means to the end of science as *phronesis* is means to the end of virtue. Both are intellectual virtues. Finally and most importantly, returning to Rule 1, the relation between the means (the method) and the end (forming true and sound judgments about whatever comes before the mind) is such that the sciences are themselves reciprocally related or interconnected to one another: in solutions to problems, one science without another science is no science at all (e.g., optics without physics and mathematics): "It must […] be acknowledged that all the sciences are so closely interconnected that it is much easier to learn them all together than to separate one from the other. If, therefore, someone seriously wishes to investigate the truth of things [*rerum veritatem investigare*], he ought not to select one science in particular, for they are all interconnected and interdependent [*inter se conjunctae et a se invicem*

dependentes]” (AT 10:361, CSM 1:10). The sciences are interconnected in the way that virtues are in Aristotelian *phronesis*, such that solving problems in any one science is simply not possible unless I can solve problems in those sciences on which it depends (see Table 2.1).

Table 2.1 Aristotelian *phronesis* and the Cartesian scientific *habitus* (Analogy of Structure)

Phronesis: The end of virtue:: Method: The end of science
Phronesis renders the end determinate and devises the means by coordinating all virtues on a case by case basis: The method renders the end determinate and devises the means by coordinating all sciences on a case by case basis
One virtue without all other virtues is not a virtue (the unity of the virtues): One science without all the others is not a science (the unity of the sciences)

The thesis that Descartes’s method is a *habitus* directly contradicts a prominent interpretation of Descartes’s method championed by J. L. Beck and Jean-Luc Marion in their respective commentaries on *Rules*.[6] Beck and Marion argue that Descartes’s method is not a *habitus*, since, as we have ourselves seen in Chapter 1, scientific *habitus* are distinguished by their respective objects (e.g., geometry by continuous magnitude; arithmetic by discrete magnitude, etc.), such that there are as many scientific *habitus* as there are objects of science. Scientific *habitus* entail irreducibly many individually united sciences, not the unity of the sciences as an interconnected totality. To understand the Beck–Marion Thesis more clearly, I must cite at length the passage from Rule 1 on which they base their interpretation:

> Whenever people notice some similarity between two things, they are in the habit [*consuetudo*] of ascribing to the one what they find true of the other, even when the two are not in that respect similar. Thus they wrongly compare the sciences, which consist wholly in knowledge acquired by the mind, with the arts, which require some bodily aptitude and practice [*aliquem corporis usum habitumque*]. They recognize that one man cannot master all the arts at once and that it is easier to excel as a craftsman if one practices only one skill; for one man cannot turn his hand to both farming and harp–playing, or to several different tasks of this kind, as easily as he can to just one of them. This has made people come to think that the same must be true of the sciences as well. Distinguishing the sciences by the differences in their objects [*pro diversitate objectorum ab invicem distinguentes*], they think that each science should be studied separately, without regard to any of the others. But here they are surely mistaken. For the sciences as a whole are nothing other than human wisdom, which

> always remains one and the same, however different the subjects to which it is applied [*scientiae omnes nihil aliud sint quam humana sapientia, quae semper una et eadem manet, quantumvis differentibus subjectis applicata*], it being no more altered by them than sunlight is by the variety of things it shines on. Hence there is no need to impose any restrictions on our mental powers [*ingenia*]; for the knowledge of one truth does not, like skill in one art, hinder us from discovering another; on the contrary, it helps us (AT 10:359–360, CSM 1:9).

Here is Beck's interpretation of this passage:

> Descartes attacks the Aristotelian, and Scholastic, distinction between the arts and the sciences […]. To learn "an art," such as that of agriculture, involves necessarily the acquisition of some special bodily skill, some physical *habitus* implying training, practice, and the building up of certain reflexes. It is clear that, given the normal span of human life, no single individual, however gifted, can master all the arts, or even a small proportion of the arts. […] But, Descartes maintains, it is not the same in the case of science. To learn a science in no way depends upon, or entails, any specialized training of the body.[7]

Marion writes more strongly:

> *Habitus*, from the Greek *hexis*, must be restricted to the arts alone, without its ever being legitimate to transfer *habitus* to the sciences and speak of a *habitus scientiarum*. […] Descartes refuses to transpose the multiplicity of *antikeimena/objecta* into a necessary multiplicity of sciences by the intermediary of a multiplicity of *habitus*.[8]

Beck and Marion argue that Descartes (1) maps the distinction between sciences and arts onto the distinction between mind and body, and (2) restricts *habitus* to the arts and the body alone, not the sciences, which are wholly acquired by the mind. By redistributing the terms of the analogy in this way, Descartes can argue that, while each art requires a separate *habitus*, each science does not. Beck and Marion conclude that there is no room for scientific *habitus* in Descartes's conception of the unity of science in *Rules*. The unity of science in Rule 1 undermines the strict correspondence consistently maintained in Aristotelian theories of science between the diversity of objects, on the one hand, and the diversity of scientific *habitus*, on the other. Descartes refers the sciences, not to their objects, but rather to a unity that transcends the unity of individual sciences precisely by transcending differences between their respective objects. There is, therefore, no need to distinguish scientific *habitus* by object and, *a fortiori*, no need for scientific *habitus* in *Rules*. Or so they argue.

The Beck–Marion Thesis rests (1) on a misunderstanding of how the concept of *habitus* functions in Aristotelian psychologies and (2) on a failure to consider all of the relevant textual evidence in *Rules*, above all

Descartes's consistent, interchangeable use of the terms *scientia*, *methodus*, and *arte* throughout the treatise (I examine the relevant textual evidence in more detail below). As we have seen in Chapter 1, Section 1.1, Aquinas distinguishes between *habitus* as *intellectual virtues* and *habitus* as *individual sciences*.[9] *Intellectus*, *scientia*, and *sapientia* are virtues that perfect the speculative part of the soul by enabling it to demonstrate conclusions from principles.[10] As intellectual virtues, these *habitus* are *not* distinguished by their *objects*, but rather by their *function in the soul*. They are not themselves identical to any one science or even the totality of the sciences (*scientiae*). The intellectual virtues Aristotle enumerates in *Nicomachean Ethics* VI are distinguished from one another by *function*, and the three theoretical sciences Aristotle enumerates in *Metaphysics* VI—metaphysics, mathematics, and physics—are distinguished from one another by *object*. The "function criterion" and the "object criterion" are not identical, but since Beck and Marion only recognize the object criterion, and since Descartes refuses to distinguish sciences by the object criterion in Rule 1, they argue that Descartes must reject *habitus* in order to assert the unity of science in *Rules*. However, in Rule 1, Descartes only rejects scientific *habitus* qua individual sciences distinguished by their respective objects, not *habitus* qua intellectual virtues. Indeed, Descartes *redefines* wisdom (*sapientia*) as an intellectual *habitus* that extends to every possible object of science and, therefore, includes the principles and demonstrations of all sciences.

I do not mean to suggest that Descartes simply inherits the Aristotelian intellectual virtues without further ado; he transforms the relation between them almost beyond recognition. As can be seen in the above-cited passage in Rule 1, Descartes *reduces* sapientia *to the totality of the sciences*, since "the sciences as a whole are nothing other than human wisdom [*scientiae omnes nihil aliud sint quam humana sapientia*]" (AT 10:360, CSM 1:9). According to Aristotle and Aquinas, by contrast, one can have wisdom without having all the sciences and vice versa. Wisdom is simply the *highest* science (metaphysics), not the *totality* of the sciences. Consequently, even when one has wisdom, one need not have the other sciences and vice versa. And even when one has both wisdom *and* the sciences, *sciences with different genera exhibit no connection among themselves on pain of violating Aristotle's ban on genus-crossing*. As Aquinas puts it: "[We] may have one science without having another," and

in any case Aristotle's ban on genus-crossing ensures that the sciences *must* be learned separately. For Descartes, *sapientia* is not identical to any one science (not even the highest science), nor yet to the mastery of the many irreducibly distinct sciences, but rather to the interconnected totality of the sciences themselves. *Sapientia* is not restricted to any one object, principle, or demonstration. It includes them all (I explain how it does so in Section 2.4). *Sapientia*, which in Aristotle and Aquinas towers above the other sciences but does not include them, is replaced by the interconnected totality of the sciences themselves. This clearly contradicts Aquinas's Aristotelian thesis that the sciences "are about diverse matters having no relation to one another. Hence we do not observe in them the connection that is to be found among the moral virtues […]."[11] For Descartes, the sciences are connected in exactly the same way as the ethical virtues are. Thus, Cartesian *sapientia* bears the same relation to the connection between the sciences that Aristotelian *phronesis* bears to the connection between the ethical virtues. The *disanalogy* here is between Cartesian *sapientia* and Aristotelian *sapientia*.

In addition to the differences between Cartesian and Aristotelian *sapientia*, one must also note that unlike Aristotelian *phronesis*, Cartesian *sapientia* is the *end*, not the *means*. The means is the method. The method *produces the connection between the sciences in solutions to particular problems*. The interconnection between the sciences must be concretely revealed in such solutions, since it is not simply given all at once in a ready-made encyclopedia. The method does not coordinate the sciences as they currently exist (e.g., in Aristotelian natural philosophy or Euclidean mathematics). It does not combine historically received sciences, but rather *constitutes these sciences themselves according to its own operations and procedures as it solves particular problems*. In this sense, whereas the unity of science (*sapientia*) is the *ratio essendi* of the method (i.e., the reason why the method can solve problems by constituting and connecting sciences), the method is the *ratio cognoscendi* of the unity of science (i.e., it reveals the unity that obtains between the sciences). "The sciences," Descartes writes in *Cogitationes privatae*, "are at present masked, but if the mask were taken off, they would be revealed in all their beauty. If we could see how the sciences are linked together, we would find them no harder to retain in our minds that the series of numbers" (AT 10:215, CSM 1:3). Thus, like Aristotelian *phronesis*, the Cartesian scientific *habitus* is an

intellectual virtue, and like the other intellectual virtues Aquinas discusses, it is not distinguished by object. Since it is an intellectual virtue, it is a *habitus*. Consequently, *pace* Beck and Marion, not only is there a *habitus scientiarum* in *Rules*—this *habitus* is the source of the unity of science. Precisely how this *habitus* manages to do what no scholastic believed it could do—extend to every possible object of science—is something I discuss in more detail in Sections 2.3–2.4.

In the meantime, there are other aspects of the Beck–Marion Thesis that must be examined. As I indicated above and discuss in more detail in Chapter 3, Section 3.4.1 and subsequent chapters, Descartes's method incorporates another central element of Aristotelian *phronesis*: sensitivity to particular cases. In this respect, the method is like an *art* (e.g., medicine or navigation); the practitioner must exercise discernment in order to apply the universal prescription to the particular case. Beck and Marion, however, deny any correspondence between art and science in *Rules*. But here the Beck–Marion Thesis rests on no textual evidence beyond Rule 1. Descartes explicitly describes his method in *Rules* as an art on numerous occasions, and so it cannot be the case that his strategy in Rule 1 is to restrict *habitus* to the arts alone. He twice describes Rule 6—where he develops the theory of order so central to the method (see Chapter 4)—as a "secret art" (*artis secretum*, AT 10:381, CSM 1:21). In Rule 8, he reassures his readers that there are limits to what can be humanly known, and that those who keep this in mind will take the view that "any lack of further knowledge […] is not at all due to any lack of intelligence or method [*artis*], and that whatever anyone else can know," they too "are capable of knowing" (AT 10:396, CSM 1:30). Cottingham correctly translates the Latin *artis* by "method" here, since the method, as Descartes himself canonically defines it in Rule 4, is supposed to yield knowledge of everything human beings are capable of knowing (AT 10:372, CSM 1:16). In Rule 10, Descartes refers to Aristotelian dialectic as an "art [*artem*]" that contributes nothing whatever to knowledge of the truth, since the latter requires reasoning from propositions known with certainty (not the probable opinions of the wise). When seen in light of Descartes's use of "art" and its variants elsewhere in the treatise, it is clear that here in Rule 11, he intends the method of *Rules* to be compared to dialectic as *one superior art to another, inferior art*, and that the distinction between Descartes's method and Aristotelian dialectic is, therefore, a distinction between *two arts*. In Rule 12, he argues that

supernaturally caused beliefs do "not fall within the scope of the method [*sub artem non cadit*]" (AT 10:424, CSM 1:47). In Rule 14, he writes that, when it comes to acquiring knowledge about physical magnitudes, it is an integral "part of the method [*artis*]" to clearly distinguish between as many dimensions of magnitude as needed (e.g., length, breadth, depth, weight, speed, etc.) (AT 10:447, CSM 1:62). Finally, in Rule 15, he unambiguously refers to the "greatest advantage of our method [*artis*]" and to what "our method [*artis*] demands" (AT 10:452, CSM 1:65). Statistically, Descartes uses the word "art" (*artis*) 32 times in *Rules*, seven times more than he uses the word "method" (*methodus*), and he frequently uses both terms interchangeably.[12] It seems to me that the relation Descartes establishes between these terms directly contradicts the Beck–Marion Thesis. At most, in Rule 1 Descartes distinguishes science from the *habitus* of those arts that *require the use of the body* (*corporis usum habitumque*, AT 10:359, CSM 1:9). Descartes only argues that intellectual *habitus* (intellectual virtues) should not be analogized to corporeal *habitus*, not that *sapientia* or *scientia* are not themselves intellectual *habitus* or virtues.

The Cartesian scientific *habitus* produced by practice in the method is an *art*, and not a determinate science or even the totality of the sciences regarded logically or propositionally; it *produces* the propositional order of science. In *Rules*, Descartes distinguishes between two senses of science: principally, as an ability to find solutions to problems (*ars inveniendi*),[13] and secondarily, as the propositional order produced by such an ability. As a *habitus*, the method is an art and is identical to science in the former sense, but not in the latter sense. This strongly distinguishes Descartes from his scholastic predecessors such as Scotus, Ockham, and Suárez, for whom scientific *habitus* simply *are* intelligible species, principles, and/or demonstrations insofar as they exist habitually in the mind. That Descartes regards science ontologically as a problem-solving ability is clear. For example, in Rule 3 he writes that "even though we know other people's demonstrations by heart, we shall never become mathematicians if we lack the intellectual aptitude to solve any given problem [*ingenio apti ad quaecumque problemata resolvenda*] […] in this case what we would seem to have learnt would not be science but history" (AT 10:367, CSM 1:13).[14] Descartes continued to maintain this conception of science as a problem-solving ability throughout his career. In a letter to Hogeland, he distinguishes between science and history in exactly the same way:

> By 'history' I understand everything which has been discovered already and is contained in books. By 'science' I mean the skill to solve every problem, and thus to discover by one's own efforts everything capable of being discovered in that science by means of our native human intelligence [*Per Scientiam vero, peritiam quaestiones omnes resolvendi, atque adeo inveniendi propria industria illud omne quod ab humano ingenio in ea scientia potest inviniri*].

(February 8, 1640, AT 3:722–723, CSMK 3:144). Science does not consist in knowing "other people's demonstrations," but rather in possessing the cognitive ability or "aptitude to solve any given problem." Those who have the ability to solve problems in the sciences have science, and those who have only knowledge of the demonstrations produced by such an ability have, not science, but *history*. Just as an art (e.g., sculpture) is not the byproduct of the art (e.g., a sculpture), but rather the ability whereby art objects are produced, so too science is not the byproduct of science (e.g., a corpus of propositions), but rather the ability whereby such propositions are produced. No doubt, the sculpture is art and the demonstration is science, but only in a secondary sense. Art exists principally in the soul of the artisan, and science exists principally in the soul of the scientist, and both exist there as *habitus*. In Descartes's case, this *habitus* disposes the mind to solve problems and produce propositions that are certain and evident. Science does not exist (not principally) in the soul habitually as a proposition or corpus of propositions, but rather as the ability whereby such propositions are effectively produced in solutions to particular problems. One acquires this ability via practice in the method. Consequently, "having" science means something radically different in Descartes than it does in scholasticism ("*habitus*" derives from the Latin "*habere*"). For Descartes, "having science" means having the relevant problem-solving ability in the mind. For Descartes's scholastic predecessors, by contrast, it means having a formality (*ratio*) or intelligible species, principle, and/or demonstration in the mind.

Other relevant differences between Descartes and his predecessors crop up here. Descartes makes clear that "knowledge of one truth does not, like skill in one [corporeal] art, hinder us from discovering another; on the contrary it helps us" (AT 10:360, CSM 1:9). In solving a problem in one science, *I perfect my ability to solve problems in other sciences.* The Cartesian scientific *habitus* is not restricted to reproducing the acts whereby it is acquired; it is capable of real extension to new acts.[15] This too distinguishes Descartes from his scholastic predecessors. As we have seen

in Chapter 1, Ockham and Suárez argue that *no* scientific *habitus* extends beyond the acts whereby it is acquired (they can only reproduce specifically identical acts), and Scotus argues the same in the case of proper scientific *habitus*. Not only does the Cartesian scientific *habitus* extend beyond the acts whereby it is acquired—that is in many respects its entire point.

On the basis of the analyses pursued above, the three principal properties that distinguish the Cartesian scientific *habitus* in *Rules* from the Aristotelian concepts of scientific *habitus* examined in Chapter 1 may now be more clearly enumerated:

(1) Like Aristotelian *phronesis*, which connects the virtues to one another, the Cartesian scientific *habitus* extends to every object of science and connects the sciences to one another. The Cartesian scientific *habitus* remains "one and the same, however different the subjects to which it is applied" (AT 10:361, CSM 1:10). This clearly distinguishes Descartes from his scholastic predecessors. As we have seen in Chapter 1, the scholastics typically distinguish scientific *habitus* by object (however different their definitions of the object(s) of science may be). By contrast, the Cartesian scientific *habitus* extends to the objects of every science, and is not distinguished by any such object.

(2) The Cartesian scientific *habitus* is not identical to any one formality or intelligible species, principle(s), and/or demonstration(s) in the mind. For Descartes, science is a *habitus* whereby problems in the sciences are solved. However propositionally complex science may be, its propositional order is the byproduct of solutions to problems by one and the same *habitus*. This *habitus* is not identical to any of the propositions produced by its means. It is an art, not a science (in the sense of a propositional totality).

(3) *The Cartesian scientific* habitus *is not restricted to merely reproducing the acts whereby it is acquired.* By learning how to solve a problem in one science, I perfect my ability to solve problems in other sciences. The Cartesian scientific *habitus* is, therefore, capable of extending beyond the acts whereby it is acquired. This undermines the principle shared by many Aristotelian scholastics that scientific *habitus* are restricted to reproducing the acts whereby they are acquired. The Cartesian scientific *habitus* is not (or not only) an effect of prior learning, but also the cause of discovery. It is fundamentally productive, not merely retentive.

2.2 The Subject, Acquisition, and Ontology of the Cartesian Scientific *Habitus*

Now that the distinguishing properties of the Cartesian scientific *habitus* have been enumerated, I can address the following three questions. Who or what is the subject of the Cartesian scientific *habitus*? How can the Cartesian scientific *habitus* be acquired? And where do such *habitus* fit into Descartes's ontology of substance, attribute, and mode? As the reader may have noticed, these are precisely the questions Aquinas would have expected an answer to based on his division of the *Treatise on* Habitus (see Chapter 1, Section 1.1). Indeed, they are questions any scholastic would have expected an answer to. To the extent that these questions bear on my interpretation of Descartes's method, they must be dealt with here—and they do bear on it, as will become clear below.

As the title of the treatise indicates, in *Rules* it is the human mind (*ingenium*) that must be directed in order to produce science. *Ingenium* is the subject of the Cartesian scientific *habitus*. What is "*ingenium*"? In many respects, Descartes's use of the term "*ingenium*" in *Rules* reflects Goclenius's definition of the term in *Lexicon philosophicum* (1613). Goclenius distinguishes between four senses of "*ingenium*." In the most general sense, *ingenium* is (1) "the innate or inborn, constitutive power of each thing [*insita vi & natura cuiusque rei*]," including animals and inanimate beings.[16] In its specific sense, following Cicero's definition of "*ingenium*" in *De oratore*, Goclenius defines *ingenium* (2) as "a power or faculty of soul for successfully and easily discovering and contriving anything, and also the power or faculty of memory [*vim feliciter ac facile inveniendi ac fingendi in homnibus, & vim memorie, teste Cicer*]."[17] In its most proper sense, *ingenium* is (3) "the constitution of the rational faculty of the soul for understanding something or discovering or teaching [*facultas rationalis animi constitutio, ad intelligendum aliquid, sive inveniendo, sive discendo*]." Goclenius further develops the most proper sense of *ingenium* when he describes it as a species of *euphuia* (*εὐφυΐα*), which Aristotle defines as naturally endowed good sense in *Nicomachean Ethics* III.5.[18] *Ingenium*, Goclenius continues, is "the good constitution by nature of the rational soul, now with respect to the true, now with respect to the good, nor is *euphuia* only of *ingenium* but also of natural judgment [*bona animi*

constitutio per naturam, tum ad verum, tum ad bonum, nec tantum est εὐφυΐα igenij, sed etiam iudicij naturalis].” *Ingenium* is also related to *eumatheia* (*ευμάθεια*), “the right constitution for learning disciplines [*recta constitutio ad disciplinas apprehendendas*]” and, therefore, to *mathesis* (*μάθησις*) (“learning” in the most general sense, not to be confused with *mathesis universalis*).[19] When it comes to the good constitution of the rational soul with respect to the true, *ingenium* is the ability for quickly finding (a) the middle term(s) of a deduction (“*celeris inventio medij termini*”) and (b) quickly finding the causes of things (“*celeris in invenienda causa rei*”). Hence its relevance to science as the principal faculty of discovery.[20] As the Conimbricenses put it: “An outstanding *ingenium* consists in a certain speed of the mind, as well as in ease of reasoning, investigating, and discovering.”[21] Similarly, Suárez writes: “Human science is acquired by acts proper to the human *ingenium* [*humanam scientiam propriis actibus humani ingenii acquisitam*].”[22] As a natural endowment, *ingenium* cannot be acquired by art, but it may be *perfected by art and exercise*. It may also be *blunted* in various ways. Toward the end of the entry, Goclenius notes that *ingenium* is (4) embodied in human individuals, and so it can vary in degrees of virtue and vice, depending on the temperament of the body, the “various dispositions of the mind, and the constitution of the organs, and of the auxiliary faculties, as of the phantasy.”[23] As such, it can be normatively evaluated as “subtle, or thick, acute or obtuse. Perspicacious or less perspicacious, quick or slow, sharp or less sharp […].”[24] He adds that *ingenium* can be overburdened by memory. Curiously, Goclenius does not identify (5) the equivalence between *ingenium*, the natural light (*lumen naturale*), and wisdom (*sapientia*) in his enumeration of the senses of *ingenium*, despite the relatively widespread identification of these three terms by Aristotelian scholastics. For example, Suárez defines the natural light of *ingenium* (*naturale lumen ingenii*) as *sapientia*, and he defines the latter as the *habitus* whereby the simplest, most certain, most evident things are known.[25]

As can already be seen from this brief survey, the lexical field covered by the term “*ingenium*” is incredibly rich. An inventory of Descartes’s use of “*ingenium*” in *Rules* reveals that he most often employs the term in sense (4), which includes senses (1)–(3) and (5), but anchors them in human

individuals whose *ingenium* vary in degrees of perfection and are, therefore, subject to normative evaluation and habituation.[26] This is important because, as we have seen in Chapter 1, *habitus* can only exist in subjects that can be determined to operation in a variety of ways. This is clearly the case with *ingenium*. Nevertheless, it is curious that Descartes insists on *ingenium* (as opposed to, say, the intellect) as the subject of *habitus*. Why *ingenium*? For Descartes, *ingenium* includes more than the intellect because the intellect is not the only faculty that plays a role in Cartesian science. It is the entire *vis cognoscens*, which includes the intellect, the imagination, and even the senses insofar as they are subject to rational control, that must be deployed in solutions to problems in the sciences.[27]

How does *ingenium* acquire the Cartesian scientific *habitus*? In a word: in degrees by practice. In *Rules*, Descartes frequently employs the verbs *assuesco*, which means "to use or accustom oneself to something, to habituate,"[28] and *exerceo*, which means "practice, training, exercise." *Assuesco* refers, not to *habitus* as an already constituted mode inhering in the mind, but rather to the process of habituation by means of prescribed acts, and as we have seen in Chapter 1, *habitus* are acquired by acts. He also employs verbs like *excolo*, which refers more specifically to the *cultivation* and, therefore, *perfection* of the mind via practice in the method.[29] To be sure, the operations of the method—intuition, deduction, and enumeration—are natural operations of the human mind. The method cannot teach one how to perform these operations: "The method cannot go so far as to teach us how to perform the actual operations of intuition and deduction, since these are the simplest of all and quite basic. If our intellect were not already able to perform them, it would not comprehend any of the rules of the method, however easy they might be" (AT 10:372, CSM 1:16). Nevertheless, these operations do not produce science unless one has mastered their coordinated deployment in solutions to particular problems: we must "make our employment of intuition and deduction more skillful [*industria possimus aptiores reddi ad illas exercendas*] and at the same time […] cultivate two special mental faculties [*duas praecipuas ingenij facultates excolere*]: viz., perspicacity [*perspicacitatem*] in the distinct intuition of particular things [*res singulas*] and sagacity [*sagacitatem*] in the methodical deduction of one thing from another (AT 10:400, CSM 1:33; translation modified). To solve problems in the sciences, one needs more than the natural operations of the human mind; one needs a method, which

produces the intellectual virtues of perspicacity and sagacity. These two virtues—perspicacity in intuition and sagacity in deduction—prepare the mind to solve problems in the more advanced sciences. In the case of intuition, I must "acquire through practice the ability [*usu capacitatem acquirunt*] to make perfect distinctions between things, however minute and delicate," in which case I become perspicuous (Rule 9, AT 10:401, CSM 1:33). In the case of deduction, I must "grow accustomed" (*assuescamus*) to solving problems by reducing them to their simplest component parts, so that I may methodically deduce the solution, in which case I acquire the virtue of sagacity (Rule 10, AT 10:405, CSM 1:36). One must acquire the Cartesian scientific *habitus* in order to master the coordinated deployment of the operations of the method in solutions to particular problems, and to do so more and more effectively, no matter how complex the problem may be.[30]

In *Rules*, *ingenium* is the subject and operator of the method, and it acquires the Cartesian scientific *habitus* via practice. This practice produces an enduring, stable quality or "durable mode" of operation in *ingenium*, and so becomes an authentic *habitus*. As we have seen in Chapter 1, according to Aristotle and the scholastics, ontologically *habitus* are qualities that are stable and difficult to change. There are no qualities in Descartes's ontology, neither in *Rules* nor elsewhere. In Descartes's canonical statement of his ontology in *Principles* I. 60–2, there are only substances, attributes (principal and common), and modes (see AT 8A:28–30, CSM 1:213–15). Where do *habitus* fit into this ontology? Descartes does not have an explicit ontology in *Rules*, but it is clear that he repudiates Aristotle's categories, including the category of quality (see AT 10:381, CSM 1:21). To understand where *habitus* fit into Descartes's ontology, one must turn to Descartes's mature ontology. In this ontology, scientific *habitus* are modes of mind. In a letter to Regius, Descartes writes:

> Nor do we deny dispositions [*habitus*], but we divide them into two kinds. Some are purely material and depend only on the configuration or other arrangement of the parts. Others are immaterial or spiritual, like the states of faith, grace and so on which theologians talk of; these do not depend on anything bodily, but are spiritual modes inhering in the mind, just as movement and shape are corporeal modes inhering in the body (January 1642, AT 3:503–4, CSMK 3:208).

All *habitus* inhering in the mind—be they naturally acquired or supernaturally infused—are "spiritual modes." Descartes's reference to the

habitus “which the theologians talk of” has extra-theological, properly ontological implications. These *habitus*—faith, grace, etc.—are durable modes, unlike the purely ephemeral mental modes (e.g., the thought of a unicorn). Descartes’s accommodation of such *habitus* indicates, moreover, that there is room in his theory of modes specifically for a distinction between modes that *perfect the attribute of thought* and those that *merely modify it without perfecting it*. In this sense, the Cartesian scientific *habitus* is not merely a mode; it is a durable spiritual mode that *perfects* the principal attribute (operation) of the mind.

Given that the scholastic debate about the unity of scientific *habitus* is primarily a debate about the ontology of scientific *habitus* (i.e., whether sciences are only one simple *habitus* in the intellect or composed of many really distinct *habitus*), one may well wonder how Descartes’s ontology of scientific *habitus* places him in this debate. Descartes never addresses this question. Nevertheless, one can argue that insofar as scientific *habitus* are modes of the same substance, and modes are not really distinct from one another nor from the substance in which they inhere, at most there can only be many modally distinct scientific *habitus* in the mind. There cannot be any really distinct scientific *habitus* in the mind, since the real distinction only pertains to distinctions between substances. Furthermore, as I will show in more detail in Chapter 3, the Cartesian scientific *habitus* is based entirely on one operation: intuition. All other operations are either composed of intuitions (deduction) or prepare problems such that they may be solved by means of intuition (enumeration). Even the intellectual virtues of perspicacity and sagacity are, at best, modes of modes, since they only perfect intuition, which is an operation (mode) of thought. Intuition, moreover, is an entirely simple operation whose perfection can only come in degrees. Thus, insofar as the Cartesian scientific *habitus* only perfects and maximally extends intuition to every knowable object, the Cartesian scientific *habitus* remains entirely simple. Descartes can argue in this way because the Cartesian scientific *habitus* is not identical to any proposition or demonstration; the only reason why Scotus, Ockham, and Suárez argue that sciences are composed of many *habitus* is that they *identify* each such *habitus* with a principle or demonstration that exists in the intellect. Descartes’s position in the debate is closest (but not identical) to that of Aquinas: the Cartesian scientific *habitus* is simple, and is increased by degrees, not parts. The Cartesian scientific *habitus* does not resemble the

propositions it produces, and so it cannot be reduced to or multiplied by them. And as I will show in more detail in Section 2.5 the Cartesian scientific *habitus* also cannot be multiplied by the objects it deals with, since they all share a common property in virtue of which it relates to them in the same way.

2.3 Suspending Aristotle's Ban on Genus-Crossing

In Section 2.2, I argued *contra* Beck and Marion that in *Rules* Descartes does not reject, but rather redefines *sapientia* and *scientia* as intellectual virtues modeled on Aristotelian *phronesis*. What, then, *is* Descartes rejecting when he diagnoses the false analogy between arts and sciences in Rule 1? As we have seen in Chapter 1, in *Posterior Analytics* I. 7 Aristotle bans genus-crossing in the sciences (i.e., employing the principles of one science in order to demonstrate conclusions in another science that has a different genus). Given Aristotle's ontological definition of sciences as *habitus* in the soul, one implication of Aristotle's ban on genus-crossing is that one scientific *habitus* connects to another only when the one relates to the other as subalternating to subalternate (e.g., the subalternation of optics to geometry). In short, Aristotle's ban on genus-crossing in the sciences restricts scientific *habitus* to the genus it deals with plus the sciences that deal with the same genus in some respect. Even in these cases, however, subalternation is not unity; the two sciences remain distinct.[31] Thus, the possibility of one scientific *habitus* that extends to every possible object of science is positively ruled out from the very beginning. As we have seen in Section 2.1 above, even Aristotelian *sapientia* does not unify the sciences; the principles, conclusions, and objects of each science remain distinct even when one has Aristotelian *sapientia*. The unifying power of Aristotelian *sapientia* is, therefore, aggressively limited by Aristotle's ban on genus-crossing. This is why Beck and Marion (mistakenly, in my view) argue that Descartes abandons scientific *habitus* altogether in *Rules*. It seems to me, however, that it is the ban on genus-crossing that is Descartes's primary target in Rule 1, not scientific *habitus*. Indeed, the concept of scientific *habitus* does not depend on Aristotle's ban on genus-crossing; Descartes can suspend the latter while retaining the former. This makes perfect sense,

since Descartes conceives science in *Rules* as a problem-solving ability that extends to every possible object of science. The restriction of a scientific *habitus* to one and only one object, which Descartes describes in Rule 1 as a needless restriction on our mental powers, is not imposed by the definition of scientific *habitus* as such, but rather by the logical requirements of Aristotle's theory of demonstration in *Posterior Analytics* and the ontology these requirements presuppose. By suspending Aristotle's ban on genus-crossing, Descartes can subtract the concept of scientific *habitus* from the scope restrictions imposed by Aristotle's ban on genus-crossing and, therefore, assert that the sciences are an interconnected totality. Thus, in Rule 1, Descartes is not throwing out the baby with the bathwater; he does not deny that wisdom and science are *habitus*, but rather retains and transforms these *habitus* by subtracting them from the broader theory of science in which they had hitherto been embedded.

2.4 Supertranscendental Extrinsic Denomination: The Simple Natures

While Descartes's suspension of Aristotle's ban on genus-crossing may open the door to the unity of the sciences, it does not by itself unify the sciences. Descartes needs a theory of the object of science in order to demonstrate that one scientific *habitus* can indeed extend to every possible object of science. As we have seen in Sections 2.1–2.3 above, Descartes does not distinguish scientific *habitus* by object in *Rules*. Clearly, this does not mean that the sciences have no objects, but only that the Cartesian scientific *habitus* transcends differences between these objects. How can this be done? Descartes can only transcend differences between the objects of the sciences by demonstrating that these objects share a common property. As we will see in more detail below, Descartes transcends differences between the objects of the sciences by bracketing their real essences (as defined by some specification of Aristotle's categories) and reducing them to what can be intuited by the intellect alone. Whatever cannot be intuited by the intellect is simply excluded from Descartes's ontology of the objects of science in *Rules*.

The objects of the sciences reduce to what Descartes terms "simple natures" in *Rules*. What are the simple natures? Simple natures are not propositions, but rather notions of things (*simplicium rerum notiones*, AT 10:417, CSM 1:43) that are "so clearly and distinctly [known] that they cannot be divided by the mind into others which are more distinctly known" (AT 10:418, CSM 1:44). The simple natures are, as it were, the indivisible epistemic "atoms of evidence"[32] known via intuition; all other notions and propositions are composed of simple natures. Simple natures are, therefore, the principal objects of intuition. They divide into three classes: intellectual (e.g., knowledge, doubt, ignorance, volition, etc.); material (e.g., extension, shape, motion, etc.); and common (e.g., existence, unity, duration, as well as common notions "whose self-evidence is the basis for all the rational inferences we make," such as "Things that are the same as a third thing are the same as each other," AT 10:419, CSM 1:45). The intellectual simple natures define the essence of mind (one of the objects of Descartes's metaphysics) and the material simple natures define the essence of body (the object of Descartes's mathematics and natural philosophy).[33]

Descartes's theory of simple natures plays an enormously important role in his method.[34] First, the simple natures "are self-evident and never contain any falsity" (AT 10:420, CSM 1:45), and there is nothing in them "beyond what we intuit or reach in our thinking" (ibid.). Second, "it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another" (AT 10:422, CSM 1:46). Third, "the whole of human knowledge consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things" (AT 10: 427, CSM 1: 49). The theory of simple natures effectively ensures the unrestricted scope of intuition and deduction vis-à-vis any and all objects of science, from the simplest to the most complex.

Descartes's theory of simple natures is based on the principle of "extrinsic knowability." Let me explain what this means. Many early modern scholastics, including Suárez, distinguish between the intrinsic knowability and the extrinsic knowability of a thing.[35] A thing is intrinsically knowable when it has a real essence that can be known by the intellect. Only real beings (possible or actual) can be intrinsically known, because only real beings have real (non-contradictory) essences and as such

are apt to exist.[36] A thing is extrinsically knowable, however, when the denomination "knowable" only means that the thing can be an object of an intellectual act, whether or not it has a real essence and so is apt to exist. Beings of reason (e.g., negations, such as not-blue; privations, such as blindness; and impossible beings, such as chimeras or goat stags) are only extrinsically knowable, since they have no being beyond the intellectual acts whereby they are known. Their entire being consists in their being known, whereas real beings can actually exist. All beings—both real beings and beings of reason—can be objects of the intellect, and so in this sense all beings are extrinsically knowable. Real beings, however, in addition to being extrinsically knowable, are also intrinsically knowable by their real essence.[37] In early modern scholastic metaphysics, extrinsic knowability is not really regarded as knowledge in the proper sense; when a thing is a real being, then it is intrinsically knowable by its real essence. Extrinsic knowability is only introduced in order to explain the type of epistemic access the intellect has to beings of reason, i.e., how beings of reason become objects of the intellect despite the fact that they cannot have real being.

In *Rules*, Descartes makes a monumental decision: *he prioritizes the extrinsic knowability of real beings over their intrinsic knowability*. All things are intuited, not as they are in reality according to their real essences, but rather as they are relative to the intellect alone. Consider the material simple natures. In reality (*a parte rei*, AT 10:418, CSM 1:44), according to Aristotelian ontologies any material substance has per se hylomorphic unity, and so is not an accidental composition but essentially "one single and simple entity" (AT 10:418, CSM 1:44). "*Yet with respect to our intellect* [*respectu vero intellectus nostri*]," Descartes writes, "we call it a composite made up of these three natures [extension, shape, and motion], because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject" (ibid.). All things are composed of simple natures only in relation to the human intellect, not as they are in reality according to their real essences. A body is extrinsically denominated as a "composite" only "with respect to our intellect." A body is intrinsically denominated as "one single and simple entity" "in reality," beyond its relation to the intellect. Descartes does not mention "extrinsic knowability" in *Rules*, but it is clear that the theory of simple natures rests on the relation all things have to the intellect

alone, and that such a relation is a purely extrinsic denomination whose content—the thesis that all things are composed exclusively of simple natures—has no foundation *a parte rei*. The distinction between intrinsic and extrinsic knowability in *Rules* simply *is* the distinction between things as they are in reality and *those very same things* as they are with respect to our intellect. For Aristotelians, prioritizing extrinsic knowability as Descartes does in *Rules* would have made absolutely no sense. Why settle for extrinsic knowability when one can have knowledge of a real essence? For Descartes, by contrast, *extrinsic knowability is the only way to ensure that the objects of science, however complex, are reducible to simple natures, which are completely subject to the principal operation of the method: intuition*.[38]

Historians of late scholastic metaphysics have argued that early modern scholastic metaphysics takes a decisive turn away from "being qua being" toward the "knowable" or the "intelligible" as the proper object of metaphysics.[39] They regard this turn as radical because earlier scholastics, such as Aquinas, Henry of Ghent, Scotus, and Suárez regarded the "knowable" as too inclusive an object. According to Suárez, metaphysics only has real being as its object (*ens in quantum ens reale*), while the "knowable" and the "intelligible" in general embraces both real beings and beings of reason. Even non-beings are extrinsically knowable or intelligible in the sense that they can be objects of the intellect. "Knowability" and "intelligibility" are "supertranscendentals" because, while transcendentals such as "one,""good," and "true" embrace all and only real beings in any category, supertranscendentals embrace both real beings and beings of reason. Supertranscendentals transcend even the transcendentals. Thus, the turn in early modern metaphysics consists in its becoming a supertranscendental science of anything whatever (*aliquid*, which embraces both real beings and beings of reason), whereas before metaphysics remained a transcendental science of being qua real being.[40]

Descartes's theory of simple natures in *Rules* should definitely be understood as a decisive moment in this history. However, it is not clear that his contribution to this history has been properly appreciated. Descartes's contribution consists, not in the priority assigned to knowability in his theory of the object of science in *Rules*, but rather in the fact that he is the first to have prioritized the *extrinsic knowability* of *real beings*. It is real beings (e.g., real bodies), *not* beings of reason, that are known according to

the simple natures, even if these beings are not known according to their real essences (Descartes does not deny that they have such essences in *Rules*; he simply disregards them as unavailable to intuition). This distinguishes Descartes from all late scholastics who, while they define the object of metaphysics much more broadly than previous scholastics, saw no reason whatever to bracket the intrinsic knowability of real beings. Even if these scholastics regard the object of metaphysics as the knowable or intelligible, this is only because they regarded these concepts as the most common concepts under which anything whatever (*aliquid*) falls, not because they set out to bracket the intrinsic knowability of real beings and prioritize their extrinsic knowability. As I mentioned above, any such prioritization of extrinsic knowability would have seemed absurd to them. To prioritize knowability or intelligibility as the object of metaphysics and include beings of reason in the science of metaphysics is one thing, but to prioritize the extrinsic knowability of real beings—i.e., to regard them as knowable *only* extrinsically, as beings of reason are—is quite another.[41]

Indeed, in *Disputationes metaphysicae* Suárez explicitly *rejects* extrinsic denomination as a strategy for establishing the unity of *any* science. Insofar as objects are known by irreducibly different formalities (*ratio formalis*), it is meaningless to assert that a science is “one” simply because its object is extrinsically denominated as “knowable” (*scibilis*) in relation to the intellect alone. As Aristotle puts it, that would be equivalent to saying that the object of a science is simply the scientifically knowable.[42] Everything is scientifically knowable in this sense, but clearly no science is individuated by declaring that its object is the scientifically knowable. Objects do not become objects of one science simply because they are the terminus of an intellectual act. As we have seen in Chapter 1, according to Suárez, objects must share a formality in their real being beyond their relation to the intellect in order to be objects of one science. They must be intrinsically knowable *ex parte objecti*, not simply extrinsically knowable *sub esse scibili*.[43] Descartes, by contrast, *openly extrinsically denominates all real beings as knowable in relation to the intellect alone*. The objects of the different sciences are simply replaced by the simple natures, and all things are denominated as “composed” of simple natures only in relation to the intellect. Thus, in *Rules*, the Cartesian scientific *habitus* extends to all objects, not insofar as they exist in reality (*a parte rei*), but rather insofar as they can be intuited by the intellect alone (*respectu intellectus nostri*).

In fact, in *Rules* Descartes does precisely what Suárez accuses Aquinas of doing. Earlier in *Disputationes metaphysicae*, Suárez argues that Aquinas's analogy between the unity of a faculty and the unity of scientific *habitus* proves too much. If one accepts Aquinas's analogy, then just as one simple faculty (the intellect) can extend to all objects, so too can one simple scientific *habitus* extend to all objects. Both Aquinas and Suárez deny that there can be any such scientific *habitus*. Consequently, Suárez rejects the analogy.[44] In Rule 1, however, Descartes argues precisely that universal wisdom (*sapientia*)—the highest intellectual virtue (*habitus*)—extends to all objects, and yet "remains one and the same, however different the subjects to which it is applied, it being no more altered by them than sunlight is by the variety of things it shines on" (AT 10:360, CSM 1:9). It is one and the same *habitus* that extends to all possible objects of science. And now we know why: all things share the extrinsically denominated, supertranscendental property of being knowable (intuitable) by the human intellect. Their real essence is too obscure to be intuitable, and so it must be bracketed entirely. Cartesian *sapientia* does not extend to all objects under any ontological formality or transcendental property. It extends to them under a supertranscendental property more universal—because less ontologically robust—than even the transcendentals themselves. The unity of science in *Rules* is based on supertranscendental extrinsic denomination alone.

2.5 The Unity of Science in *Rules*

But is it not the case that the unity of science in *Rules* consists in the *logical* interconnectedness of the sciences, such that they are all parts of a single deductive chain? This image is certainly strongly suggested by Descartes's language in Rule 1. As we have seen, Descartes describes the sciences as interconnected and interdependent (*inter se esse connexas; inter se conjunctae & a se invicem dependentes*, AT 10:361, CSM 1:10). He also contends that "it is much easier to learn them [the sciences] all together than it is to separate one from the other" (AT 10:361, CSM 1:10). Clearly, it would not be easier to learn the sciences together unless their principles and conclusions were in some sense interconnected and interdependent, such

that the unity of science consists in the *hierarchical logical order which obtains between these principles and conclusions*, closer to what Ockham believed. This certainly seems to mesh well with Descartes's conception of the unity of science in *Principles*, where metaphysics provides the principles, and the remaining sciences are deduced from these principles.[45] Why then bother with *habitus* and focus on the fact that it extends to every possible object of science?

The fact is that it is not clear how much propositional unity Descartes ultimately requires or can even achieve in *Rules*. On the one hand, Rule 1 seems to suggest that there is a propositional order in which every scientific principle and conclusion has its assigned place. But Descartes never explicitly spells things out in quite this way, and the remainder of the treatise does not seem to yield even the possibility of such propositional unity. Nowhere in *Rules* does Descartes insist that there is one science whose principles have priority over the principles and conclusions of all other sciences. It is not clear that there can even be a *prima philosophia* in *Rules*; Descartes does not determine the relation between metaphysics and physics in *Rules*. In fact, *metaphysics and physics constitute irreducibly diverse propositional series*. Descartes does not assert that the intellectual simple natures (the objects of metaphysics) are higher in epistemic rank than the material simple natures (the objects of physics). At most, hierarchical order only obtains between the intuition of the material simple natures (extension, shape, and motion) and the more specialized branches of natural philosophy. But the logical order that obtains between the propositions of natural philosophy remains local; the principles of physics do not in turn depend on the principles of metaphysics, but rather on the authority of intuition alone. Beyond the local logical order that obtains between sciences that deal with the same class of simple natures, Descartes does not assert that the principles and conclusions of every science can be deduced from the principles of *one* science. There is no "system" of science in *Rules*, if by "system" one means the hierarchically ordered deductive chain of the totality of scientific propositions. Consequently, in *Rules* there can be no tree of philosophy, in which metaphysics is the roots, physics the trunk, and mechanics, morals, and medicine the branches (*Principles*, AT 10:14, CSM 1:186). Descartes simply had not yet expressly decided in favor of such an arborescent conception of the unity of science.[46] In *Rules* "interdependence" can mean only that those sciences that deal with the

same class of simple natures exhibit hierarchical logical order, not that all sciences depend on one science. Descartes does not assign *any* science the role he would later assign to metaphysics.

In *Rules*, the unity of science consists, not in the existence of a foundational science from which all other sciences may be deduced, but rather in the fact that the objects of the sciences (the simple natures) share a common property—intuitability—and the fact that there is one scientific *habitus* based on intuition that extends to them all. Descartes insists, not on the existence of an hierarchically ordered "system" of science (as he later would in *Principles*), but rather on the fact that "the whole of human knowledge consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things" (AT 10:427, CSM 1:49). The unity of science consists in the fact that there is one *habitus*—one problem-solving ability—that extends to the objects, principles, and conclusions of every science, however logically interconnected (or not) these sciences may themselves be. This is not to deny that there is indeed a high degree of logical interconnectedness among the branches of natural philosophy (physics, meteorology, optics, physiology, biology, and medicine), but only to deny that the principles of physics are deduced from the principles of metaphysics (or any other science). In short, the unity of science in *Rules* is habitual, not propositional or logical.

2.6 Rule 1 in the Cambridge Manuscript

In the Cambridge Manuscript, the entire first paragraph of Rule 1_{AT} as well as the discussion of simple natures in Rules 8_{AT} and 12_{AT} are absent.[47] Descartes does not diagnose the false analogy between arts and sciences. He does insist that the sciences are interconnected, and that it is easier to learn them together than to study each one separately. But he does not assert that the "sciences as a whole are nothing other than human wisdom, which always remains one and the same, however different the subjects to which it is applied, it being no more altered by them than sunlight is by the variety of the things it shines on" (AT 10:360 CSM 1:9). In short, he does not assert that the unity of science consists in one scientific *habitus* that extends to all

possible objects of science. The simplest and, to my mind, best explanation is that Descartes had not yet solved the problem of the limits of knowledge in Rules 8_{CM} and 12_{CM}, and so did not yet have a theory of simple natures enabling him to assert that one *habitus* extends to every possible object of science. Without a theory of simple natures, Descartes could not confidently assert the thesis that the sciences should not be distinguished by object in Rule 1, nor could he assert the thesis that there is one *habitus* that extends to all possible objects of science. He could not diagnose the false analogy between the arts and the sciences in Rule 1. To diagnose the false analogy, he would need to assert that sciences should not be distinguished by their objects, and that assertion depends on the theory of simple natures. It seems to me that Descartes could only have written the first paragraph of Rule 1_{AT} *after* he had developed his theory of simple natures in Rules 8 and 12, which he had not yet done in the Cambridge manuscript.[48]

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0003

[1] Russell 2014, 208.

[2] Aristotle 1984, 2:1800 (*Nicomachean Ethics* VI 5.1140^{a}25–30).

[3] Russell 2014, 213.

[4] Aristotle 1984, 2:1808 (*Nicomachean Ethics* VI 13.1145^{a}1–2).

[5] Readers eager to see how this works in practice may turn to Chapters 3, Sections 3.4.1 and 3.7; Chapter 5; and Chapter 10.

[6] See Beck 1952, 14–16 and Marion 1975, 25–30.

[7] Beck 1952, 15.

[8] Marion 1975, 26–9.

[9] Aquinas 1965, 166–7 (*Expositio super librum Boethii De trinitate* q. 5, art. 1); trans. Aquinas 1986, 15–16.

[10] For Aristotle and Aquinas, *phronesis/prudentia* is also an intellectual virtue, because it resides in the practical intellect, whose deliberations determine the will to action. See Aristotle 1984, 2:1800–1 (*Nicomachean Ethics* VI.5) and Aquinas 1882–, 6:374 (*ST* I–II, q. 58, art. 3); trans. Aquinas 1945, 2:444.

[11] See Aquinas 1882–, 6:419 (*ST* I–II, q. 65, art. 1); trans. Aquinas 1945, 2:497.

[12] See Armogathe and Marion 1976, 9, 74.

[13] See AT 10:372, CSM 1:16; AT 10:375, CSM 1:18; AT 10:384, CSM 1:23; AT 10:387, CSM 1:24; AT 10:408, CSM 1:38; AT 10:429, CSM 1:51. On the history of *ars inveniendi*, see Kisiel 1980. On *ars inveniendi* in the sixteenth century (especially in Ramus), see Bruyère 1984 and

Robinet 1996. On Descartes's concept of *ars inveniendi*, see Robinet 1999 and the remarks in Beck 1952, 171; Schouls 2000, 82; and Rabouin 2009, 214. For salient criticisms of Robinet's attempted *rapprochement* between Ramus and Descartes on *ars inveniendi*, see de Buzon 2013, 34–58.

[14] On Descartes's mathematics as a "disposition of the mind which allows one to solve problems and thus discover mathematical truths," see Mancosu 1996, 107.

[15] See, e.g., Rule 6: "…on the basis of our knowledge of the most simple and primary things we can make many discoveries, even in other disciplines, through careful reflection and discriminating inquiry" (AT 10:387, CSM 1:24).

[16] All citations are from Goclenius 1613, 241. I largely follow Sepper's translations in Sepper 1996, 87–91.

[17] Goclenius's reference is to Cicero's *De oratore* I.25. Cicero writes that while *ingenium* is a natural endowment (*dona naturae*), and so cannot "be derived from art," it may be "polished" (*limare*) or perfected by art (ibid.). Cicero frequently emphasizes the role played by practice or exercise (*exercitatio*), which perfects *ingenium*. See Cicero 1942, 80–1. Kambouchner 2009 underlines the importance of Cicero's *De oratore* in Descartes's theory of *ingenium*. For an excellent survey of early modern uses of the term "*ingenium*" in Latin, see Garrod et al. 2018, 19–53. For recent discussions of *ingenium* in Descartes, see Garrod and Marr 2020.

[18] See Aristotle 1984, 2:1759.

[19] See AT 10:377, CSM 1:19. On *mathesis universalis*, see Chapter 4.

[20] See Sepper 1996, 87–91 and Agostini 2020.

[21] Conimbricenses 1593, 18 (*De memoria et reminiscentia* cap. 10), cited in Agostini 2020, 152.

[22] See Suárez 1856–1878, 25:20 (*DM*, disp. 1, sec. 2.24), cited in Agostini 2020, 143.

[23] Goclenius 1613, 242. See also Conimbricenses 1597, lib. 2, q. 4, 457–64, cited in Agostini 2020, 141, n.11.

[24] Ibid. At La Flèche, report cards evaluated students's *ingenium* by the same vocabulary. See Sepper 1996, 91 and Rochemonteix 1889, 4:205–6, 348–50. See also Fowler 1999, 161–87 and Ariew 2020.

[25] See Suárez 1856–1878, 25:46 (*DM*, disp. 1, sec. 5.32), cited in Agostini 2020, 144, n. 20. The relation between *ingenium* and the *habitus* of first principles becomes even more explicit in Francisco Toledo's (Toletus) *Introductio in dialecticam Aristotelis* (1574), where Toledo argues that logical principles known by the natural light (*naturali lumine*) are such that the human *ingenium* cannot but assent to them. See Toletus 1615, lib. 4, cap. 5, 59a, cited in Agostini 2020, 144, n. 21.

[26] For a complete enumeration of occurrences of *ingenium* in *Rules*, see Armogathe and Marion 1976, 60. In Rule 12, Descartes defines *ingenium* in a more technical, narrower sense to designate *vis cognoscens* when the latter "form[s] new ideas" in the phantasy (AT 10:416, CSM 1:42). For discussion, see Chapter 6, Section 6.4.

[27] This becomes clearer in Chapter 6 and Chapters 9–10, where I discuss Descartes's theory of the faculties in *Rules* and show how it functions in solutions to problems in mathematics and optics, where both the intellect and the imagination must be employed.

[28] Descartes also uses the negative *desuescere*, which Cottingham appropriately translates as "get out of the habit of […]" (AT 10:375, 20, CSM 1:18). See also Rule 4, AT 10:371, CSM 1:16, where Descartes compares those who investigate the truth haphazardly to those who are poorly habituated, "who are accustomed [*assuescunt*] to walking in the dark […]."

[29] For a complete enumeration of these occurrences, see Armogathe and Marion 1976, 10, 38–9.

[30] I discuss these intellectual virtues and how they are acquired in more detail in Chapter 3, Section 3.6.

[31] See Chapter 1, Section 1.5.

[32] Hamelin 1921, 86.

[33] For more discussion, see Chapter 7.

[34] For more discussion, see Chapters 7–11.

[35] See Doyle 1984, 123, n. 10 and Doyle in Suárez 1995, 24. The distinction remains implicit in Suárez, but becomes explicit in Carleton (1591–1666) and Wietrowsky (1660–1737). See Carleton 1649, 70 (*Philosophia universa*, *Logica*, disp. 13, sec. 6, n. 2, cited in Doyle 2012, 77, n. 80) and Wietrowsky 1697 (discussed in Doyle 2012, 85).

[36] Both actual and possible beings have real essences, which render them "apt" to exist. Beings of reason do not have real essences and so are not apt to exist: "For an essence, said without qualification, entails a relation to being, or a capacity for that. But a being of reason is such that being cannot belong to it" (Suárez 1856–1878, 26:1017–18 (*DM* disp. 54, sec. 1.9); trans. Suárez 1995, 65–6). Suárez denies that beings of reason are objects of metaphysics because they do not share a common concept of being with real beings.

[37] See Doyle 2012, 19–69.

[38] Extrinsic knowability in *Rules* clearly comes at a cost: intuition has no ontological foundation in the things themselves. In Chapter 11, I argue that this partly explains Descartes's turn to metaphysics immediately after *Rules* in 1629.

[39] See, e.g., Courtine 1990; Doyle 1990, 1997, 2012; Aertsen 2012. These scholars disagree about which late scholastic is principally responsible for the turn toward the "knowable" or the "intelligible" as the proper object of metaphysics. Courtine reckons that Suárez initiated the turn, while Aertsen contends that Suárez remained committed to being qua real being (*ens in quantum ens reale*) as the proper object of metaphysics, and that the turn occurred after Suárez. I am more inclined to accept Aertsen's interpretation, but in my case what really matters is that the turn toward the knowable or the intelligible as the proper object of metaphysics actually occurred, not who initiated the turn. The authors cited (especially Doyle and Aertsen) provide ample evidence that, beginning in the seventeenth century, early modern scholastic metaphysics becomes a supertranscendental science of the knowable or the intelligible.

[40] See Aertsen 2012, 635–57.

[41] Courtine 1990, 489 recognizes that Descartes embraces what he (Courtine) regards as a late scholastic prioritization of knowability or intelligibility in *Rules*, but Courtine does not seem to recognize that what really distinguishes Descartes from the scholastics who came before and after him in *Rules* is that Descartes *prioritizes the extrinsic knowability of real beings over intrinsic knowability.* To my knowledge, no scholastic before or after Descartes suspends the intrinsic knowability of real beings according to their real essences. Even if they recognize the *possibility* of the extrinsic knowability of real beings, they never *prioritize* it over the intrinsic knowability of real beings. For detailed discussion of extrinsic knowability in early modern scholastic metaphysics both before and after Descartes, see Doyle 2012. In none of the authors Doyle discusses (Suárez; Timpler; Carleton; Wietrowsky) does one find a prioritization of the extrinsic knowability of real beings over the intrinsic knowability of real beings.

[42] See Aristotle 1984, 2:1613 (*Metaphysics* V.15 1021^a31–1021^b2).

[43] Suárez 1856–1878, 26:713–14 (*DM* disp. 44, sec. 11.64). See also Suárez 1856–1878, 26:232 (*DM* disp. 31, sec. 2.10) and Doyle 1991, 327–8.

[44] Suárez 1856–1878, 26:712 (*DM* disp. 44, sec. 11.58). See also Doyle 1991, 325.

[45] For an interpretation along these lines, see Beck 1952, 14. Similarly, Schuster 2013, 251 argues that in *Rules* all “rationally obtainable truths subsist in a network of deductive linkages,” which he describes as “Descartes’s latticework vision of the unity of the sciences.” Schuster cites Gilson in Descartes 1987a, 158–60, who also sees in *Rules* “*l’idée de l’unité systématique des sciences*,” which he states is identical in both *Rules* and *Principles*. McRae 1957, 36 describes the unity of science in both *Rules* and *Discourse* as “a single deductive system,” as does Garber 2001, 48 and 1992, 49–50. This is perhaps true after Descartes’s turn to metaphysics in 1629, but not in *Rules*.

[46] I discuss the reasons behind Descartes’s turn to an arboresecent conception of the unity of science after *Rules* in more detail in Chapter 11, Sections 11.3–11.4.

[47] See *CM* fo.2^{r}.

[48] For more discussion, see Chapter 8, Section 8.3.

PART II

THE OPERATIONS AND CULTURE OF THE METHOD

3

The Operations of the Method

Intuition, Deduction, and Enumeration

3.1 The Principle of Proportionality

In Part I, I argued that Descartes decisively transforms the scholastic debate about the unity of science by establishing one scientific *habitus* that extends to every possible object of science. How this *habitus* actually works, however, remains to be seen. Clearly, one cannot understand how the Cartesian scientific *habitus* works unless one understands the operations it deploys. There are three such operations: intuition, deduction, and enumeration. In this chapter, I reconstruct these operations in detail and discuss how they are deployed in solutions to particular problems in the more advanced sciences (Sections 3.2–3.4.3). My purpose is not to systematically reconstruct solutions to these problems (I do that in Chapters 5–7 and Chapters 9–10), but rather to introduce Descartes's taxonomy of problems (perfectly and imperfectly understood problems) (Section 3.5) and show that the method must be adapted to the problem at hand, on the model of Aristotelian *phronesis* discussed in Chapter 2 (Section 2.1). I also discuss how the human *ingenium* acquires the intellectual virtues that perfect intuition and deduction—perspicacity and sagacity, respectively—in solutions to the simplest problems in the arts and sciences (Section 3.6). I conclude by discussing Descartes's understanding of the order of operations in solutions to problems (Section 3.7). In the remainder of this section, I lay down the principle behind Descartes's distinction between the operations of the method. This principle, which I term the "principle of proportionality,"

also determines how problems must be prepared in order to be solved by the method.

As I mentioned in Chapter 2, Section 2.2, the operations of the method —intuition, deduction, and enumeration—are natural operations of the human mind. They cannot be acquired, and the method cannot teach one how to perform them (see AT 10:372, CSM 1:16). These operations do not produce science unless one has mastered their coordinated deployment in solutions to particular problems. To be sure, “everyone can mentally intuit that he exists, that he is thinking, that a triangle is bounded by just three lines, and a sphere by a single surface, and the like” without a method (Rule 3, AT 10:368, CSM 1:14), but these are what Descartes terms “simple propositions,” not “problems.” Everything that can be known is either a simple proposition or a problem: “[We] divide everything that can be known into simple propositions [*propositiones simplices*] and problems [*quaestiones*]” (Rule 12, AT 10: 428, CSM 1: 50). “[S]imple propositions,” Descartes writes, “must occur to us spontaneously; they cannot be sought out” (Rule 12, AT 10:428–9, CSM 1:50). One does not need a method in order to intuit simple propositions. Similarly, one does not need a method in simple deductions; whenever it is “a matter of drawing a single deduction from a single, simple fact […] that can be done without the aid of rules” (Rule 12, AT 10:429, CSM 1:51). Problems are more complex; in the case of problems “it is […] a matter of deriving a single fact which depends on many interconnected facts, and of doing this in such a methodical way that no greater intellectual capacity is required than is needed for the simplest inference” (Rule 12, AT 10:429, CSM 1:51). To solve problems in the sciences, one needs more than the natural operations of the human mind; one needs a method. One must acquire the Cartesian scientific *habitus*.

The principle that determines both Descartes’s distinction between the operations of the method and how problems must be prepared in order to be solved by the method is the principle of proportionality between the natural operations of the human mind and the problems solved by their means.[1] The principle of proportionality requires that problems be reducible to a form in which the natural operations of the human mind can effectively solve them. The simple natures play a central role in securing this principle in any given case. As we have seen in Chapter 2, Section 2.4, in *Rules* Descartes extrinsically denominates all things relative to the intellect as either simple natures or composed of simple natures. Since (1) all things are

either simple natures or composed of simple natures, and (2) all simple natures can be intuited by the intellect, it follows that (3) all things can be objects of intuition and deduction. Consequently, (4) every problem can be solved by means of these operations and objects. The theory of simple natures eliminates the possibility of there being any problems that the operations of the method are not equipped to solve. The principle of proportionality ensures that every object and, therefore, every problem is without exception "totally adapted to human cognitive capacities" (AT 10:404, CSM 1:35). It eliminates the possibility of any disproportion between operations, objects, and problems.

Based on the principle of proportionality, each operation of the method is defined by the complexity of the object(s) it can accommodate. Intuition is the simplest act of the human mind, and the objects of intuition (the simple natures) are absolutely simple. Intuition, however, cannot accommodate more than a small number of simple natures in any one act. Deduction is a more complex operation, since it is composed of a series of intuitions. Deduction is needed whenever the solution to a problem requires combining many simple natures, whose relations to one another must nevertheless be intuited.[2] Finally, enumeration is required when the problem is too complex to be solved by means of either intuition or deduction alone. Enumeration ensures that no problem is so complex that it escapes solution by the human mind: "For nothing can be so many-sided or diffuse that it cannot be encompassed within definite limits or arranged under a few headings by means of the method of enumeration" (AT 10:398, CSM 1:31). However complex a problem may be, enumeration can always "cut it down to size" so that it may be solved by means of a finite number of operations. I should state at the outset that while I discuss these operations according to their relative simplicity or complexity, *the order of simplicity and complexity does not necessarily reflect the order of operations in solutions to problems*. In many (perhaps most) cases, enumeration comes first in the order of operations and organizes the problem such that it may then be solved by intuition and deduction whenever appropriate. This becomes clearer in Sections 3.4–3.4.3 and 3.7.

3.2 *Facie ad faciem*: Intuition

In Latin, the verb *intueor* is closely related to the verb *videor*, and means "to look upon; to regard; look closely at; gaze at," and in a more extended sense "to contemplate; consider; give attention to," and even "to admire; wonder at." Augustine employed *intueri* and *intuitus* to describe the act whereby the mind "sees" or apprehends eternal truths.[3] In the thirteenth century, the Franciscan Matthew of Aquasparta (c. 1237–1302) employed *intueri* and *intuitus* to describe the cognition of singular things present to the mind (e.g., this tree standing before me), and this sense of intuition determined how Scotus and Ockham would later employ the term.[4] Scotus and Ockham regarded the intuitive cognition of singular things present to the mind (typically but not always by the intellect via the senses) as a simple apprehension, which is *immediate* (requires no phantasm), *self-evident* (non-inferential), and *always true*. Drawing on St. Paul's First Epistle to the Corinthians (where Paul describes the beatific vision of God after death),[5] the Conimbricenses vividly describe intuitive cognition as cognition "face to face" (*facie ad faciem*). As I will show in more detail in Section 3.2.1, Descartes's theory of intuition is very different from the theories of intuition employed by his predecessors. Nevertheless, even in Descartes's case intuition remains an act in which an object (in his case, a simple nature) is seen "face to face" before the mind in immediate self-evidence.

Descartes defines intuition in Rule 3 as follows:

> By "intuition" [*intuitum*] I do not mean the fluctuating testimony of the senses or the deceptive judgment of the imagination as it badly composes [*male componentis imaginationis judicium fallax*], but the conception of a clear and attentive mind [*mentis purae et attentae… conceptum*], which is so easy and distinct [*tam facilem distinctumque*] that there can be no room for doubt about what we are understanding [*intelligimus*]. Alternatively, and this comes to the same thing, intuition is the indubitable conception of a clear and attentive mind born solely from the light of reason [*mentis purae et attentae non dubium conceptum, qui a sola rationis luce nascitur*]. […] Thus, everyone can mentally intuit that he exists, that he is thinking, that a triangle is bounded by just three lines, and a sphere by a single surface, and the like (AT 10:368, CSM 1:14; translation slightly modified).

While Descartes frequently refers to propositions as objects of intuition, he does not regard propositions as the principal objects of intuition.[6] The principal objects of intuition are simple natures, and as we have seen in Chapter 2, Section 2.4, simple natures are not propositions, but rather notions (*simplicium rerum notiones*, AT 10:417, CSM 1:43). It will therefore be best to anchor Descartes's definition of intuition in the theory

of simple natures. As we have seen in Chapter 2, Section 2.4, simple natures are notions that are known "so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known" (AT 10:418, CSM 1:44). Simple natures are the simplest objects of intuition; all other notions and propositions are composed of simple natures, but not vice versa. For example, extension can be exhibited in spatial intuition (in the imagination), where I see a body extended in length, width, and breadth. As a simple nature, extension is notionally basic or primitive. Why? First, because not only are all bodies extended, but no corporeal property (e.g., motion or shape) can exclude extension, while extension can be intuited entirely on its own. Second, because when the intellect divides extension into other notions, these notions are mere abstractions that turn out to *presuppose* extension. Extension cannot be notionally divided into length, width, and breadth, since the length of a body cannot be imagined unless it is extended in space, and the width and depth of a body also cannot be imagined unless they too are extended in space. Consequently, extension is not "composed" of these notions, and so it cannot be defined by them. Indeed, because the simple natures are indivisible, they cannot be defined at all. They can only be intuited.[7]

Descartes's theory of simple natures informs each part of his definition of intuition in Rule 3. Since it is indivisible, a simple nature has no parts. Consequently, "if we have even the slightest grasp of it in our mind [*minimum quid mente attingamus*] […] it must follow that we have complete knowledge of it [*totam illam cognoscere*] […]" (AT 10:420, CSM 1:45). In other words, my conception of a simple nature is (1) always *distinct*; it has one and only one indivisible object, which is sharply separated from every other object.[8] My conception of a simple nature is also (2) always *easy*; simplicity entails ease of conception precisely because there are no parts in a simple nature that escape my conception or that might be confused with one another or that require effort in order to be distinguished from one another. My conception of a simple nature is (3) *attentive* because whenever I conceive a simple nature, I am entirely focused on one indivisible notion, "[devoting my] whole attention to the simplest and easiest of matters" (AT 10:401, CSM 1:33). This contrasts with letting my "thinking be distracted [*distrahunt*, from *distraho*, literally "to pull asunder; tear in pieces; divide"] by many different objects at the same time" (AT 10:401, CSM 1:33; my emphasis). Finally, my conception

of a simple nature is also always (4) *clear*; attentiveness entails clarity because the latter is nothing but the manifest presence of an object to the mind.[9] From these considerations, it is clear that acts of intuition depend principally on the *simplicity of their objects—the simple natures*.[10] It is, therefore, simply not possible to separate act and object in any interpretation of Descartes's theory of intuition in *Rules*.[11]

In Rule 12, Descartes writes that intuitions of the simple natures "are all self-evident and never contain any falsity" (AT 10:420, CSM 1:45). It has been argued that in both *Rules* and other texts Descartes's claim that intuitions never contain any falsity means that intuitions can be neither true nor false, and that Descartes restricts truth and falsity to judgments alone.[12] I see no reason to deny that intuitions are always true, or to claim that truth is restricted to judgment alone in *Rules*. The more natural (and more literal) interpretation of Descartes's claim that intuitions never contain any falsity is that intuitions are always true, while judgments can be either true or false. In this respect, Descartes follows tradition. Like most scholastics (including Suárez and the Conimbricenses), Descartes regards simple apprehensions (in his case, apprehensions of the simple natures in intuition) as antecedent to judgment and as always true. The Conimbricenses very clearly distinguish between simple apprehensions, which are always true, and judgments, which are complex, and which can be either true or false.[13] Simple apprehensions, they argue, are analogous to images of things. If an image fails to represent something in any respect, then in that respect it is *not* an image of *that thing*. Consequently, insofar as they do represent things, images can only represent them *as they are*, such that conformity or adequation between a simple apprehension of the intellect and the thing obtains without fail. Nearly every scholastic denied the possibility of falsity in simple apprehensions. They referred to truth in simple apprehension as *material* truth (which pertains only to simple apprehensions, prior to judgment) and distinguished it from *formal* truth (which pertains only to judgment).[14] In *Rules*, there is no daylight between intuition and truth. Since the intuition of simple natures is self-evident and never contains any falsity, only propositions can be doubted.

3.2.1 Descartes's Interlocutors in the Definition of Intuition in Rule 3

The negative parts of Descartes's definition of intuition are no less important than the positive parts, since they reveal further important differences between Descartes and his scholastic predecessors on scientific *habitus* not previously discussed in Chapter 2, Section 2.1. For most scholastics, intuitive cognition is cognition via the senses, whereby one has knowledge that a singular thing exists. To see Socrates is to intuitively know that he exists. Clearly, this is not how Descartes defines intuition, since he explicitly distinguishes intuition from the "fluctuating testimony of the senses." For the scholastics, *intuitive cognition plays no substantive role in science*, since science is not concerned with the existence of singular things. Science is based rather on what they term "abstractive cognition," or cognition that abstracts from the existence of singular things. Abstractive cognition includes the cognition of universals (e.g., "animal," "rational animal," etc.); the cognition of contingent propositions about non-present singular things; necessary propositions, such as principles (e.g., a definition) and demonstrations, which depend on abstractive cognition of the universals that compose them.[15] For Scotus and Ockham, abstractive cognition is broader in scope than intuitive cognition: the latter only has present and existing singular things as its objects, whereas the former can have singular things that are not present before the senses, universals, and propositions as its objects.[16] This means that the scientific *habitus* acquired when one learns a principle or demonstration are dispositions to have *abstractive*, not *intuitive*, cognitions. The Cartesian scientific *habitus*, by contrast, is *exclusively intuitive without being sensory*. For Descartes, intuition can indeed have singular things as its objects, such as one's existence or intellectual acts. However, intuition is a purely intellectual act; the senses do not deliver intuitions, and the intellect does not have intuitions via the senses. Whereas purely intellectual intuition, while possible, is hardly the norm in scholastic theories of intuitive cognition,[17] in *Rules* intellectual intuition is not only possible, it is the *only* certain and evident cognition there is. The intuition that, according to Descartes's predecessors, human beings cannot have on earth and that God always has—purely intellectual intuition—descends from heaven to earth in *Rules*. Furthermore,

Descartes expands intuition to include the intuition of notions (simple natures), propositions composed of simple natures (both contingent and necessary), and even entire deductions (which are composed of propositions that are themselves composed of simple natures). For Descartes, unlike his predecessors, intuitive cognition is broader in scope than abstractive cognition, which is not really cognition at all. Since everything ultimately reduces to the simple natures, everything can be intuited provided that it does not combine simple natures that cannot be combined (e.g., "My doubt is square shaped"). Intuitive cognition is the *only* type of cognition Descartes recognizes in science. It alone is certain and evident, and abstractive cognition has no place in science at all. For Descartes, everything in science must be "face to face" without exception (see Table 3.1). Descartes completely undermines the scholastic thesis that science is based on abstractive cognition alone.

Table 3.1 Descartes and the scholastics on intuition

Scotus Ockham Suárez	*Cognitio abstractiva*	Universals (e.g., "Animal"); necessary propositions (e.g., "All humans are rational"); contingent propositions (e.g., "Socrates is white," when Socrates is not around).
Conimbricenses	*Cognitio intuitiva*	Present, existing things (e.g., Socrates). Purely intellectual intuition of my own mental acts (Scotus, Ockham, et al.); purely intellectual intuition only possible after death (Aquinas, Suárez, the Conimbricenses, et al.).
Descartes	*Intuitus*	Simple notions (e.g., my own mental acts; extension); necessary and contingent propositions composed of simple natures. Purely intellectual (e.g., "I exist") or intellectual aided by the imagination (e.g., "No shape lacks extension.").

As we have seen, Descartes also distinguishes intuition from the "deceptive judgment of the imagination as it badly composes." What is initially odd about Descartes's claim about the "deceptive judgment of the imagination" is that it is not immediately clear that anybody previously regarded the imagination as a faculty that could judge. Why distinguish one's own concept of intuition from a concept of intuition nobody seems to have embraced? There are, in fact, three components to the thesis Descartes denies here: that the imagination has intuitions; that the imagination makes judgments; and that these judgments are cases of intuition. That the imagination has intuitions is a thesis shared by many scholastics: Scotus,

Ockham, and Suárez all argue that whatever a lower power can do, a higher power can do.[18] Since the senses can have intuitions, and the imagination is a higher power than the senses, the imagination can have intuitions too, as can the intellect. Furthermore, Suárez appears to have maintained that, in addition to being able to have intuitions, the imagination can make judgments. Did he regard such judgments as cases of *intuition*? Yes. The type of judgment Suárez has in mind here is not *discursive*; it does not consist in assenting to or denying a proposition. It is rather what Suárez calls a judgment *in actu exercito*. This is the act whereby a sense knows that it senses in *simple apprehension*. For example, in seeing the color blue, I concomitantly know that I am seeing the color blue. When the internal sense receives sensible species from the five external senses, it receives them precisely as "marked" by the "concomitant self-knowledge" that comes with them.[19] This, it seems to me, explains why Descartes is at pains to insist that by "intuition" he means *neither* the fluctuating testimony of the senses *nor* the deceptive judgment of the imagination as it badly composes.[20] Scotus, Ockham, and Suárez opened the door to intuitive cognitions by the phantasy or imagination—a door that Descartes needed to close.[21]

3.3 *Illatio*: Deduction

As I stated in Section 3.1, Descartes distinguishes each operation of the method by the complexity of the object(s) it can accommodate. Intuition is the simplest act of the human mind, and the objects of intuition (the simple natures) are absolutely simple. Intuition, however, cannot accommodate more than a small number of simple natures in any one act.[22] Deduction is a more complex operation, since it is composed of a series of intuitions. Deduction is needed whenever the solution to a problem requires combining many simple natures, whose relations to one another must themselves be intuited. In Rule 11, Descartes writes that "two things are required for mental intuition: first, the proposition must be clearly and distinctly understood; second, the whole proposition must be understood all at once, not successively [*tota simul et non successive intelligatur*]" (AT 10:407, CSM 1:37; translation slightly modified). I have already discussed

the first requirement in Section 3.2 above. Descartes introduces the second requirement because any proposition that must be understood by a series of successive acts is too complex to be absorbed in a single act of intuition. He is not referring to single propositions that are indefinitely long (like a sentence with no determinable end) but rather to *propositions whose intuition depends on intuiting many other propositions*. Descartes introduces the requirement that intuited propositions be understood all at once in order to distinguish intuition from deduction.

Descartes defines deduction in Rule 3 as the "inference of something as following necessarily [*necessario concluditur*] from some other propositions which are known with certainty […] provided they are inferred from true and known principles through a continuous and uninterrupted movement of thought [*continuum et nullibi interruptum cogitationis motum*] in which each individual proposition is clearly intuited" (AT 10:369, CSM 1:14–15). It is clear from Descartes's definition that he regards deduction as preserving, not only *certainty*, but also *logical necessity*.[23] I explain the necessity requirement below. Before doing so, I must address a potential misunderstanding about the certainty requirement. The necessary relation between any two propositions in a deduction is in principle always the same (necessity does not come in degrees), but the certainty transmitted from the initial intuition to the remainder of the propositions in a deduction may come in degrees: "Because it is simpler, it [intuition] is more certain [*certior*] than deduction" (AT 10:368, CSM 1:14).[24] This is partly due to the difference between evidence and self-evidence in Descartes's theory of deduction in *Rules*. The simplest intuition in a deduction must always be self-evident. The intuition of deduced propositions, however, is not self-evident; their evidence depends on their relation to the propositions from which they have been inferred. Deduction provides (mediate) evidence, while intuition provides (immediate) self-evidence.[25] The mediacy of deduction means that "we are aware of a sort of movement or a sequence" in deduction, but not in intuition, and that "deduction in a sense gets its certainty from memory" (AT 10:370, CSM 1:15). In the simplest cases, both the immediate self-evidence of intuition and the mediate evidence of deduction can enjoy the same degree of certainty. However, the certainty of the remote propositions in a deduction is reduced in proportion to the complexity of the deduction. Reducibility to intuition is a requirement that only simpler deductions can satisfy.

Consequently, while the immediate self-evidence of intuition always enjoys the same degree of certainty, the mediate evidence of deduction can have varying degrees of certainty. When the complexity of the deduction is such that the propositions can only be intuited successively, then the deduction cannot be reduced to intuition. The deduction then becomes irreducibly sequential, and the sequentiality of the deduction means that an auxiliary faculty—memory—must intervene in order supplement intuition. As we will see in Sections 3.4.2 and 3.6, Descartes believes that one can perfect one's capacity to reduce complex deductions to intuition (i.e., one can actually expand the scope of intuition), but however perfect one's capacity to reduce deduction to intuition becomes, there is a limit defined by the complexity of the deduction itself, which in many cases is simply too great for such a reduction to be possible.

What does the necessary connection between two or more propositions in a deduction consist in?[26] To answer this question, it is best to begin with the simplest cases of deduction first (simple propositions) and then transfer what can be learned from these cases to more complex cases (problems).[27] Descartes provides useful examples of the simplest cases of deduction in Rule 12, where he writes that "when we deduce [*deducamus*] that nothing which lacks extension can have a shape," "we intuit that the conjunction of the one with the other is wholly necessary […]" (AT 10:425, CSM 1:48). The "necessary conjunction" is one that I "see" whenever I intuit a shape in my imagination; any shape I imagine will necessarily be extended, and I can intuit, not only the *shape*, but also *the necessary connection between shape and extension* whenever I imagine a shape.[28] The relevant proposition is deduced from the intuition of shape in the imagination, where I see that I cannot intuit a non-extended shape. However, I can intuit shapeless extension. Consequently, I deduce that shape depends on extension, but not vice versa. What I see here is a necessary order between the simple natures. Deduction is the operation in which such order is revealed. This is a clear case in which deduction is both made via intuition and reducible to intuition. Similarly, when "Socrates […] says that he doubts everything, it necessarily follows that he understands at least that he is doubting, and hence that he knows that something can be true or false, etc.; for there is a necessary connection between these facts and the nature of doubt" (AT 10:421, CSM 1:46). Doubt is an intellectual simple nature, and the act of doubt reveals a necessary connection between doubt,

knowing that one doubts, and knowing the difference between truth and falsity.

As can be seen in these two examples, the necessity in Cartesian deductions is based on relations between simple natures, such that *some cannot be intuited independently of others.* There is, moreover, *a hierarchy of epistemic priority or dependency between the simple natures, such that the intuition of some simple natures depends on the intuition of others, but not vice versa.* Relatedly, the necessity in the deductions is "material," not "formal," since the deductions are based on the intuitable *contents* of notions and the *relations that obtain between these contents*, not *formal* relations between propositions in which the relevant contents are not considered. Finally, the "necessary connections" between simple natures are immediately *seen* in intuition *without the intermediary of a middle term*. It would be absurd to require that Socrates lay down the premises, "Anyone that doubts understands that he is doubting" and "I am doubting," in order to deduce "I understand that I am doubting." On the contrary, the premises and conclusions in this syllogism can only be known *because the act of doubt intuitively reveals that doubting, knowing that one doubts, and knowing the difference between truth and falsity are necessarily connected*. In other words, the *premises of the syllogism depend on the intuition*, *not vice versa*. Consequently, the syllogism is *otiose*.[29] In *Rules*, logical forms (which depend on a middle term) play no role in deduction; intuition alone does, since "each individual proposition" in a deduction, and the relation between them, must be "clearly intuited."

The examples of deduction discussed above are so simple that they are wholly reducible to intuition: "For if we have deduced one fact from another immediately, then provided the inference is evident, it already comes under the heading of true intuition" (AT 10:389, CSM 1:26). Moving on to more complex cases (problems), the necessity in deduction is not an order between simple natures, *but rather between simple and composite natures*, such that the composite natures are wholly determined by the simpler natures that compose them. For example, the solution to any problem in mathematics or natural philosophy depends on the material simple nature of extension. However, in order to solve a problem in mathematics, I must intuit, not only extension as such, but also relations between a series of determinate extended magnitudes. Suppose the problem is to determine the value of x and y in the following series of proportionals:

2:6 = *x*:*y* = 162:486. To determine the value of *x* and *y*, I must determine their relations to the other magnitudes in the series (i.e., I must compare them to one another). I see that 6 = 3•2, and proceed to 6•3 = *x* and *y* = 3*x*, yielding *x* = 18 and *y* = 3•18 = 54. The necessity of the deduction consists in the fact that the more complex magnitudes can only be known in relation to the simpler magnitudes (or the relations between the simpler magnitudes), *which the more complex magnitudes contain*. In this series, 54 contains 2•3 = 6 and 6•3 = 18, and 18•3 = 54 is epistemically wholly determined by the other relations in the series because *it can only be known on their basis, and so it necessarily follows from them by the same measure in which they follow from one another*.[30] This is why Descartes repeatedly insists that no problem can be solved until the relevant knowns have been enumerated and ordered according to their relative simplicity and complexity (see Section 3.4).

As we will see in more detail in Chapter 4, Descartes models deduction on geometrical and algebraic analysis, not Euclidean geometry (see AT 10:377, CSM 1:19).[31] Euclidean geometry is precisely the model that Descartes *rejects* in *Rules*, not only in mathematics, but in science in general. In Euclidean geometry, which Descartes describes in *Second Replies* as employing the method of "synthesis," one deduces conclusions from axioms. In geometrical and algebraic "analysis," by contrast, one assumes that the problem is already solved (by assigning variables to unknown magnitudes) and then works backwards in order to determine the value of these variables on the basis of their relation to the known magnitudes. The synthetic method of demonstration employed in Euclidean geometry "demonstrates the conclusion clearly and employs a long series of definitions, postulates, axioms, theorems and problems," while the analytic method of demonstration employed in geometrical and algebraic analysis "shows the true way by means of which the thing in question was discovered" (AT 7:155, CSM 2:110). As the example discussed above demonstrates, Descartes clearly models deduction on analysis, not synthesis. The point is indeed to "show the true way by means of which the thing in question was discovered [*inventa*]," since Cartesian deduction is a central component of the method considered as an *ars inveniendi*.

3.4 *Enumeratio, sive inductio*: Enumeration

After intuition and deduction comes enumeration, which is arguably the most complex operation of the method. In his interpretation of "enumeration or induction [*enumeration, sive inductio*]" in *Rules,* Beck correctly observes that Descartes employs these terms in different ways to refer to what are in fact distinct methodological operations.[32] To ensure clarity, these operations must be clearly distinguished from one another. The best way to distinguish them from one another is by the order in which they must be executed in solutions to problems. Enumeration$_1$ is the preliminary operation whereby problems are reduced to their simplest component parts and these parts are ordered in such a way that they may then be solved by intuition and deduction whenever appropriate. Enumeration$_1$ organizes problems *prior* to intuition and deduction. Enumeration$_2$, by contrast, is a *non-deductive inference* from many propositions (either in one continuous inferential chain or many discrete inferential chains) that is irreducible to intuition and, therefore, dependent on memory in some measure. Finally, enumeration$_3$ is another species of *non-deductive inference* based on *the construction of classes and considerations of class membership and exclusion* according to locally specifiable criteria.

3.4.1 Enumeration$_1$: Reduction and Order

In Rule 7, Descartes writes that "[…] enumeration, or induction, consists in an exploration of everything relating to the problem at hand, an exploration which is so careful and accurate that we may conclude with certainty and evidence that we have not inadvertently overlooked anything [*omnium quae ad propositam aliquam quaestionem spectant, tam diligens et accurata perquisitio, ut ex illa certo evidenterque concludamus, nihil a nobis perperam fuisse praetermissum*]" (AT 7:388, CSM 1:25; translation modified).[33] Enumeration$_1$ *precedes* both intuition and deduction by determining *the conditions relevant to the solution of the problem*:

> In every problem […] there has to be something unknown – otherwise the inquiry would be pointless. Nevertheless this unknown something must be delineated by definite conditions, which point decidedly in favor of one direction of inquiry rather than another. These conditions should, in our view, be gone into right from the very outset. We shall do this if we concentrate our mind's eye on intuiting each individual condition distinctly, looking carefully to see to what extent each condition delimits the unknown object of our inquiry (Rule 13, AT 10:434–5, CSM 1:54).

Descartes writes that "the order in which things are enumerated can usually be varied; it is a matter for individual choice," but that "if our choice is to be intelligently thought out we should bear in mind what was said in Rule 5" (AT 10:391, CSM 1:27). In Rule 5, he famously writes:

> The whole method consists entirely in the ordering and arranging of the objects on which we must concentrate our mind's eye if we are to discover some truth. We shall be following this method exactly if we first reduce complicated and obscure propositions step by step to simpler ones, and then, starting with the intuition of the simplest ones of all, try to ascend through the same steps to a knowledge of all the rest (AT 10:379, CSM 1:20).

Enumeration$_1$ organizes problems by reducing "complicated and obscure propositions step by step to simpler ones." Enumeration$_1$"cuts problems down to size" or divides them so that they are *cognitively manageable* and can be solved by a *finite number of operations*.[34] It orders the relevant problems such that each problem may be solved by means of intuition and/or deduction whenever appropriate, according to a prescribed order. The problems must be ordered according to their relative simplicity and complexity, since the solution to some problems depends on the solution to other, simpler problems, but not vice versa. Intuition and deduction simply operate on the problem as organized and ordered by enumeration$_1$. The *structure of the deduction is determined by enumeration$_1$*, since enumeration$_1$ determines the order in which the problem must be solved, and solving each problem according to this order simply *is* deduction. Without such order, the mind cannot deploy intuition and deduction. It quickly loses track and knows neither where to begin nor where to end its inquiry.

Two clear examples of enumeration$_1$ can be found in Rule 8, where Descartes discusses (1) how to deduce the law of refraction and the shape of the anaclastic lens (see Figure 3.1), and (2) what he terms the "noblest example," or the problem of the limits of human knowledge. I will discuss both of these problems in much more detail in Chapter 10 and Chapters 5–

7, respectively. Here, I discuss them only as examples of how Descartes employs enumeration$_1$ in order to reduce problems to their simplest component parts and establish order among the parts.

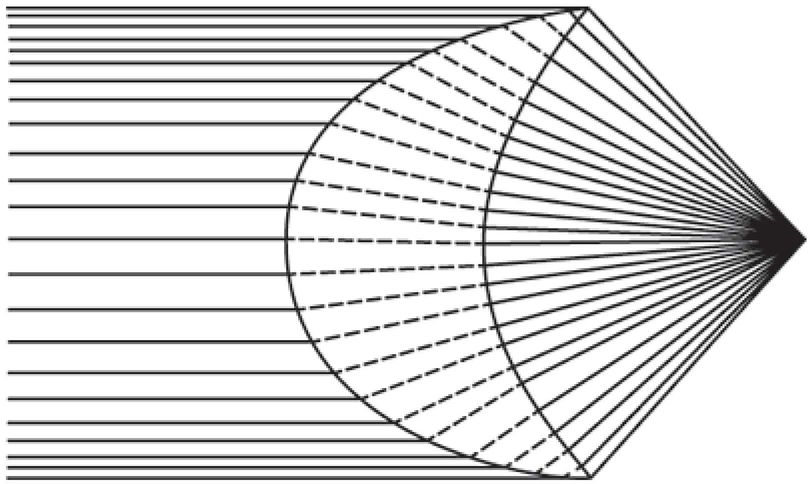


Figure 3.1 Anaclastic lens (ellipse)

Parallel rays of light refracted toward a common point (AT 6:194)

The problem of the anaclastic is a complex problem, and it clearly cannot be immediately solved by means of either intuition or deduction. The problem must, therefore, be reduced to a series of simpler problems, and these problems must be ordered according to their relative simplicity and complexity by means of enumeration$_1$. The purpose behind the reduction is to render the problem amenable to solution by means of intuition and deduction. As can be seen above (Table 3.2), the most complex problem in the series—the problem of the anaclastic itself—can be reduced to, and so depends on, all of the other problem below it. Descartes reduces the problem of the anaclastic to a series of five simpler problems.

Table 3.2 The structure of Descartes's deduction of the anaclastic lens (see Garber 2001, 37)

Enumeration$_1$:
Q1 What is the shape of a line (lens) that focuses parallel rays of light to the same point?
Q2 What is the relation between the angle of incidence and the angle of refraction (i.e., the law of refraction)?
Q3 How is refraction caused by light passing from one medium to another?
Q4 How does a ray of light penetrate a transparent body?
Q5 What is the nature of the action of light?
Q6 What is a natural power?
Intuition and deduction:
Q1 A natural power is…
Q2 The nature of the action of light is…
Q3 A ray of light penetrates a transparent body by…
Q4 Refraction is caused by light passing from one medium to another when…
Q5 The relation between the angle of incidence and the angle of refraction is…
Q6 The shape of the line (lens) that focuses parallel rays of light to the same point is…
Q1 must be solved by means of intuition, and the solution to Q2–Q6 must be deduced from the solution to Q1 in order.

As this example illustrates, intuition and deduction can only be performed *after* enumeration$_1$ has reduced and ordered the problem. Intuition and deduction start where enumeration$_1$ ends: the simplest problem in the series is solved by means of intuition, while the more complex problems are solved by means of deduction and enumeration. *The order of the deduction merely inverts the order of enumeration$_1$*. Deduction does not produce order. On the contrary, it presupposes order as already produced by another, antecedent operation: enumeration$_1$.[35]

In Rules 8 and 12, Descartes reduces the problem of the limits of knowledge to a series of simpler problems (see Table 3.3), and remarks that "this seems to me to be a complete enumeration [*enumeratio…completa*] and to omit nothing which is within the range of human endeavor" (AT 10:411, CSM 1:39). Similarly, at the end of Rule 12, he concludes his solution to the problem by reminding his readers that "we have explained distinctly and, I think, by a sufficient enumeration [*sufficientem enumerationem*] what at the outset we were able to present only in a

confused and rough-and-ready way, viz., that there are no paths to certain knowledge of the truth accessible to men save manifest intuition and necessary deduction” (AT 10:425, CSM 1:48; translation modified). To solve the problem of the limits of knowledge, the operator of the method must “make a precise enumeration of all the paths to truth which are open to men, so that he may follow one which is reliable. There are not so many of these that he cannot easily discover them by means of a sufficient enumeration” (AT 10:396, CSM 1:30). What enumeration$_1$ provides is the possibility of an exhaustive classification of and reflection on the human cognitive faculties and the objects of science, or pursuing “every humanly accessible path which leads to knowledge of the truth” (AT 10:399, CSM 1:32). After everything relevant to the solution of the problem has been “encompassed within definite limits” and “placed under a few headings” (AT 10:398, CSM 1:31), all that remains to be done is to examine each problem within the limits required by the solution. In the case of the problem of the limits of knowledge, Descartes reduces the problem into two parts: the human cognitive faculties, and the objects of science (see Table 3.3). Each faculty and object must be evaluated relative to Descartes’s definition of science as “certain and evident cognition” in Rule 2. Any operation or object that does not satisfy this definition must be excluded.

Table 3.3 Descartes’s reduction of the problem of the limits of human knowledge in Rules 8 and 12

Q1 What is human knowledge and what is its scope?	
Q2 What is the subject of knowledge?	Q7 What is the object of knowledge?
Q3 What faculties do they have?	Q8 What presents itself to us spontaneously?
Q4 What are the operations of these faculties?	Q9 How can one thing be known on the basis of something else?
Q5 Which operations produce science?	
Q6 Which operations do not produce science?	
Q10 What conclusions can be drawn from each of these?	

Both of the enumerations$_1$ in Tables 3.2 and 3.3 are well-ordered in accordance with Rule 5. But there are important differences between them. In the case of the anaclastic, Descartes’s enumeration$_1$ is well-ordered

because the most complex problem is reduced to its simplest component parts, ending with the simplest problem of all ("What is a natural power?"), which can be solved by means of intuition. In the case of the problem of the limits of knowledge, the complex problem is reduced into two simpler, principal parts, each of which is then subdivided into other, simpler parts. In this case, however, *there is no single intuition that constitutes the solution to the simplest problem in the series because there is no one simplest problem*. The problem of the limits of knowledge cannot be reduced to a single, simplest problem, at least not in the sense in which the problem of the anaclastic can. The case of the anaclastic is one in which the simplest problem ("What is a natural power?") can be solved by intuiting the material simple natures. In the problem of the limits of knowledge, by contrast, the problem in Q7–9 is not (or not only) to *intuit* the simple natures, but rather to *sufficiently enumerate* them*:* intellectual or spiritual, material, and common. This is an enumeration$_3$ (see Section 3.4.3 below). Furthermore, intuition and deduction cannot be employed in the solution to the problem of the limits of knowledge because *part of the problem (Q6) is to determine which operations produce science, such that intuition and deduction must themselves be shown to be the only operations that produce science in the solution to the problem*.[36] Intuition is only employed to *verify* that each simple nature is indeed intuitable, but it is not employed to demonstrate that no other operation besides intuition and deduction can produce science. Any such demonstration requires an enumeration$_3$ of the class of candidate operations in which intuition is but one member. Finally, the problem of the limits of knowledge is simply *too complex* to be solved by intuition and deduction. As we have seen in Section 3.3 above, deduction is only possible when all of the propositions in the deduction are connected to one another and the deduction can be reduced to intuition. Descartes's solution to the problem of the limits of knowledge consists in a series of inferences (conclusions) drawn from *many disconnected propositions*. Any inference from many disconnected propositions is an enumeration$_2$ (see Section 3.4.2 below). Descartes solves the problem of the limits of knowledge in Rules 8 and 12 by means of enumeration$_{1–3}$ because these operation(s) *and no others* can solve the problem.

The differences between Tables 3.2 and 3.3 reveal that while the "order" that defines enumeration$_1$ always requires the reduction of a problem to its

simplest component parts, *the degree of simplicity of the component parts and the operations required to solve them cannot be measured by any sense of "order" that transcends particular problems and their internal complexity.*"Order" cannot function uniformly in all problems *without ceasing to be efficacious.* When it comes to the simplicity of the component problems, "simplicity" is a relative term; the simplicity of the simplest component problem is only so relative to the complexity of the problem of which it forms a part. As we have seen, the complexity of the problem of the limits of knowledge is such that the problem can only be solved by means of enumeration$_{1–3}$. There is no absence of order, nor even imperfect order in Descartes's reduction of the problem; the problem has been reduced as far as it can be reduced. Descartes's method is no less efficacious in ordering the problem of the limits of knowledge than it is in ordering the problem of the anaclastic in Rule 8.[37] It is only different.

To sum up, in any suitably complex problem, the problem must be reduced and ordered by enumeration$_1$. The problem must be reduced to its simplest component parts, and these parts (problems) must be ordered according to their epistemic dependency relations (relative simplicity and complexity). Each component problem must be solved individually by the relevant operation. In problems that can be solved by intuition and deduction (e.g., the problem of the anaclastic), these operations only intervene *after* enumeration$_1$ has reduced and ordered the problem. In certain cases (e.g., the problem of the limits of knowledge), the problem is such that intuition and deduction cannot be employed. These problems must be solved by enumeration$_{1–3}$ alone.

****

3.4.1.1 Clarity and Distinctness in Problems: Relevant and Irrelevant Conditions

Enumeration$_1$ is far more complex than a cursory reading of *Rules* would seem to suggest. Since enumeration$_1$ enumerates only *relevant* conditions, it clearly requires *considerations of relevant and irrelevant conditions.* Furthermore, once the relevant conditions have been isolated, *one must impose constraints on how to interpret them.* Descartes discusses these

requirements and how best to satisfy them in Rule 13, where he identifies two ways in which an enumeration$_1$ is "liable to go wrong."

> For in this context the human mind [*humana ingenia*] is liable to go wrong in one or other of two ways: it may assume [*assumendo*] something beyond the data [*datum*] required to define the problem, or on the other hand it may leave something out. We must take care not to suppose [*supponamus*] more than the data, and not to take the data in too narrow [*strictiora*] a sense (AT 10:435, CSM 1:54; translation modified.).

As examples of assuming something beyond the conditions required to define the problem, Descartes discusses the problem of the Tantalus cup (a problem in what was known in the seventeenth century as "recreational mathematics") (see Figure 3.2) and the problem of planetary motion (a problem in astronomy).[38] In the problem of the Tantalus cup, Descartes writes that at "first glance, it might seem that the artistry here lay entirely in the construction of the figure of Tantalus, when in fact that is merely a coincidental feature and by no means a factor that defines the problem. The whole difficulty," Descartes continues, "is this: how must the bowl be constructed if it lets out the water as soon as, but not before, it reaches a fixed height?" (AT 10:436, CSM 1:55). Once the wine reaches Tantalus's hands, the wine drains out of the bowl via the canal hidden beneath Tantalus. The figure of Tantalus has nothing to do with the fact that the wine drains out of the bowl, and to believe that it does is to be distracted by an irrelevant condition that undermines the possibility of a solution to the problem. In this case as in more complex cases, to assume something beyond the conditions required to define a problem is to enumerate$_1$ irrelevant conditions. When irrelevant conditions are included, then the problem becomes obscure and confused. It is not only *ideas* that must be clear and distinct in Descartes; *problems* must also be clear and distinct, such that what the problem is about is manifest and each condition relevant to its solution is sharply separated from the rest, with no admixture of anything that does not properly define or constitute a part of the problem. Irrelevant conditions (which are "conditions" in name only) render problems obscure and confused, and so direct the mind to search for a solution to the problem in the wrong place. They make problems into blind alleys.

As is always the case in Descartes, a major source of gratuitous assumption is, of course, "ingrained prejudice," which predisposes the mind

to interpret the relevant conditions in a definite way. This becomes especially clear in Descartes's discussion of the problem of planetary motion. Here, one may interpret the motion of the stars on the basis of the assumption that they revolve around an earth that is "motionless and fixed at the center of the universe […] just because that is how from our infancy it appeared to us to be. That assumption," Descartes writes, "should be called into doubt so that we may consider what in the way of certainty our judgment may attain on this matter" (AT 10:436, CSM 1:55). This is surely excellent advice, but how can it be implemented when one *does not yet know that the belief in question* is *a gratuitous assumption*? Sadly, assumptions do not wear their gratuitousness on their sleeve. There may not be any reason to call an assumption into doubt until *the dilemmas it produces in the course of the solution to a problem undermine one's ability to find a solution*. Loss of confidence may occur when it becomes clear that the assumption is simply "in conflict with many observations <made recently> (especially the waxing and the waning phases of light which are observed on Venus just as they are on the moon" (*Principles* III. 16, AT 8A:85, CSM 1:250). Descartes provides no explicit criteria for when a theory should be regarded as empirically disconfirmed, and there are well-known instances in which he himself disregards empirical data that seem to conflict with the principles of his physics. These tend to be instances in which the accommodation of the data would require abandoning or revising the principles. But in instances in which these principles are not at stake, Descartes not only admits but demands a high degree of sensitivity to empirical data. Here, the elimination of assumptions adopted due to ingrained prejudice does not occur prior to the solution of a problem, but rather in the course of the solution, precisely when one discovers that the problem cannot be solved unless the assumption is abandoned. One does *not* (or not necessarily) initially enumerate$_1$ *all and only* those conditions that are relevant to the solution of a problem. In many cases, one may have to exclude irrelevant conditions in the course of the solution, as Descartes also does in *Meteorology* VIII when he discovers that the number of refractions light undergoes in water is totally irrelevant to the production of the colors of the rainbow (see AT 6:329–30, Descartes 201: 335). It frequently happens that one only discovers *in the course of an inquiry* that a condition previously assumed to be relevant turns out not to have been relevant at all.

Conversely, one can also commit what Descartes terms a "sin of omission," which occurs when one fails "to take account of some condition necessary for defining a problem, a factor which is either explicitly stated in it or is in some way implied by it" (AT 10:436, CSM 1:55). This can occur either when one simply fails to enumerate$_1$ some condition relevant to the solution of a problem, or when one interprets a relevant condition too narrowly. As an example of interpreting a condition too narrowly, Descartes discusses "the riddle of the Sphinx about the animal which is four-footed to begin with, then two-footed, and later on becomes three-footed" (AT 10:433, CSM 1:53). Like problems in mathematics, the riddle contains everything needed to solve it; no relevant condition has been excluded, and no irrelevant condition has been included. Nevertheless, the solution—human beings, who are four-footed as infants, two-footed as adults, and three-footed in old age (they rely on a cane)—is not immediately clear, and can only be deduced once the word "footed" is interpreted less narrowly than is typical.

What is a case of interpreting the data too narrowly in a properly scientific context? In the problem of magnetism, one might mistakenly infer that magnetism is due, not to the mechanical interaction between extended bodies too small to be seen by the eye, but rather to a force or soul in the magnet. Such an inference would appear to be warranted on the basis of observation, since the bodies in a magnetic interaction do not seem to be in contact with one another. The English scientist William Gilbert (1544–1603), Descartes writes, believed that "the magnet contains some kind of entity the like of which our intellect has never before perceived," in which case Descartes argues that "it is pointless to hope that we shall ever get to know it simply by reasoning; in order to do that, we should need to be endowed with some new sense, or with a divine mind" (AT 10:436, CSM 1:55).[39] According to Gilbert, magnets—including the earth—have souls that act on other bodies at a distance, and these souls explain planetary orbits. This "assumption," Descartes writes, "should be called into doubt so that we may then consider what in the way of certainty our judgment may attain on this matter" (ibid.). For Descartes, the ontology of body—the material simple natures of extension, shape, and motion—places constraints on any possible interpretation of the relevant data; all effects of the magnet must be deduced solely from the extension, shape, size, and motion of the (in this case) microscopic bodies that produce these effects: "But if we

perceive very distinctly that combination of familiar entities [material simple natures] which produces the same effects which appear in the magnet, then we shall credit ourselves with having achieved whatever it is possible for the human mind to attain in this matter” (AT 10:439, CSM 1:57). Unlike souls in matter, the material simple natures are very much cut to the measure of the human mind: they are one and all intuitable by the human mind. Enumeration$_1$ interprets the data according to the simple natures. In this way, whatever the data may be, anybody who employs the method will know the limits within which to interpret them. For Descartes, ontology always constrains interpretation in science.

The problems discussed above are simple and, therefore, eminently clear examples of problems whose solution requires the clarification of one or more terms in which the problem is expressed (riddle of the Sphinx) and problems whose solution requires the (progressive or, whenever possible, immediate) inclusion and exclusion of relevant and irrelevant conditions (Tantalus cup). The triviality of the examples does not belie the depth of the point they illustrate. On the contrary, as we will see in more detail in Section 3.6, Descartes consistently appeals to riddles, the crafts, and recreational mathematics in order to extract important methodological precepts, which can then be applied to more complex problems.

Figure 3.2 Tantalus cup (Doračić 2017 and Delauncey 2017, respectively)

3.4.2 Enumeration$_2$: Irreducibly Complex Linear and Non-Linear Inference and the Expansion of Intuition

Once a problem has been reduced to its simplest component parts via enumeration$_1$, one must solve each one individually. Clearly, any such solution depends not only on intuition, but also on inference. As we have seen above in Section 3.3, in simpler cases, deduction is reducible to

intuition. Since, however, the number of inferences required in the solution to any problem always depends on the complexity of the problem, in more complex problems the inferences are not reducible to intuition. In Rule 7 and also in Rule 11, Descartes defines enumeration$_2$ as any inference in which there are either (a) simply too many propositions in one continuous inferential chain, or (b) many discrete inferential chains, such that the dependence of the conclusion on the inferential chain(s) from which it is deduced cannot be intuited all at once. Unlike enumeration$_1$, enumeration$_2$ does not determine the conditions relevant to the solution of a problem (a determination that must be made prior to intuition and deduction), but rather constitutes *a distinct species of inference defined by its complexity and irreducibility to intuition*:

> But when our knowledge of something is not reducible to simple intuition and we have cast off our syllogistic fetters, we are left with this one path [enumeration$_2$], which we should stick to with complete confidence. […] [If] we infer one proposition from many disconnected propositions [*multis et disjunctis*], our intellectual capacity is often insufficient to enable us to encompass all of them in a single intuition; in which case the certainty which the above operation [enumeration$_2$] allows must suffice (Rule 7, AT 10:389, CSM 1:26).

> In [Rule 3] we contrasted [intuition] with deduction, and in [Rule 7] only with enumeration, which we defined as an inference drawn from many disconnected facts [*illationem ex multis et disjunctis rebus collectam*]. […] [We] are supposing that the deduction is made through intuition when it is simple and transparent [*simplex et perspicua*], but not when it is complex and involved [*multiplex et involuta*]. When the latter is the case, we call it "enumeration" […] since the intellect cannot simultaneously grasp it as a whole, and its certainty in a sense depends on memory, which must retain the judgment we have made on the individual parts of the enumeration if we are to derive a single conclusion from them taken as a whole (Rule 11, AT 10:407–408, CSM 1:37).[40]

These passages clearly indicate that Descartes's theory of inference in *Rules* cannot be reduced to a theory of deductive inference (inference drawn from many connected propositions); Descartes's theory of inference in *Rules* also includes non-deductive inferences drawn from "many disconnected propositions," which are not only acceptable in science, but—in the case of more complex problems—absolutely indispensable.[41] The complexity of the inference(s) that yield the solution to a problem is essentially correlated to the *complexity of the problem*. More complex problems have more parts; the more parts a problem has, the more inferences the solution requires. Furthermore, since the parts of a problem must be ordered according to their relative simplicity and complexity, each component problem not only

adds to the total *number* of problems, but also becomes more *internally complex* insofar as it depends on, contains, or presupposes the solution to the simpler component problems. This is clear in the problem of the anaclastic discussed in Section 3.4.1. Memory intervenes, not only because the number of problems (and, therefore, inferences), has outstripped what intuition can retain in one act, but also because as each component problem becomes more internally complex, it becomes more (but not totally) resistant to intuition.

Descartes provides examples of enumeration$_2$ in the case of a complex linear inference in Rules 7 and 11:[42]

> If, for example, by way of separate operations, I have come to know first what the relation between the magnitudes A and B is, and then between B and C, and between C and D, and finally between D and E, that does not entail my seeing what the relation is between A and E; and I cannot grasp what the relation is just from those I already know, unless I recall all of them. So I shall run through them several times in a sort of continuous movement of the imagination [*continuo quodam imaginationis motu*], simultaneously intuiting one relation and passing on to the next, until I have learnt to pass from the first to the last so swiftly that memory is left with practically no role to play, and I seem to intuit the whole thing at once. In this way our memory is relieved, the sluggishness of our *ingenium* redressed, and its capacity in some way enlarged [*hoc enim pacto, dum memoriae subvenitur, ingenij etiam tarditas emendatur, ejusque capacitas quadam ratione extenditur*] (AT 10: 387–388, CSM 1: 25; translation slightly modified).[43]

The example reveals that in many (perhaps most) complex linear inferences containing more than two relations, intuition is more or less difficult depending on *how many relations the mind tries to intuit at once.* While the mind may intuit the relation between members that are immediately related to one another with an equal degree of facility, it cannot do this when the relation between the members is mediated by other relations, above all not when these other relations are not present to mind, as is the case when the mind tries to intuit the relation between the first and final members (magnitudes, in this case). To intuit the relation between the first and final members, one must also intuit *every intermediate relation*. In all such cases, intuition is placed under more pressure and encounters limits that can only be supplemented by *repetition*.[44] This initially requires many discrete inferences, not one act. The certainty of these inferences depends on memory. However, the inferences can be repeated in what Descartes terms a "continuous movement of thought [*cogitationis motu*]," which maximally reduces the role played by memory, so that "I seem to intuit the whole thing

at once" (AT 10:388, CSM 1:25). The initial limits on intuition can either be completely overcome or only considerably reduced. Repetition of the deduction enables the mind to maximally reduce the many discrete and successive intuitive acts to *one continuous act*. The successivity of a complex inference composed of many discrete intuitions is reduced to the simultaneity of one continuous intuition. The repetition of an enumerative inference *expands the scope of intuition*, and so *perfects* the human *ingenium*. The reduction of the role played by memory is accompanied by a corresponding expansion of the mind's ability to include more and more magnitudes in *one and the same intuitive act.* As Descartes puts it in Rule 11: "These two operations [intuition and enumeration$_2$] aid and perfect each other [*se mutuo juvent et perficiant*]; they do this so thoroughly that *they seem to coalesce into a single operation* [*adeo ut in unam videantur coalescere*], through a movement of thought, as it were, which involves carefully intuiting one thing and passing on at once to the others," effectively facilitating "a more certain knowledge [*certius cognoscendam*] of the conclusion in question" (AT 10:408, CSM 1:38; translation modified; my emphasis). Hence Descartes's conclusion that "one cannot fail to see that in this way the sluggishness of the mind [*ingenij*] is redressed and its capacity even amplified [*amplificari capacitatem*]" (AT 10:409, CSM 1:38; translation modified).

The reduction of the role played by memory in enumeration$_2$ is not so much its *elimination* as its *enhancement*, i.e., its expanded capacity to retain more and more relations *in the present*: "As we have said, conclusions which embrace more than we can grasp in a single intuition depend for their certainty on memory, and since memory is weak and unstable, it must be refreshed and strengthened through this continuous and repeated movement of thought [*revocari debet et firmari per continuum hunc et repetitum cogitationis motum*]" (AT 10:408, CSM 1:38). There is, in the end, no difference between elimination and enhancement; they are inversely proportional, such that the more memory is enhanced, the less it supplements intuition as an external aid that enables the mind to recall propositions no longer present to mind and the more it internally expands the "width" of intuitive presence by enabling the mind to retain these very same propositions in the present. The point here is to *exploit* memory as an instrument, not to *eliminate* it altogether.

As has by now become clear, in *Rules* Descartes requires that the *lived experience of evidence*—the evidence provided by intuition and deduction—be *refreshed in the present* whenever the need arises. Evidence that is not present is not clear, and what is not clear cannot be distinct. In the case of complex inference, this must be done by means of enumeration$_2$. In any scientific inference, the conclusion must be intuited together with *the totality of its (non-formal) logical conditions*, otherwise the conclusion produces neither certainty nor, therefore, evidence. *Conclusions severed from the totality of their logical conditions are propositions effectively dead to science.* Nevertheless, however much enumeration$_2$ may expand the limits of intuition, this operation suffers from the same limitation as intuition itself: it remains irreducibly *finite*. There is a limit here beyond which enumeration$_2$ simply cannot cross. Many inferences are simply too complex to be reduced to intuition. This means that not every inference can be reduced to intuition. The reduction of complex linear inference to intuition remains a regulative ideal in Cartesian science. There is a limit to how many relations I can embrace in any one act of intuition.

3.4.3 Sufficient, Complete, and Distinct Enumeration$_3$: Inference and Class Construction

Unlike enumeration$_2$, enumeration$_3$ is an inference based on the construction of classes and considerations of class inclusion and exclusion. In Rule 7, Descartes distinguishes between three subspecies of enumeration: complete, distinct, and sufficient enumerations (AT 10:389–390, CSM 1:26). While these subspecies pertain to enumeration$_{1–3}$, he introduces them via a series of examples of enumeration$_3$, and I will follow his lead here.[45] All enumerations, Descartes writes, must be sufficient:

> [This] operation [enumeration] should be "sufficient," because it can often be deficient and hence liable to error. For sometimes, even though we survey many points in our enumeration which are quite evident, yet if we make even the slightest omission, the chain is broken and the certainty of the conclusion is entirely lost. Again, sometimes we do cover everything in our enumeration, yet fail to distinguish one thing from another, so that our knowledge of them all is simply confused. The enumeration should sometimes be complete, and sometimes distinct, although there are times when it need be neither

(AT 10:389–390, CSM 1:26).[46]

Sufficient enumerations$_3$ are required in problems whose complexity is such that "we [must] arrange all of the relevant items in the best order, so that for the most part they fall under definite classes," in which case "it will be sufficient if we look closely at one class, or at a member of each particular class, or at some classes rather than others," which will allow us "never pointlessly [to] go over the same ground twice" and "review quickly and effortlessly a large number of items which at first sight seemed formidably large" (AT 10:391, CSM 1:27). Descartes provides four examples of problems that can be solved via enumeration$_3$ in Rule 7. Each example illustrates that the completeness and distinctness requirements must only be satisfied when the solution to the problem demands more than sufficiency:

(1) In order to solve the problem of "how many kinds of corporeal entity there are […]," he writes, "I shall not assert that there are just so many, and no more, unless I have previously made sure I have included them all in my enumeration and have distinguished one from another" (AT 10:390, CSM 1:26). This enumeration$_3$ is both complete (no class of body has been excluded) and distinct (all classes have been distinguished from one another).

(2) In order to solve the problem about "whether a natural power can travel instantaneously to a distant place, passing through the whole intervening space," one must "make an enumeration of all the other natural powers, in the hope that a knowledge of some other natural power" will help one "understand this one, if only by way of analogy" (AT 10:402, CSM 1:34; AT 10:395, CSM 1:29). Here, I enumerate$_3$ the class of "all other natural powers" in order to determine whether one of them (in this case, the action of light) may be analogized to any of the others (in this case, the motion of a stick). Despite the presence of the quantifier "all," Descartes really only requires *one* case in which a natural power (or force) can instantaneously travel to a distant place in order to determine whether such cases are indeed *possible*.[47] Membership in the same class licenses *analogical inferences* about one member based on another member. Consequently, here enumeration$_3$ need not be complete, but only sufficient

and distinct (whatever natural powers are enumerated$_3$ must be distinguished from one another).

(3) In order to solve the problem of whether the rational soul is corporeal, "there is no need for the enumeration to be complete; it will be sufficient if I group all bodies together in order to demonstrate that the rational soul cannot be assigned to any of these" (AT 10:390, CSM 1:26–7). Here, *class exclusion* licenses an inference that the member of *one* class is not a member of *another* class. Since the difference between mind and body does not depend on any differences between bodies themselves, there is no need for the enumeration of bodies to be complete. The enumeration is not complete, but only sufficient and distinct.

Finally, (4) in order to solve the problem of whether "the area of a circle is greater than the area of any other geometrical figure whose perimeter is the same length as the circle's," I "only need to demonstrate that this fact holds for some particular figures," in which case "I shall be entitled to conclude by induction that the same holds true in all the other cases as well" (AT 10:390, CSM 1:27).[48] Here, the class is only partially constructed, and licenses the inference that one member has a property no other member can have. The enumeration$_3$ is neither complete nor distinct, but only sufficient. This example is particularly instructive because the sufficiency of the enumeration$_3$ is required by the fact that *not all members of the relevant class can even be enumerated$_3$ in principle*. The class of geometrical figures equal in perimeter to a given circle is indefinite, and so the possibility of a complete enumeration$_3$ is excluded in principle by the very nature of the problem. I neither can nor need to enumerate$_3$ all geometrical figures equal in perimeter to a given circle in order to solve the problem. I only need to demonstrate that the circle has more area than *some* particular figures equal in perimeter to the circle. Why? In the case of objects satisfying a definite property (here, the property of being equal in perimeter to a given circle), *the property defines every member of the class, and so the complete enumeration$_3$ of each member would be entirely superfluous.* The *condition* that defines class membership suffices, so the actual construction of the class is unnecessary.[49]

It may seem that enumeration$_{1–3}$ need not be regarded as a part of the method, since these operations are either preparatory to the operations of

intuition and deduction (in the case of enumeration$_1$) or are themselves species of deduction (in the case of enumeration$_{2–3}$). However, as we have seen, enumerations$_{2–3}$ are not species of deduction, but rather non-deductive species of inference, both of which are required in complex problems whose solution cannot be discovered via intuition. Enumeration$_1$, moreover, is the operation without which complex problems cannot even be solved by means of intuition and deduction. Descartes clearly insists that enumeration, in all three of its functions, is a distinct operation. Thus, not only is enumeration a part of the method—it is the operation without which the method could not be applied to anything but the simplest problems. That is why it is so important to reconstruct, in detail, the diverse roles played by enumeration in Descartes's method. Otherwise, it becomes incredibly difficult to see how the method could actually be applied.[50]

3.5 Problems: Definition and Taxonomy

In Sections 3.1–3.4, I discussed the basic operations of the method: intuition, deduction, and enumeration$_{1–3}$. These operations must be employed in solutions to problems. Problems are simply any question the answer to which can be either true or false (AT 10:432, CSM 1:53). The solution to such problems produces certain and evident cognition and, therefore, science. In Rule 13, Descartes writes that we "must enumerate [enumeration$_3$] the different kinds of problems, so that we may determine what we have the power to achieve in each kind" (ibid.). In Rule 12, Descartes distinguishes between perfectly understood problems and imperfectly understood problems:

> As for problems, […] some can be understood perfectly, even though we do not know the solutions to them, while others are not perfectly understood. […] The division between perfectly understood and imperfectly understood problems is one that we have introduced quite deliberately: its purpose is partly to save us from having to mention anything which presupposes an acquaintance with what follows, and partly to enable us to set forth first those matters which in our view have to be tackled first if we are to cultivate our mental powers [*ingenia excolenda*]. We must note that a problem is to be counted as perfectly understood only if we have a distinct perception of the following three points: first, what the criteria are which enable us to recognize what we are looking for when we come upon it; second, what exactly is the basis from which we ought to deduce it; third, how it is to be proved that the two

> are so mutually dependent that the one cannot alter in any respect without there being a corresponding alteration in the other. So now that we possess all the premises, the only thing that remains to be shown is how the conclusion is to be found. This is not a matter of drawing a single deduction from a single, simple fact, for, as we have already pointed out, that can be done without the aid of rules; it is, rather, a matter of deriving a single fact which depends on many interconnected facts [*multis simul implicatis dependens*], and of doing this in such a methodical way that no greater intellectual capacity is required than is needed for the simplest inference. Problems of this sort are for the most part abstract, and arise almost exclusively in arithmetic and geometry […]. [Those] who desire a perfect mastery of the latter part of my method (which deals with the other sort of problem [imperfectly understood problems]) should be advised that a long period of study and practice is needed in order to acquire this technique (AT 10:429–30, CSM 1:51).

Descartes intended to devote the third part of Rules—Rules 25–36—to imperfectly understood problems, but the treatise abruptly ends at Rule 21.[51] Nevertheless, the distinction between perfectly understood problems and imperfectly understood problems is important (1) because the solution to each type requires different operations and procedures, and (2) because the order in which one learns the method is such that one must learn how to solve perfectly understood problems before solving imperfectly understood problems. Perfectly understood problems arise for the most part in arithmetic and geometry because the conditions needed to solve problems in these sciences are provided in the statement of the problem. Suppose the problem is to determine the value of the mean proportionals *x* and *y* in the series 2:6 = *x:y* = 162:486. To determine the value of *x* and *y*, I must determine their relations to the other members in the series. The criteria which "enable us to recognize what we are looking for when we come upon it" are provided in the statement of the problem, such that the distinction between solutions and non-solutions can be distinctly perceived: *x* and *y* must be such that they bear the same relation to the other members in the series that these other members bear to one another. The "basis from which we ought to deduce" the solution—the numerical values provided in the series—is also distinctly perceived. Finally, how the solution depends on the deductive basis is obvious: any change in the values of the numbers changes the values of the unknown numbers. When one is faced with the problem of finding mean proportionals, then, one does not need to further define the problem or enumerate_1 any conditions relevant to its solution beyond what is already provided in the statement of the problem. One can proceed immediately to the solution of the problem. This does not mean that the solution will always be easy or obvious, but only that the *criteria*

that define the solution, together with the *deductive basis* from which the solution may be inferred, are distinctly perceived. The problem is perfectly understood because it is clear and distinct.

The three conditions that define perfectly understood problems are satisfied almost exclusively in arithmetic and geometry. In natural philosophy, these conditions are not typically satisfied. For example, in the case of the anaclastic (see Section 3.4.1 above and Chapter 10), I only know that I am looking for the shape of a lens from which parallel rays of light are refracted toward a common point (see Figure 3.1). I do not initially know what conditions are relevant to the solution of the problem. These conditions are not provided in the statement of the problem; they must be found by enumeration$_1$. To determine these conditions, I must enumerate$_1$ the simpler component parts of the problem. Enumeration$_1$ thus plays an essential role in solutions to imperfectly understood problems. In all such cases, intuition and deduction can only be performed after enumeration$_1$ has reduced the problem to its simplest component parts and ordered each part according to its relative simplicity or complexity.[52]

Descartes's distinction between perfectly and imperfectly understood problems is not absolute. Descartes recommends learning how to solve perfectly understood problems first, not only because these problems are less complex than imperfectly understood problems, but also because *all imperfectly understood problems can in principle be reduced to perfectly understood problems*. Once enumeration$_1$ has reduced the problem to its simplest component parts and ordered the parts according to their relative simplicity and complexity, then everything needed to solve the problem has been discovered, and "it is easy to see […] how imperfect problems can all be reduced to perfect ones – as I shall explain at greater length," presumably in Rules 13–24 (AT 10:431, CSM 1:52). Even though Descartes never completed the third part of *Rules* (Rules 25–36), he did define its subject: solutions to imperfectly understood problems. He also indicated the manner in which such problems must be solved: they must be reduced to perfectly understood problems, the subject of the second part of *Rules* (Rules 12–24). Thus, enumeration$_1$ plays a central role in the unity of the treatise as a whole. As the operation whereby imperfectly understood problems can be reduced to perfectly understood problems, enumeration$_1$ ensures that the problems discussed in the third part of *Rules* (problems that

typically arise in natural philosophy) can in principle always be translated into the form of problems addressed in the second part of *Rules* (problems that typically arise in mathematics). Perhaps this is what Descartes meant when he later remarks in *Discourse* III that he applied the method to problems in mathematics as well as to "certain other problems [in natural philosophy] which I could put into *something like* mathematical form […]" (AT 6:29, CSM 1:125; my emphasis). By "mathematical form," he does not mean a deductive chain of propositions; that is how solutions, not problems, may (sometimes) look, whereas here it is a matter of putting *problems* into mathematical form. Rather, he means an imperfectly understood problem reduced to a perfectly understood problem.

3.6 Perspicacity and Sagacity: Two Intellectual Virtues or *Habitus*

Now that the operations of the method and the problems to which they may be applied have been covered in Sections 3.1–3.5, I turn to how the operations may be *perfected* in solutions to problems. Descartes frequently insists in *Rules* that the operations of the method must not be immediately applied to problems in the more advanced sciences. For the novice, problems in these sciences are too complex, and while it is "a common failing of mortals to regard what is more difficult as what is more attractive," the proper way to perfect the natural operations of the human mind is by practice in "the most insignificant and easiest of matters [*res minimas et maxime faciles*], [dwelling] on them long enough to acquire the habit of intuiting the truth distinctly and perspicaciously [*assuescamus veritatem distincte et perspicue intueri*]" (Rule 9, AT 10:400, CSM 1:33; translation modified). What are the "most insignificant and easiest of matters" Descartes refers to here? The so-called "feminine arts" (e.g., embroidery) and recreational mathematics (e.g., number games), about which I will have more to say below. By practicing intuition and deduction in these arts and popular sciences, one acquires two intellectual virtues or *habitus*: perspicacity and sagacity.[53]

> In this and the following Rule [Rules 9 and 10, respectively] we shall proceed to explain how we can make our employment of intuition and deduction more skillful [*industria possimus*

aptiores reddi ad illas exercendas] and at the same time how to cultivate two special mental faculties [*duas praecipuas ingenij facultates excolere*]: viz., perspicacity [*perspicacitatem*] in the distinct intuition of particular things [*res singulas*] and sagacity [*sagacitatem*] in the methodical deduction of one thing from another (AT 10:400, CSM 1:33; translation modified).

The message of this Rule [Rule 10] is that we must not take up the more difficult and arduous issues immediately, but must first tackle the simplest and least exalted arts, and *especially those in which order prevails* – such as weaving and carpet-making, or the more feminine arts of embroidery, in which threads are interwoven in an infinitely varied pattern. Number-games and any games involving arithmetic, and the like, belong here. It is surprising how much all these activities exercise our minds [*ingenia exerceant*], provided of course we discover them for ourselves and not from others. For, since nothing in these activities remains hidden and they are totally adapted to human cognitive capacities [*nihil in illis maneat occultum, et tota cognitionis humanae capacitati aptentur*], they present us in the most distinct way with innumerable instances of order, each one different from the other, yet all regular. […] We must therefore practice these easier tasks first, and above all methodically, so that by following accessible and familiar paths we may grow accustomed [*assuescamus*], just as if we were playing a game, to penetrating always to the deeper truth of things. In this way we shall gradually find – much sooner than we might expect – that it is just as easy to deduce, on the basis of evident principles, many propositions which appear very difficult and complicated (Rule 10, AT 10:404–5, CSM 1:35–6; my emphasis).

In the case of intuition, I "acquire through practice the ability [*usu capacitatem acquirunt*] to make perfect distinctions between things, however minute and delicate," in which case I become perspicuous (Rule 9, AT 10:401, CSM 1:33). Perspicacity is acquired via practice in the so-called "feminine arts." In the case of deduction, I "grow accustomed" to solving problems by reducing them to their simplest component parts via a well-ordered enumeration$_1$, so that I may methodically deduce the solution, in which case I acquire the virtue of sagacity (Rule 10, AT 10:405, CSM 1:36). Sagacity is acquired via practice in recreational mathematics. These two virtues—perspicacity in intuition and sagacity in deduction—prepare the mind to solve problems in the more advanced sciences. How, exactly? Can Descartes's recommendation to practice solving problems in these arts and popular sciences really be taken seriously? Absolutely. As I will show below, these activities present no resistance whatever to human cognitive capacity, but are so easy and simple that they paradigmatically exhibit acts of intuition, deduction, and enumeration$_{1–3}$ as well as the simplicity all problems must ultimately be reduced to in order to be soluble by the human mind.

The principle behind recreational mathematics is to discover, behind a wondrous effect (e.g., the problem of the Tantalus cup, discussed in Section 3.4.1.1 above), the means whereby it is produced. To solve the problem, one

must work backwards by regressing from the problem to its conditions of solution via enumeration$_1$. Relevant conditions must be distinguished from irrelevant conditions, and the problem is so designed as to *conceal* the relevant conditions as much as possible. The procedure whereby such problems are solved is in principle *no different* from the procedure employed in the solution to *any* problem. The only difference between problems in recreational mathematics and problems in the more advanced sciences is their *complexity, not the principle behind the procedure required in order to solve them.* The capacity to solve problems via regress to their conditions of solution (and, therefore, to the rules whereby they are solved) is precisely what seventeenth-century recreational mathematics is all about.[54] What one acquires in solving problems in recreational mathematics is the intellectual virtue or *habitus* of sagacity.

The uniquely Cartesian concept of sagacity can be better isolated by distinguishing it from its Aristotelian counterpart. For Aristotle, sagacity consists in the ability to find the middle term of a syllogism. One already knows the conclusion, but must work backwards in order to find the middle term that connects the premises to the conclusion. Those who can do this more ably are sagacious.[55] For Descartes, by contrast, sagacity consists in the ability to work backwards, *not from already known conclusions, but rather from problems to their conditions of solution*. As we have seen, the ability to solve problems via regression to their conditions of solution is the *sine qua non* of Descartes's method in *Rules*. Practice in recreational mathematics exercises the mind by constraining it to autonomously discover solutions to problems in precisely this way.

Descartes's reference to the "feminine arts" is no less significant than his reference to recreational mathematics: sixteenth- and seventeenth-century embroidery manuals illustrate how the problem of reproducing a complex pattern is rendered easier by dividing the pattern into discrete units (squares) on a grid (see Figure 3.3). Each unit must be sharply separated from the others and must be sown entirely on its own, without regard to the others. The sharp (sensible) separation of the units models intuition, since intuition similarly requires the sharp (intellectual) separation of the component parts of a problem or the simple natures that compose a complex idea or notion. Because each unit is sharply separated from all the others, one can devote all one's attention to a single unit at a time, so that the mind is never distracted by too many things at once. This too models

intuition. Like mechanics, those who engage in embroidery and similar crafts “are used to fixing their eyes on a single point” and “acquire through practice the ability to make perfect distinctions between things, however minute and delicate” (AT 10:401, CSM 1:33). A cursory glance at sixteenth- and seventeenth-century embroidery manuals indicates that they do indeed present “innumerable instances of order, each one different from the other, yet all regular,” as Descartes puts it in Rule 10.[56] One sews the entire tapestry simply by following some order (in this case, one is free to choose whatever order one pleases; see AT 10:404, CSM 1:35). Reading these manuals, one can easily imagine someone practiced in embroidery coming upon a simple problem in mathematics or natural philosophy and immediately dividing the problem into as many parts as needed to solve it. Even if they could not solve the problem themselves, they would know that no complex problem can be solved unless each part of the problem is sharply separated from all the others.

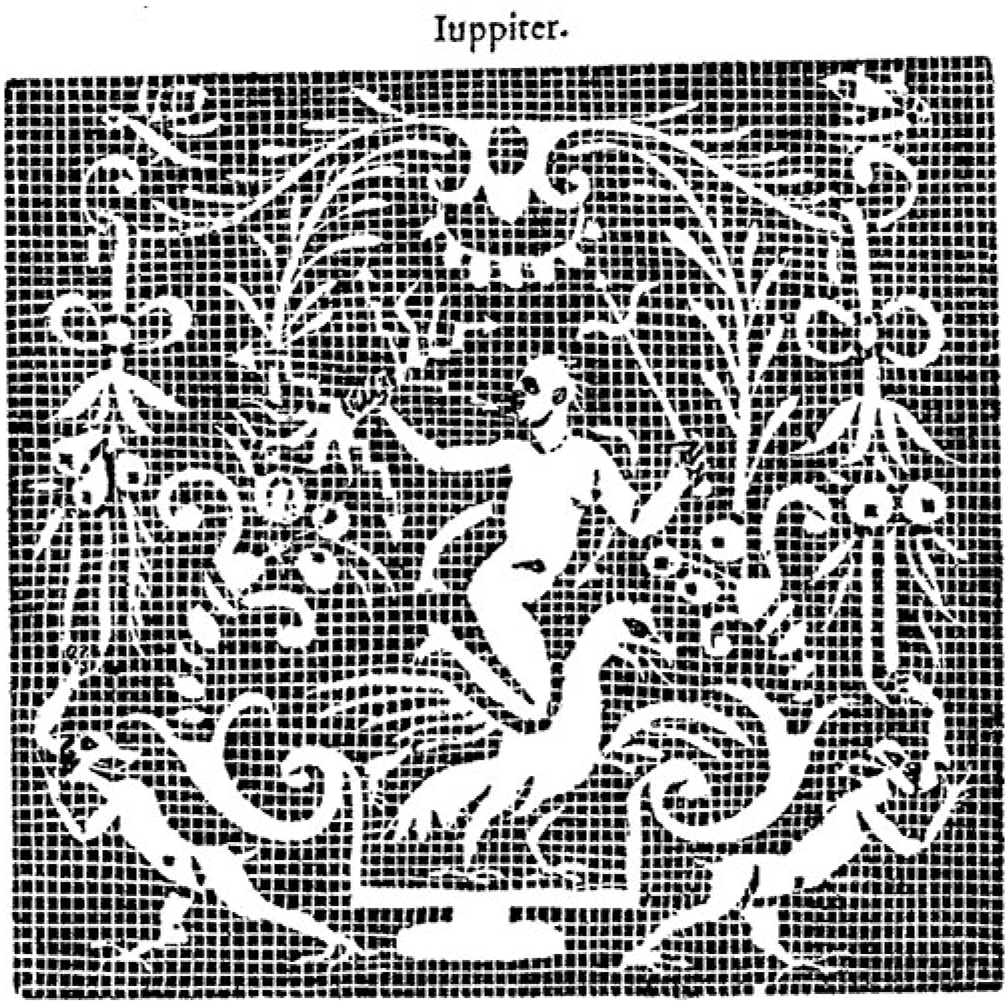


Figure 3.3 Jupiter (Zeus) (Vinicolo 1587, 65)

Like any regularly performed act, the cognitive acts deployed in solutions to problems in recreational mathematics and the so-called "feminine arts" leave "spiritual traces" (in French, *vestiges*) in the human *ingenium*, and these spiritual traces are precisely what perspicacity and sagacity ontologically *are*. As I argued in Chapter 2, Section 2.2, these spiritual traces, like the material traces in the brain that serve as the foundation of corporeal memory, are *durable spiritual modes* that *perfect* intuition and deduction, rendering them more *virtuous*.[57] Repeated practice in any methodological operation produces the corresponding intellectual virtue or *habitus*, which perfects the natural operation(s) of the mind and, therefore, facilitates their reproduction in solutions to other, more complex problems in future.

Thus, while the definition of science as certain and evident cognition in Rule 2 provides Descartes with the criteria needed to isolate intuition, deduction, and enumeration as the three principal operations of the method, these operations cannot produce science unless they are first perfected (*excolere*). They need virtues and, therefore, *habitus*. Practice in the arts and popular sciences is needed because such practice embodies one of the fundamental principles of the Cartesian scientific *habitus*: *the principle of proportionality* between the natural operations of the human mind and the problems solved by means of these operations (see Section 3.1 above). Problems in recreational mathematics and the so-called "feminine arts" are "totally adapted to human cognitive capacities," *as all other, more complex problems must be in order to be solved.* One must reduce problems to their simplest component parts, so that each problem becomes amenable to solution by one or more natural operations of the human mind. This is what it means to establish "proportionality" between problems and operations. Should there be any lack of proportionality between problems and operations, the operations encounter resistance, are frustrated in their operation, and so cannot be executed. Indeed, Descartes instructs his readers to "stop" at precisely that point where proportionality has not yet been established between problems and operations (see Rule 8 and Chapter 5–6).

3.7 The Order of Operations

As we have seen, Descartes specifies no invariant order in which intuition, deduction, and enumeration must be performed in solving problems. In any suitably complex problem, enumeration$_1$ precedes intuition and deduction and structures deduction. In some cases, such as the problem of the limits of knowledge, enumeration$_{1–3}$ either ameliorates the limitations of intuition and deduction or even replaces them entirely. Intuition may be the most important operation of the method, but it does not follow from this that it must or even can always be executed first. In all but the simplest problems, the number and proper order of operations must be determined on a case-by-case basis, such that, as Aristotle notes, "the agents themselves must in each case consider what is appropriate to the occasion, as happens also in

the art of medicine or of navigation."[58] For Descartes, one never mechanically applies the method to an already-defined problem; one must first define the problem in such a way that it becomes amenable to resolution by a definite number of operations. The interpretation of Descartes's method according to which the rules of the method completely determine how they must be applied *prior to and independently of their application to any particular problem* is foreign to Descartes's theory of method in *Rules*. Descartes resolutely opposes what Wittgenstein diagnosed as a "rails invisibly laid to infinity" picture of rule application, according to which the rule itself mechanically "traces the line along which it is to be followed through the whole of space."[59] On the contrary, for Descartes no less than Wittgenstein, "I give the rule an extension."[60] While all of the operations of the method are prescribed by rules, this does not mean that the method will mechanically direct *ingenium* in its application prior to the encounter between *ingenium* and the cognitive material it must organize. The direction of the human *ingenium* by the method is nothing but the direction of the human *ingenium* by itself. The method exists in the mind as a *habitus*, which, like any virtue, must be exercised under (and rationally responsive to) each problem, and it is the nature of the problem that determines how the method must be applied in each case.

Most interpreters of Descartes's method either explicitly assert or tacitly assume the thesis that the unity and universality of Descartes's method depends on its uniform application to all problems, be they problems in metaphysics, natural philosophy, geometry, or morals. In the Introduction, I referred to this thesis as the "Uniformity Thesis."[61] The Uniformity Thesis typically leads interpreters to (1) reduce the method to one uniform procedure; (2) identify paradigmatic applications of this procedure to problems in one or two sciences; and (3) dismiss as failures any solutions to a problem in which this procedure is not clearly exhibited. Once differences between the paradigm cases and the apparently anomalous cases arise, the Uniformity Thesis leads to denial that Descartes actually applied his method to anything but the paradigm cases. This effectively undermines Descartes's claim that "I did not restrict the method to any particular subject-matter" (*Discourse* II, AT 6: 21, CSM 1: 121) and constrains his readers to regard such claims as a merely rhetorical.

It is not my intention to suggest that the Uniformity Thesis is totally unfounded. There would seem to be good reasons for adopting it.

Uniformity in application seems required because otherwise it becomes difficult to see what the unity and universality of the method consists in. What is the difference between a method that can be applied in many ways, on the one hand, and many methods, on the other? Furthermore, it seems that the method can be universal only on the assumption that each application of the method is or must be the same. Nevertheless, the results of this chapter already indicate that the reverse is true: the unity of the method does not depend on its uniform application to all problems in all sciences, and it is incredibly difficult to see how a method that must be so applied can ever be universal. The method is "one" because it is essentially one *habitus* or ability, based on intuition and whatever operations maximally extend or enable intuition. The order and number in which these operations are deployed—the procedure—depends on factors that cannot be defined in advance because they are defined entirely by the nature of the problem. The order and number of operations can and, indeed, must vary from problem to problem. As we have seen in Section 3.4.1 and will see in much more detail in Chapters 5–7 and Chapter 10, Descartes's solution to the problem of the limits of knowledge and his solution to the problem of the anaclastic in Rules 8 and 12 are different. In both cases, Descartes reduces the problem to its simplest component parts, but he solves the former by means of enumeration$_{1–3}$, intuition, and deduction, while he solves the latter by means of enumeration$_{1–3}$ alone. Descartes clearly did not believe that he was obliged to solve the problem of the limits of knowledge by means of intuition and deduction. On the contrary, *these are the very operations he must show are the only operations that produce science in his solution to the problem*. Consequently, from the fact that not every operation of the method is employed in the solution to some problem or that the procedure employed in the solution to one problem is not employed in the solution to another problem, it does not follow that the method has not been employed *unless it can be demonstrated that the operations that were not employed should have been employed and that the same procedure would have been appropriate or even possible*. Even after they have acquired the Cartesian scientific *habitus*, "the agents themselves must in each case consider what is appropriate to the occasion," and what is appropriate to one occasion may not be appropriate to all.[62]

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0004

[1] See Dika and Kambouchner forthcoming and Kambouchner 2019.

[2] Intuition and deduction are species of "experience [*experimur*]" which Descartes defines broadly in Rule 12 as "whatever we perceive by means of the senses, whatever we learn from others, and in general whatever reaches our intellect either from external sources or from its own reflexive self-contemplation" (AT 10:422, CSM 1:46–7). What distinguishes intuition and deduction from the other species of experience is that no other species of experience produces science unless it has been subjected to these operations. On the different senses of "experience" in *Rules* and other texts, see Marion 1975, 43–7 and Clarke 1982, 17–40 (esp. 19–24).

[3] See Augustine 1968, 1:301 (*De trinitate*, lib. 9, cap. 6.9–11); trans. Augustine 2002, 30–4. See also the discussion in Biard 2011.

[4] See Biard 2011 and Yrjönsuuri 2011. It is not the case, as Beck 1952, 48 contends, that intuition was not a term of common usage among the scholastics.

[5] "For now we see through a glass darkly; then face to face [*videmus nunc per speculum in enigmate, tunc autem facie ad faciem*]" (1 Corinthians 13:12). See Gilson 1979, 53.

[6] Clarke 1982, 58 argues that "*intuitus* is restricted to understanding individual propositions [. . .] or to seeing the necessary connection between two propositions." Simple natures, however, are not propositions, but rather notions. In his later work, Descartes came to regard ideas as expressible either as terms or propositions. Ideas are neither terms nor propositions, but ideas can be expressed as either terms or propositions. Clearly, this means that propositions are not the principal objects of intuition. See his letter to Mersenne, July 1641: "It is not whether they [ideas] are expressed by terms or by propositions which makes them belong to the mind or the imagination; they can be expressed in either way. It is the manner of conceiving them which makes the difference: whatever we conceive without an image is an idea of the pure mind, and whatever we conceive with an image is an idea of the imagination" (AT 3:395; CSMK 3:186).

[7] For a systematic reconstruction of Descartes's theory of simple natures, see Chapter 7.

[8] See *Principles* I. 45: "I call a perception 'distinct' if, as well as being clear, it is so sharply separated from all other perceptions that it contains within itself only what is clear" (AT 8A:22, CSM 1:207–8). Like Nelson and Rogers 2015, I do not see any irreducible differences between Descartes's definition of intuition in *Rules* and his definition of clear and distinct ideas in *Principles*.

[9] See *Principles* I. 45: "I call a perception 'clear' when it is present and accessible to the attentive mind—just as we say that we see something clearly when it is present to the eye's gaze and stimulates it with a sufficient degree of strength and accessibility" (AT 8A:22, CSM 1:207–8). Descartes also describes intuition as an "indubitable conception." While all intuitions are indubitable, indubitability only indicates intuition, and does not constitute intuition; doubt is only possible in the *absence* of clarity and distinctness.

[10] While the simple natures are the *principal* objects of intuition, they are not the *only* objects of intuition. Propositions (i.e., combinations of simple natures) can be objects of intuition, provided that the relations between the simple natures that compose them are themselves intuited.

[11] Clarke 1982, 60 describes Descartes's theory of intuition in *Rules* as having "rather unfortunate consequences for the contemporary reader, connotations of a kind of direct, non-empirical inspection of the essence of rather suspect ontological simples." He interprets intuition without reference to the simple natures. However, the simplicity of the act of intuition depends on the simplicity of its object.

It is, therefore, difficult to see how one could dispense with the simple natures in Descartes's theory of intuition.

12 van de Pitte 1988 claims that intuition provides both clarity and distinctness and self-evidence, but not truth, which he argues Descartes regards as a property of judgments alone. See also Clark 1982, 58: "An *intuitus* is not a judgment […] and hence it has no truth-value." Van de Pitte's textual evidence for the claim that in *Rules* intuition is neither true nor false is based principally on Rule 13, where Descartes claims that "there can be no falsity in the mere intuition of things, be they simple or conjoined. In that respect they are not called 'problems' [*quaestiones*]," where by "'problems' […] we mean everything in which there lies truth or falsity" (AT 10:432, CSM 1:53). As van de Pitte reads this passage, truth and falsity can only be found in problems, and since intuitions are not problems, then truth cannot be found in intuition, but only in judgment. Van de Pitte's interpretation of the passage is logically fallacious: from the fact that there can be no falsity in intuition, it does not follow that there can be no truth in intuition, and from the fact that (solutions to) problems can be *either* true *or* false, it does not follow that intuitions can be *neither* true *nor* false. It does, however, follow that intuitions can be true, but not false.

13 See Ayers 1998, 1069–70 and Meier-Oeser 2015, 308–16. Conimbricenses [1606] 1611, 127 (*Commentari in universam dialecticam Aristotelis, in cap. IV De oratione*, q. 3, art. 2): "…*non est praetermittendum insigne discrimen inter veritatem simplicem, et complexam; quamvis enim utraque posita sit in adequatione intellectus ad rem, nihilominus veritas simplex primo, et per se consistet in adequatione repreasentationis cum obiecto repraesentato.* Unde oritur, ut nullam habeat oppositam falsitatem, quia nullius rei imago potest esse illi difformis, quoad ea, in quibus est illius imago […]," cited in Meier-Oeser 2015, 310 (my emphasis). See also Suárez 1856–1878, 25:317 (*DM*, disp. 9, sec. 1.14) "…*simplex apprehensio seu cognitio non potest habere difformitatem cum re quae est objectum ejus* […]," cited in Wells 1984, 30. See also Suárez 1856–1878, 25:283–9 (*DM*, disp. 8, sec. 3.25); ibid.: 291–2 (*DM*, disp. 8, sec.4.4); ibid.: 317–18 (*DM*, disp. 9, sec.1.14–16).

14 van de Pitte's argument (see n. 12 above) can only be accepted if it is restricted to the claim that there can be no *formal* truth in intuition, but throughout his article he draws no clear distinction between formal and material truth. Indeed, he claims that "throughout the Aristotelian-Scholastic tradition, truth had always been understood to be essentially a product of judgment," and that "Descartes would have had to express himself very clearly and very forcefully if he had intended to build a position that could stand against that tradition" (van de Pitte 1988, 454). However, as we have seen, Descartes can rely on a well-established tradition in which simple apprehensions are regarded as always true. For more on the relation between intuition and judgment in *Rules*, see Chapter 7, Section 7.9.

15 On the difference between intuitive and abstractive cognition, see Scotus 1639, 12:145 (*Quodlibetum*, q. 6, §§7–8), cited in Cross 2014, 43; Ockham 1967–1988, 1:31–2 (*Ord.*, Prol., q. 1, art. 1), trans. Ockham 1990, 23. In their commentary on *De Anima*, the Conimbricenses provide a standard definition of intuitive and abstractive cognition: "*Notitia intuitiva, quae etiam visionis dici consuevit […] est notitia rei praesentis ut praesens est; id est, cognitio qua ita rem attingimus, ut per eam cernatur praesentia objecti in se, et ex vi ipsius cognitionis, non vero quia aliunde comprobatur*," cited in Gilson 1979, 53. See Suárez 1856–1878, 3:640 (*De Anima*, lib. 3, cap. 6.10), cited by Marion in Descartes 1977, 121: "…*experimentum proprie loquendo immediate fieri in sensibus exterioribus, cum ipsa a rebus objectis immediate immutentur, et intuitive eas cognoscunt, quod est experiri, sicut tactus experitur ignem calefacere, non aliter, quam impressionem ab igne factam praesentialiter percipiendo*." See also Suárez 1856–1878, 3:655 (*De Anima*, lib. 3, cap. 12.1), cited in ibid.:"…*haec est differentia inter intellectum et sensum, quod intellectus habere possit notitiam abstractivam, sensus vero non nisi intuitivam*." On the relation between Descartes and scholastic theories of intuitive cognition in Scotus and Ockham, see Alanen and Yrjönsuuri 1997, Alanen 1999, and Alanen 2003, 12–18.

[16] See Cross 2014, 69–80 and Panaccio 2014, 58. For more details on the differences between Scotus and Ockham on intuitive cognition, see Biard 2011, Pannacio 2014, 73, n. 1 and 59, and Pannacio and Piché 2009.

[17] For Scotus and Ockham, intuitive cognitions are not typically purely sensory, but rather based on the collaboration of the intellect and the senses. While children and non-rational animals can have purely sensory intuitive cognitions, Scotus and Ockham argue that intellectual intuitions of extramental material things via the senses must be possible because otherwise the intellect could never form tensed contingent propositions about the present or syllogize on the basis of such propositions, as it evidently does. See the discussion in Scotus 1950–2013, 9:468 (*Ord.*, lib. 3, dist. 14, q. 3, n. 112), cited Cross 2014, 46, n. 4. In some cases, however, intuitive cognition can be purely intellectual, and does not rely on the senses at all. God can be intuitively cognized by means of the intellect alone, but only in the beatific vision after death. He can only be abstractively cognized *pro statu isto*. Scotus seems to regard mental acts as objects of intuitive cognition. See Scotus 1950–2013, 7:535–7 (*Ord.*, lib. 2, dist. 3, q. 2.1, n. 288–90), cited in Cross 2014, 55 and discussed in Cross 2014, 52–7. The intuition of mental acts presents a more interesting case. The early Scotus emphatically denies that purely intellectual intuition of mental acts is possible (it depends on a phantasm), but the later Scotus seems to suggest that purely intellectual intuition of mental acts is possible (without a phantasm). Ockham seems very clearly to endorse the possibility of a purely intellectual intuition of mental acts. See Ockham 1967–1988, 1:39–40 (*Ord.*, Prol., q. 1, art. 1) and the discussion in Brower-Toland 2012. However, the Jesuits—Suárez and the Conimbricenses—deny any such possibility. According to the Conimbricenses, "our soul cannot conceive of itself primarily and immediately, but is led to grasp itself through the perception of other things. For first it conceives of that whose species it has drawn from the senses […], then it reflects on its act and perceives it, and from that it cognizes both the image and the power from which it elicited the act." The mental act is not intuitively, but rather inferentially and, therefore, abstractively cognized: "[That] which is understood through its proper species is understood directly. Experience testifies, however, that we come to a knowledge of our own soul only through a reflective act, by inferring one thing from another" (Conimbricenses [1598] 1604, 505 (*In De anima*, lib. 3, cap. 8, q. 7, art. 2), cited in Pasnau 2017, 266–7. See also Suárez 1991, 1:168–78 (lib. 3, disp. 9, q. 5) and the discussion in Yrjönsurri 2000. As Pasnau 2017, 75–9 notes, Ockham and Olivi take the most Augustinian stance in the scholastic debate over the possibility of intuitive self-knowledge: the soul can enjoy direct intuition of its own mental acts. The standard Thomist line, by contrast, consistently maintained that the soul cannot directly intuit its own mental acts. It can only know them inferentially and, therefore, abstractively. For a brief overview the debate about self-knowledge from the thirteenth to the seventeenth century, see Pasnau 2017, 77, n. 8. For systematic reconstructions, see Putallaz 1991 (Putallaz focuses on the thirteenth century) and, more recently, Rode 2015. The Stoics also made room for purely intellectual impressions (see Laertius 2018, VII. 51, 770).

[18] Suárez 1991, 1:36 (lib. 3, disp. 8, q. 1, n. 17) and the discussion in Heider 2017a, 144.

[19] See Heider 2017b, 145. As Heider points out, judgment *in actu exercito* is "nothing but simple apprehension." On judgments *in actu exercito* in Suárez, see also Heider 2016, 190 and Perler 2014, 269.

[20] Marion in Descartes 1977, 119–25 interprets "deceptive judgment of the imagination as it badly composes" as a reference to the Stoic theories of cataleptic impressions. The Stoics regard cataleptic impressions as causally necessitating affirmative judgment or assent, but they do not *identify* cataleptic impression and judgment in the way in which Marion seems to suggest.

[21] While Descartes's definition of intuition excludes both the senses and the imagination, this does not mean that these faculties cannot *aid* the intellect in intuition. For Descartes, the senses can deceive, but only when I combine sensory experience and the judgment that the contents of such

experiences resemble their causes. Otherwise, the senses do not deceive. See Rule 12, AT 10:423, CSM 1:47 and *Sixth Replies*, AT 7:436, CSM 2:294–5. Similarly, the imagination deceives, not when it composes, but only when it "composes badly [*male componentis*]." Otherwise, the imagination does not deceive. On the contrary, in mathematics, it is the *intellect* that deceives when it is unaided by the imagination (see Rule 14, AT 10:442–443, CSM 1:59). As we will see in more detail in Chapters 5–8 and Chapters 9–10, according to Descartes no faculty is intrinsically deceptive. Everything depends on whether the *right* faculties are being employed in order to intuit the *right* objects.

[22] See AT 10:389, CSM 1:26.

[23] Normore 1993, 449 argues that Cartesian deductions preserve certainty, but not necessity. Descartes clearly insists that deductions preserve necessity in his definition of deduction. Other interpretations of Cartesian deduction as a species of inference that preserves both certainty (or some degree of certainty) and necessity can be found in Gaukroger 1989 and Nelson and Rogers 2015.

[24] Consequently, it cannot be the case that deductions preserve the same degree of certainty, as Normore 1993 contends. See also Beck 1952, 84, who claims that deduction is "as certain and infallible as intellectual intuition," when it is clear that Descartes asserts the contrary.

[25] Beck 1952, 83 places undue requirements on deduction when he describes "each of the steps [of the deduction as] being self-evident" or argues that deduced propositions are "equally clear and distinct self-evident [*sic*]" (ibid.). Even in the case of deductions that can be reduced to intuition, the proposition that depends on the one or two others that precede it is only evident in relation to the others, not per se.

[26] Gaukroger 1989, 51 argues that "inference cannot be analyzed on Descartes's view because it is simple and primitive." I agree with Wong 1982, Marion 1992, and Nelson and Rogers 2015 that Descartes's theory of inference in *Rules* can be analyzed, and that it should be analyzed in relation to Descartes's theory of simple natures.

[27] On the distinction between simple propositions and problems, see Section 3.1 above.

[28] I discuss necessary conjunctions between simple natures in more detail in Chapters 7–8.

[29] On Descartes's critique of Aristotelian logic, see Gaukroger 1989; Normore 1993; Charrak 2005; Mehl 2005; Cassan 2015; Nelson and Rogers 2015; Douglas 2017; Nelson 2017.

[30] With appropriate caveats, the same can be said about deduction in natural philosophy. For a concrete case, see Chapter 10.

[31] Gaukroger 1989, 51 argues that "given the way in which Descartes presents the distinction between intuition and deduction, the obvious model is a geometrical one, in which we grasp certain axioms, and so on, and deduce from these geometrical theorems." See also Beck 1952, 47 and Normore 1993. Gaukroger describes the difference between analysis and synthesis in rich detail, but argues that Descartes's "procedure in algebra leads him to reject synthesis and he generalizes this to a wholesale rejection of deduction" (87). On Gaukroger's interpretation, Descartes *models deduction on synthesis and distinguishes both from analysis* (see Gaukroger 1989, 85–8), whereas it seems clear to me that Descartes *models deduction on analysis and distinguishes both from synthesis*. The happy consequence here is that there is no need to assert that Descartes ever embraced "a wholesale rejection of deduction," as Gaukroger claims, since *the rejection of synthesis does not entail the rejection of deduction*. On Descartes's relation to the history of analysis, see Hintikka and Remes 1974 and Dubouclez 2013. For more discussion, see also Chapter 4.

[32] See Beck 1952, 143. See also Weber 1964, 48–57; Marion 1975, 103–13; Smith 2010, 67–113.

[33] See also *Discourse* II (AT 6:19, CSM 1:120 and AT 6:21, CSM 1:121).

[34] Enumeration$_1$ is referred to in the literature by a number of names, including "reduction" (Garber 2001, 36) or, rather misleadingly, "analysis" (Clarke 1982, 179; Schuster 2013, 254). The latter term is inappropriate because in Descartes "analysis" refers to a method of demonstration (see AT 7: 155, CSM 2: 110), which includes not only the reduction of a problem to its simplest component parts, but also the solution of the problem. Analysis and synthesis are not parts of the same method, but rather two distinct methods (see Garber 2001, 35, n. 4). "Reduction" avoids this problem, but since it is employed by Descartes as a *definiens* rather than as a *definiendum* —"reduction" is part of the definition of "enumeration," not vice versa—it seems better to employ "enumeration" rather than "reduction." The fact that the terms "reduction" and "analysis" are employed in the literature instead of "enumeration" seems principally due to the fact that "enumeration," despite its importance in Descartes's method, rarely plays a substantial role in interpretations of Descartes's method, which typically focus narrowly on intuition and deduction alone (i.e., operations that can only be performed *after* enumeration$_1$).

[35] Descartes does not deduce the solution to the problem of the anaclastic in *Rules*; he only proposes the deductive order in which the problem would have to be solved. Following his proposed order, I carry out the deduction myself in Chapter 10.

[36] In *Rules*$_{\text{CM}}$, Descartes is explicit about the fact that "there are some problems in which the whole method consists solely in this operation [enumeration] [*Et nonnullae sunt difficultates, ad quas tota Methodus in hac sola Operatione consistit*] […]" (CM fo. 8^{r}). See AT 10:391, CSM 1:27. For more detailed discussion, see Chapter 5.

[37] Garber 2001, 34–9; Schuster 2013, 248–57; and Clarke 1982, 165–80 deny that Descartes really applies his method to the problem of the limits of knowledge in *Rules*. I discuss their interpretations in more detail in Chapter 5.

[38] I discuss the role played by recreational mathematics in Descartes's method in Section 3.6.

[39] See Gilbert cited in Kelly 1965, 60. The thesis that the magnet contains a soul can be traced as far back as Thales. See Aristotle, *De Anima* 1.2 405^{a}19–21, cited in Curd and McKirahan 2011, 15.

[40] The second sentence in this passage may seem to suggest that enumeration$_2$ should be regarded, not as distinct from deduction, but rather as a species of deduction. As the third sentence makes clear, however, Descartes regards any "complex and involved" inference that is not reducible to intuition as an enumeration$_2$. In *Rules*, deduction is *not* the genus, but rather a *species* of the genus "inference." Enumeration$_2$ is a distinct species of inference, which Descartes emphasizes by assigning it a distinct term. Using "deduction" to cover both deduction and enumeration$_2$ is likely to generate confusion about which species of inference Descartes is referring to.

[41] Gaukroger 1989 does not discuss enumeration$_2$ (or any other sense of enumeration) in his interpretation of Descartes's theory of inference in *Rules*.

[42] For an example of a complex non-linear inference, see Chapter 5, where I discuss Descartes's solution to the problem of the limits of knowledge.

[43] In the title of Rule 7, Descartes writes that "every single thing relating to our undertaking must be surveyed in a continuous and wholly uninterrupted movement of thought [*cogitationis motu*]" (AT 10:388, CSM 1:25). But in the second part of the paragraph, he refers instead to a "continuous movement of the imagination [*imaginationis motu*]." In his edition, Crapulli (Descartes 1966, 22) replaces *imaginationis motu* with *cogitationis motu*. More recently, Beyssade and Kambouchner (Descartes 2016, 691, n. 134), basing their choice on both the Amsterdam and the Hanover manuscripts, retain *imaginationis motu*. (The relevant passage is not contained in the Cambridge

manuscript.) I agree with Beyssade and Kambouchner that *imaginationis motu* is better, and the principal manuscripts certainly recommend it. But the most important reason (as Beyssade and Kambouchner also point out) is proper to Descartes's theory of the faculties in *Rules*: the example Descartes discusses here is about *magnitudes*, and since the imagination must aid the intellect whenever the latter intuits magnitudes, this constrains Descartes to introduce *imaginationis motu* as a *special case* of *cogitationis motu*. For more details on Descartes's theory of the faculties in *Rules*, see Chapters 7–8.

44 Beck 1952, 88–9 (citing Joachim 1957, 45, n. 1, Hamelin 1921, 82, and Kemp Smith 1902, 33) correctly argues that "the relation of deduction and intellectual intuition definitely confirms the fundamental identity of the twofold activity of mind." He also (Beck 1952, 143) describes enumeration$_2$ as "a verification of the logical steps already traversed in a deductive process." This suggests that the inference is first performed by deduction, and only verified by enumeration$_2$. This may be so in some cases, but it is not necessarily so. In many cases, enumeration$_2$ is both inference and verification: the inference produced via enumeration$_2$ is simply repeated over and over with ever more facility. As Descartes makes clear in Rules 7 and 11, any inference that is too complex to be absorbed in one act of intuition simply *is* an enumeration$_2$.

45 For examples of sufficient enumeration$_1$, sufficient enumeration$_2$, complete enumeration$_1$, and other examples of sufficient enumeration$_3$, see Chapter 5, Sections 5.2–5.3; Chapter 7, Section 7.2; and Chapter 10, Section 10.3.

46 Weber 1964, 55 mistakenly equates completeness and sufficiency and mistranslates *defectiva* (AT 10:389, line 27) as *incomplète*, effectively denying that an enumeration can be both incomplete and yet sufficient. For an identical criticism, see Marion 1975, 105, n. 99.

47 I discuss this analogy in more detail in Chapter 10, Section 10.4.

48 The demonstration is cited and discussed in Costabel [1982] 2013, 45–6. See also Polya 1954, 1:168–9.

49 For a valuable interpretation of enumeration in *Rules*, see Smith 2010. Smith's interpretation of enumeration is very broad, and seems to confuse a number of operations that Descartes distinguishes from one another in *Rules*. For example, Smith claims that enumeration renders ideas clear and distinct by revealing the relations of epistemic priority and posteriority between the simple natures that compose them (see ibid., 67–69, 89). For Descartes, however, these epistemic relations are not revealed by means of enumeration, but rather by means of intuition and deduction (AT 10:421, 424–5; CSM 1:45 and 47–8). Descartes only ever mentions intuition and deduction when discussing the relations (necessary and contingent conjunctions) between simple natures, never enumeration. See Chapters 7 and 8.

50 Concrete examples of enumeration in action can be found in Chapter 5, Section 5.2; Chapter 7, Section 7.2; and Chapter 10, Section 10.3.

51 I explain why in Chapter 9.

52 This is not to suggest that complex problems in mathematics require no reduction to their simplest component parts; they do. The difference is that these parts can be enumerated$_1$ based solely on the statement of the problem, whereas in natural philosophy this is not the case. On mathematics in *Rules*, see Chapter 9. See also Sasaki 2003, 159–205; Bos 2001, 261–71; and Rabouin 2009, 251–347.

53 See also Beyssade 1979, 144 and Beyssade and Kambouchner in Descartes 2016, 698, n. 182.

54 As Budnick 2018, 63–4 aptly describes it: "*Cet exercice très formateur développe l'esprit d'analyse* et la sagacité […]. *En recréant les phénomènes ou en en révélant les principes*

mathématiques sous-jacente, les récréations permettent d'appréhender, même de façon superficielle, les nombres et les mesures qui régissent les lois de la nature. […] Les récréations physiques en tant que spectacle agréable pour les sens le sont aussi pour l'esprit puisqu'il s'agit de découvrir les effets cachés de la nature qui sont reproduites à petite échelle ainsi que les nouveautés scientifiques dans un cadre ludique" (my emphasis). Unlike newcomers, the most advanced recreational mathematicians "sont capables de comprendre tous les problèmes et leurs démonstrations mais aussi d'inventer leurs propres règles, des variantes voire de nouveaux problèmes" (my emphasis). Recreational mathematics plays an important role in seventeenth-century mathematics and seems to have had a reasonably wide appreciation. See, e.g., de Méziriac 1612 (*Problèmes plaisans et delectables, qui se font par les nombres*); Leurechon 1624 (*Récréations mathématique composées de plusieurs problèmes plaisans et facetieux*) (For an in-depth analysis of the origins, sources, and authorship of this text, see Heeffer 2006); and Descartes's close associate in Paris, Claude Mydorge in Mydorge 1630 (*Examen du livre des Récréations mathématiques*), which provides analyses of problems in Leurechon 1624. The problem of the Tantalus cup discussed in Section 3.4.1.1 above can be found in Leurechon 1624, 32–3 and Mydorge 1630, 75–7. Mydorge's book also includes riddles and tricks in arithmetic; geometry; physics; optics; mechanics; music; and cosmography. On the likely influence of Mydorge's *Examen* on Descartes, see Hattab 2009, 90–1. For more recent examples of recreational mathematics, see, e.g., Averbach and Chein 2000. On recreational mathematics in France during the seventeenth century, see Chabaud 1994 and Budnik 2018. On the history of recreational mathematics and its role in the history of mathematics, see Singmaster 1994.

55 See Kapp 1975; Marion 1975, 155–61; Gaukroger 1989, 32.

56 See, e.g., Vinciolo 1587. Frederico de Vinciolo was a Venitian lace-maker and pattern-designer in the court of Henry II. His book, *Les singuliers et nouveaux pourtraicts*, which was written and published in French, appeared in many editions between 1587–1623.

57 See Descartes's letter to Regius (January 1642, AT 3:503–504, CSMK 3:208), cited in Chapter 2, Section 2.2. See also letter to Mesland, May 2, 1644, where Descartes specifically refers to purely spiritual "traces which remain in the mind itself," traces that are "wholly different from" the material "traces left in the brain after an image has been imprinted on it" (AT 4:114, CSMK 3:233). See also Rodis Lewis 1950, 98–100.

58 Aristotle 1984, 2:1744 (*Nicomachean Ethics* II. 2 1104^{a}7–9).

59 Wittgenstein 1958, 85.

60 Wittgenstein 1978, 331.

61 The Uniformity Thesis has determined most interpretations of Descartes's method from the late nineteenth century up to the present. See, e.g., Liard 1880, 574; Boyce-Gibson 1898b, 348; Keeling [1934] 1968, 79; Schouls 1980, 63; Clarke 1982, 165–80; Garber 1992, 34–44; Garber 2001, 34–9; Dear 2000, 157–61; and Schuster 2013, 215–20. (Scholarly consensus on the Uniformity Thesis neither requires nor entails agreement about Descartes's method and its relation to Cartesian science in all respects.) The Uniformity Thesis has also determined most interpretations of "the" scientific method (Cartesian and other) by most historians and philosophers of science. See, e.g., Bachelard 1934, 135–79 and 165–6; Bachelard 1972, 35–44 (and the discussion in Lecourt 1968, 72); Koyré 1956, 15; Kuhn 1977, 137, 150; Feyerabend 1975, 14; Putnam 1981, 192; Schuster and Yeo 1986; Shapin 2018, 94–5. See also my discussion in the Introduction.

62 Aristotle 1984, 2:1744 (*Nicomachean Ethics* II. 2 1104^{a}7–9).

4

The Culture of the Method

The Methodological Function of *Mathesis Universalis*

4.1 *Mathesis Universalis*: The Second Degree of the Cartesian Scientific *Habitus*

In this chapter, I argue that *mathesis universalis* is an essentially *propaedeutic* science and explain the role that practice in *mathesis universalis* plays in the formation of the Cartesian scientific *habitus*. As we have seen in Chapter 3, in recreational mathematics, I learn how to abstract problems from subject-matters and enumerate$_1$ all and only conditions relevant to the solution of a problem, so that once all of the relevant conditions have been enumerated$_1$, the problem can be solved. Practice in the so-called "feminine arts" and recreational mathematics makes one perspicuous in intuition and sagacious in deduction, but only up to a point. As intellectual virtues or *habitus*, perspicacity and sagacity *come in degrees*. One can be more or less perspicuous or sagacious; intellectual virtues can be more or less perfect. The so-called "feminine arts" and recreational mathematics only produce what I term the "first degree" of the Cartesian scientific *habitus*. This prepares the human *ingenium* for the "second degree," which can only be acquired by practice in *mathesis universalis*. The principal function of *mathesis universalis* is to shore up and intensify perspicacity in intuition and sagacity in deduction. It is only after one has acquired the second degree of the Cartesian scientific *habitus* that one can "graduate" to properly methodological problems and problems

in the more advanced sciences, beginning with the problem of the limits of knowledge (Chapters 5–8) and then problems in advanced mathematics (Chapter 9) and natural philosophy (Chapter 10).

In Section 4.2, I discuss Descartes's definition of *mathesis universalis* as the science of "order and measure," and I argue that, contrary to an historically longstanding and widespread tendency to interpret *mathesis universalis* in expansive terms as either identical to Cartesian mathematics as a whole or (even more expansively) to Descartes's method, in *Rules* *mathesis universalis* refers to one of the simplest and easiest mathematical sciences, well-known since antiquity: the theory of proportions.[1] In Section 4.3, I argue that reflection on the operations needed to solve problems about continuous and mean proportionals yields Descartes's theory of relatives and absolutes in Rule 6, which any well-ordered enumeration$_1$ integrates as it reduces problems to their simplest component parts and orders them according to their relations of epistemic dependency (relative simplicity and complexity, such that simpler problems must be solved prior to the more complex ones). In Section 4.4, I argue that *mathesis universalis* also yields a classification of different ways in which perfectly understood problems may be solved. In Section 4.5, I show how *mathesis universalis* builds on the skills learned via practice in recreational mathematics (discussed in Chapter 3, Section 3.6). In Section 4.6, I show how *mathesis universalis* unifies mathematics, and in Sections 4.7–4.7.1, I delve into the debate about the relation between *mathesis universalis*, Descartes's mathematics, and the method. I also draw on the Cambridge manuscript in order to bolster the principal arguments developed in Sections 4.2–4.5 and propose a revision in standard accounts of the controversial and rather complex chronological genesis of Rule 4.

4.2 The Science of Order and Measure

As I briefly indicated in Section 4.1, since the nineteenth century, *mathesis universalis* has been regarded by historians and historically-oriented philosophers as an originally Cartesian enterprise defined by the "mathematization of nature."[2] This enterprise, they argue, has no precedent in the history of science prior to the seventeenth century, and it commands

the entire development of modern science as a program shared by all philosophers and scientists after Descartes. Scholars influenced by these interpretations have argued that in Descartes *mathesis universalis* is simply another name for the method, which embraces both mathematical and non-mathematical sciences.[3] Others have argued that *mathesis universalis* embraces a more restricted terrain: Cartesian mathematics, which they then interpret in a variety of ways.[4] Below, I argue that in *Rules mathesis universalis* refers to one of the simplest and easiest mathematical sciences, well-known since antiquity: the theory of proportions.[5] Once the argument is complete, it will be easier to evaluate where it stands in the debate about *mathesis universalis* in Section 4.7.

In the course of his reflections on the unity of mathematics in Rule 4, Descartes concludes that arithmetic, geometry, astronomy, music, optics, and mechanics are all "called branches of mathematics" because their "exclusive concern" is "with problems of order and measure" (AT 10:377–8, CSM 1:19). It is "irrelevant whether the measure in question involves numbers, shapes, stars, sounds or any other object whatever" (ibid.). In a crucial paragraph, he continues:

> This made me realize that there must be a general science [*generalem…scientiam*] which explains all the points that can be raised concerning order and measure [*ordinem et mensuram*] irrespective of the subject-matter, and that this science should be termed *mathesis universalis* – a venerable term with a well-established meaning – for it covers everything that entitles these other sciences to be called branches of mathematics. How superior it is to these subordinate sciences both in utility and simplicity is clear from the fact that it covers all they deal with, and more besides; and any difficulties it involves apply to these as well, whereas their particular subject-matter involves difficulties which it lacks. Now everyone knows the name of this subject and without even studying it understands what its subject-matter is. So why is it that most people painstakingly pursue the other disciplines which depend on it, and no one bothers to learn this one? No doubt I would find that very surprising if I did not know that everyone thinks the subject too easy, and if I had not long since observed that the human intellect always bypasses subjects which it thinks it can easily master and directly hurries on to new and grander things (AT 10:378, CSM 1:19–20).

All problems in mathematics are essentially problems about order and measure. But what, exactly, are order and measure? Descartes's definition of *mathesis universalis* as the science of order and measure is *not* original, and he does not even present it as such. On the contrary, he explicitly refers to *mathesis universalis* as a "venerable term with a well-established meaning," and he points out that "everyone knows the name of this subject and without even studying it knows what its subject-matter is." In the

sixteenth and seventeenth centuries, everyone would have known that "order and measure" refers to the theory of proportions.[6] Indeed, the only (oblique) reference to *mathesis universalis* in Descartes's corpus after *Rules* —in *Discourse* II—describes his mathematical research in the 1620s (while writing *Rules*) and is explicitly defined by the theory of proportions: "Nor did I have the intention of trying to learn all the special sciences commonly called 'mathematics.' For I saw that, despite the diversity of their objects, they agree in considering nothing but the various relations or proportions that hold between these objects" (see AT 6:19–20, CSM 1:120–1). Thus, in order to understand what Descartes means by "order and measure," one must understand what proportional relations are.

Proportional relations obtain between both discrete and continuous magnitudes (e.g., 3, 6, 12, 24, etc.; line, square, cube, etc.). Consequently, proportional relations are prior to or transcend the distinction between arithmetic, geometry, and the subalternate branches of mathematics whose principles depend on one of these two sciences (music, in the case of arithmetic; astronomy, optics, and mechanics, in the case of geometry). There are two types of problem in *mathesis universalis*. One must either (1) extend a series of continued proportionals (e.g., given 3 and 6, find 12, 24, etc.) or (2) find one or more mean proportionals (e.g., given 3 and 12, find one mean proportional between them, or given 3 and 24, find two mean proportionals between them, etc.) (see Rule 6, AT 10:384–7, CSM 1:23–4).[7] In both types of problem, the magnitudes are one and all *measurable by a unit*, which defines the relation between each member in the series. The unit measure makes it possible to compare the magnitudes to one another, and the *serial order* of the magnitudes depends on *how many times they contain the unit measure*. The unit measure is the simplest relation in the series, and each subsequent member in the series is more complex relative to the unit measure. Consequently, in *mathesis universalis* "order and measure" refers to magnitudes ordered according to their relative simplicity and complexity as defined by their relation to the unit measure. The unit defines the measure, and the order of magnitudes is defined by the relation each magnitude has to the unit measure. The unity of mathematics consists in the fact that relations between all magnitudes can be expressed in terms of order and measure (see Section 4.6 below).

But it is not only the unity of mathematics that interests Descartes here. What also interests him is the *methodological import* of *mathesis*

universalis, i.e., the role *mathesis universalis* plays in the scientific formation of *ingenium.* This is clear from the way in which he determines the place of *mathesis universalis* in the order of sciences that must be learned in order to acquire the Cartesian scientific *habitus* in Rule 4:

> Aware of how slender my powers are, I have resolved in my search for knowledge of things to adhere unswervingly to a definite order, always starting with the simplest and easiest things and never going beyond them till there seems to be nothing further which is worth achieving where they are concerned. *Up to now, therefore, I have devoted all my energies to this universal mathematics* [*Mathesim universalisem*], *so that I think I shall be able in due course to tackle the somewhat more advanced sciences, without my efforts being premature* (AT 10:378–9, CSM 1:20; my emphasis).

Here, Descartes insists that *mathesis universalis* is the first science one must learn in order to acquire the Cartesian scientific *habitus* and eventually solve problems in the "somewhat more advanced sciences." Why? Because *mathesis universalis* is the "simplest and the easiest," and the simplest and easiest sciences must be learned prior to the more advanced ones. The simplicity and ease of *mathesis universalis* vis-à-vis the special mathematical sciences consists in the fact that it "covers all they deal with, and more besides; and any difficulties it involves apply to these as well, whereas their particular subject-matter involves difficulties which it lacks" (AT 10:378, CSM 1:19). That *mathesis universalis* is the "simplest and easiest" science clearly entails that it is *not to be counted among the more advanced sciences*, contrary to a widespread tendency to regard it as equivalent to Descartes's method or to Cartesian science as such.[8] What is new about *mathesis universalis* in *Rules* is neither the name of the science nor the definition, but rather the way Descartes presses *mathesis universalis* into a regime of cognitive practice defined by a definite order of learning, which ultimately terminates in the *habitus* deployed in solutions to problems in "the somewhat more advanced sciences." After practice in the so-called "feminine arts" and recreational mathematics, but before practice in the more advanced sciences, the human *ingenium* advances to *mathesis universalis.* As a propaedeutic science, practice in *mathesis universalis* further perfects the natural operations of the human *ingenium*, which are deployed in solutions to any problem, be they mathematical or non-mathematical. This is nowhere clearer than in Rules 5–6, where Descartes defines the type of order required in solutions to any problem.

4.3 First Lesson: The Theory of Order in Rule 6

As we have seen in Chapter 3, Section 3.4.1, in Rule 5 Descartes requires "[reducing] complicated and obscure propositions step by step to simpler ones, and then, starting with the intuition of the simplest ones of all, try to ascend through the same steps to a knowledge of all the rest" (AT 10:379, CSM 1:20). With the exception of the simplest problems, however, "the order that is required here is often so obscure and complicated that not everyone can make out what it is" (AT 10:380, CSM 1:21). Thus, rather than leap immediately into problems that have such "obscure and complicated" order, one must begin by "sagaciously reflecting" on deductions that have already been effectuated in order to observe the relations between their parts. As Descartes puts it in Rule 6:

> In order to distinguish the simplest things from those that are complicated and to set them out in an orderly manner, we should attend to what is most simple in each series of things in which we have directly deduced some truths from others, and should observe how all the rest are more, or less, or equally removed from the simplest (AT 10:381, CSM 1:21).

Descartes writes "we should attend to what is most simple" in a deduction that has *already been effectuated*. This is important because practice in *mathesis universalis* will not *consist* in actually *finding* continuous and mean proportionals (even if it involves finding them), but rather in "sagaciously reflecting" on the epistemic relations that obtain between the members of *given* proportional series.[9] This is confirmed later in Rule 6, where Descartes once more insists that we must accustom ourselves "to reflecting with some discernment on the minute details of the things we have already perceived [*assuescamus ad minima quaeque ex ijs, quae jam ante percepimus, cum quadam sagacitate reflectere*]" (AT 10:384, CSM 1:23). Finally:

> We must not begin our studies by investigating difficult matters. Before tackling any specific problems we ought first to make a random selection of truths which happen to be at hand, and ought then to see whether we can deduce some other truths from them step by step, and from these still others, and so in logical sequence. This done, we should reflect attentively on the truths we have discovered and carefully consider why it was we were able to discover some of these truths sooner and more easily than others, and what these truths are. This will enable us to judge, when tackling a specific problem, what points we may usefully concentrate on discovering first (AT 10:384, CSM 1:23).

In all of these passages from Rule 6, the message is the same: do not leap immediately into solving problems in the sciences; sagaciously reflect on the epistemic relations that compose available deductions; determine the order of dependence between these relations; identify why, on the basis of these relations, some truths can be more easily discovered than others. The examples Descartes provides in Rule 6 are all drawn from the theory of proportions (see AT 10:384–7, CSM 1:23–4); they are examples of relations between continuous and mean proportionals. The theory of proportions (or rather the proportional series themselves) paradigmatically exhibits the order prescribed in Rule 6. As we have seen in Chapter 3, Section 3.3 and in Section 4.2 of this chapter, in a series of mean or continuous proportionals (3:6 = 6:12 = 12:24, etc.), the relation between the simplest member and all subsequent members is such that the more complex relations can be deduced from the simplest relations. Furthermore, how far removed the more complex members are from the simplest member can be clearly discerned. Finally, which relations are easier to discover is also clear: it is much easier to discover the relation between 3 and 6 than between 3 and 24. The lesson drawn from the theory of proportions in Rule 6 is that *the parts of any problem must have the same relations to one another as the members in a series of proportionals*. This similarity in structure enables Descartes to develop criteria whereby the "obscure and complicated order" contained in a problem can be rendered transparent to the mind, such that the mind may then solve the problem via intuition and deduction.

In the first part of Rule 6, Descartes lays out his famous theory of absolutes and relatives, according to which "all things can be arranged serially into various groups, not insofar as they can be referred to some ontological genus (such as the categories into which the philosophers divide things), but insofar as some things can be known on the basis of others" (AT 10:381, CSM 1:21). Each part or condition of a problem must be placed in a series according to its relative simplicity or complexity, so that "when a difficulty arises, we can see at once whether it will be worth looking at any others first, and if so which ones and in what order" (ibid.). The core of the theory of absolutes and relatives is contained in the following passages:

> In order to be able to do this correctly [viz., distinguish between the simple and the complex and subject them to order], we should note first that everything, with regard to its possible usefulness to our project, may be termed either "absolute" or "relative" – our project being,

> not to inspect the isolated natures of things, but to compare them with each other so that some may be known on the basis of others. Thus when a difficulty arises, we can see at once whether it will be worth looking at any of the others first, and if so which ones and in what order.
>
> (AT 10:381, CSM 1:21)

> I call "absolute" whatever has within it the pure and simple nature in question; that is, whatever is viewed as being independent, a cause, simple, universal, single, equal, similar, straight, and other qualities of that sort. I call this the simplest and the easiest thing when we can make use of it in solving problems.
>
> (AT 10:381–382, CSM 1:21)

> The "relative," on the other hand, is what shares the same nature, or at least something of the same nature, in virtue of which we can relate it to the absolute and deduce it from the absolute in a definite series of steps. The concept of the "relative" includes other terms besides, which I call "relations": these include whatever is said to be dependent, an effect, composite, particular, many, unequal, dissimilar, oblique, etc. The further removed from the absolute such relative attributes are, the more mutually dependent relations of this sort they contain. This Rule points out that all these relations should be distinguished, and the interconnections between them, and their natural order, should be noted, so that given the last term we should be able to reach the one that is absolute in the highest degree, by passing through all the intermediate ones.
>
> (AT 10:382, CSM 1:21–2)

Distinguishing between the absolutes and relatives in any problem constitutes an enumeration$_1$. I need to reduce the problem to its simplest component problems and determine the order in which to solve them (e.g., in the problem of the anaclastic, discussed in Chapter 3, Section 3.4.1). As the first of the above-cited passages indicates, the act that constitutes enumerations$_1$ is *comparison*. One must compare things according to their relative simplicity and complexity in order to determine their place in the series. The reduction of a problem (1) to a series of simpler problems (2) via acts of comparison that are (3) defined by relations of absolute and relative complexity, which can be discovered by intuiting (4) epistemic dependencies (what needs to be known on the basis of what) occurs prior to and as a condition of the solution to any problem. (1)–(4) describe a *practical technique* that constitutes the Cartesian scientific *habitus*, not a theoretical epistemology.[10]

The required order is clearly epistemic, not ontological. For example, cause and effect are ontologically correlative: all causes produce some effect, and all effects proceed from some cause.[11] Epistemically, however, cause and effect are not correlative, "for if we want to know what the effect is, we must know the cause first, and not vice versa" (AT 10:383, CSM

1:22). While all effects proceed from some cause and all causes produce some effect, knowledge of an effect does not produce knowledge of a cause. When the intellect compares things in order to solve a problem, it compares them as they are relative to the intellect; it is only relative to the intellect that one problem must be solved prior to another. In themselves, things may not have the same relations of priority and posteriority that they have in relation to the intellect. This principle of "epistemic asymmetry"—the fact that some things can only be known on the basis of others, but not vice versa—defines Descartes's concept of order in Rule 6. Relative to the intellect, the ontologically correlative relation between cause and effect is an epistemically asymmetrical relation between an "absolute" and a "relative." More broadly, in relation to the intellect *everything in any problem* is either absolute or relative, since problems simply *are* composed of parts that epistemically depend on others. In Chapter 2, Section 2.4, I argued that in *Rules* Descartes extrinsically denominates all things as simple or complex in relation to the intellect alone. In his theory of absolutes and relatives, he extends this extrinsic denomination to include any and all relations between "everything, with regard to its possible usefulness to our project," which is precisely to solve problems in the sciences (AT 10:381, CSM 1:21).

4.4 Second Lesson: Direct and Indirect Deductions

As I argued in Section 4.3, the methodological function of *mathesis universalis* consists *not* (or not principally) in solving problems about proportions, but rather in "sagaciously reflecting" on relations between the members of a proportional series. These series illustrate Descartes's theory of order—his theory of absolutes and relatives—in Rule 6.[12] Now, I will discuss how such reflection also yields a classification of different types of (perfectly understood) problem and the procedures whereby they may be solved. As we have already seen in Chapter 3, Section 3.5, classifications of problems and solution procedures are central to Cartesian scientific practice. This is especially clear in Descartes's mathematics. From his earliest research in mathematics up to the *Geometry*, Descartes always insisted on classifying different types of problem and correlating them to

the procedures (equations, constructions) whereby they may be solved.[13] One encounters a problem, determines what type of problem it is (its degree of complexity), and proceeds to solve it by means of the corresponding equations and acceptable constructions. In many cases, more complex problems can be reduced to simpler ones, and so solved by the very same procedures typically employed to solve these simpler ones. As we will see below, this art of problems is no less present in *Rules*. The Cartesian scientific *habitus*, as a problem-solving disposition, never tackles any problem head-on. Before solving any problem, I must determine what type of problem it is in order to invent the procedure whereby it may be solved by means of a finite number of operations (AT 10:381, CSM 1:21).

We have already encountered Descartes's distinction between perfectly and imperfectly understood problems in Chapter 3, Section 3.5. All problems about proportions are perfectly understood, but this does not mean that they all can be solved in the same way. Descartes distinguishes between problems that can be solved "directly" and problems that can only be solved "indirectly" (AT 10:386–7, CSM 1:24). The examples Descartes chooses to illustrate this distinction are $3:6 = x:12$ and $3:x = x:12$, respectively. As the reader can see, the proportional series is identical in both cases; the difference between them hinges on which magnitudes are known and which are not. In the first example, the solution to the problem is easier to deduce: I immediately see that $6 = 2 \cdot 3$, and so all I need to do to determine the value of x is to determine what magnitude 12 doubles: viz., 6. The relation between the first two magnitudes is given in such a way that their relation to the second two magnitudes can be seen quite naturally. In the second example, the solution is a bit harder to deduce: "I […] observe that given the magnitudes 3 and 6, I easily found a third magnitude which is in continued proportion, viz., 12, yet, when the extreme terms 3 and 12 were given, I could not find just as easily the mean proportional 6" (AT 10:385, CSM 1:23). "If we look into the reason for this," Descartes continues, "it is obvious that we have here a quite different type of problem from the preceding one. For if we are to find the mean proportional, *we must attend at the same time to the two extreme terms and the ratio between them, in order to obtain a new ratio by dividing this one*" (AT 10:385, CSM 1:23–4; my emphasis). In other words, I must multiply the two extremes ($12 \cdot 3 = 36 = x^2$) and divide it ($x = \sqrt{36}$) in order to obtain the mean proportional, $x = 6$.[14] "This is a very different task from that of finding a third magnitude,

given two magnitudes in continued proportion" (ibid.). The difference between the first and second examples is that only *one act* of intuition is required in order to find a third magnitude given two magnitudes in continued proportion, whereas *two acts* of intuition are required in order to find a mean proportional given two extreme magnitudes. What emerges here is that Descartes defines the complexity of a problem by the *number of operations* required to solve it, not only separately, but also "at the same time."

A further difference between direct and indirect deductions is that in problems that can be solved directly, the series can continue indefinitely *but the degree of complexity always remains the same, however many unknowns there are*: "The nature of the problem is no different when we are trying to find three, four, or more magnitudes of this sort, since each one has to be found separately and without regard to the others" (AT 10:385, CSM 1:23). I can continue the series of continued proportionals 3, 6, 12, etc., without increasing the complexity of the deduction, since however many unknown magnitudes there are, every subsequent magnitude is only determined by the magnitude that immediately precedes it. All other magnitudes can be disregarded, so that the extension of the series never requires more than one act at a time.This is not the case in problems that can only be solved indirectly, such as the problem of finding one or more mean proportionals. The more magnitudes between the extremes there are, the more complex the problem becomes. For example, $3:x = x:y = y:24$ and $3:x = x:y = y:z = z:48$. Regarding the first example, Descartes writes: "Here, we have another sort of problem again, an even more complicated one than either of the preceding ones. We have to attend not just to one thing or two but to three different things at the same time [*simul est attendendum*], if we are to find a fourth" (AT 10:386, CSM 1:24). Once more, the complexity of the problem depends on the *number of operations* that must be performed *at the same time* in order to solve it. The more relations the mind must attend to, the more stress is placed on intuition. The fourth and final example is instructive in yet another way. This problem initially has the appearance of being more complex than any of the previous three. Descartes shows, however, that it can be reduced to a problem similar in form to the second problem ($3:x = x:12$) by dividing it into three separate problems, one for each mean proportional: $3:x = x:48$, $3:x = x:12$, and $12:x = x:48$. After carrying out the relevant operations, the value of x in each of these

proportional series is 12, 6, and 24, respectively, so that 3:6 = 6:12 = 12:24 = 24:48. This example provides a clear case in which more complex problems can be reduced to less complex problems, and solved in exactly the same way as the latter problems are.[15] The scale of complexity never becomes prohibitive; at a certain level more complex problems can be solved by means of procedures required in solutions to simpler ones. This reveals the fundamental unity of the simple and the complex in Descartes's theory of problems: since the complex is composed of the simple, it is always reducible to its simplest parts. Since no problem, however complex, resists such reduction, the possibility of solving every problem by means of intuition and deduction is secured in advance.[16]

Following Descartes's recommendation to "sagaciously reflect" on these examples, one can see that different types of problems do indeed require different types of solution, and that the human mind must discern the type of problem in order to identify the required solution: "From these examples I realize how in our pursuit of knowledge of a given thing we can follow different paths [*diversas vias*], one of which is much more difficult and obscure than the other" (AT 10:386, CSM 1:24).

After discussing these examples from *mathesis universalis*, Descartes writes:

> All of this is so clear as to seem almost childish; nevertheless when I think carefully about it [*attente reflectendo intelligo*], I can see what sort of complications are involved in all the questions one can ask about the proportions or relations between things, and in what order the questions should be investigated. This one point encompasses the essential core of the entire science of pure mathematics [*purae Mathematicae*] (AT 10:384–385, CSM 1:23).

We are now in a better position to understand what Descartes means here. First, Descartes's examples illustrate that the solution to all problems in mathematics requires abstracting problems from subject-matters and comparing pure magnitudes to one another in order to deduce unknown magnitudes from known magnitudes. Second and no less importantly, Descartes's examples illustrate his conception of science as a *habitus* that enables one to classify different types of problem and their corresponding solutions. As I indicated above, throughout Descartes's career the underlying conception of science consistently remains one in which problems must be classified according to their degree of complexity and the appropriate solution to each type must be determined.[17] This is precisely

what Descartes had hoped to accomplish in his program for a "completely new science" in his letter to Beeckman in 1619 and what he actually accomplishes in *Geometry*. The same conception of science as a problem-solving *habitus* is clearly reflected in Rule 3, where Descartes writes that "we shall never become mathematicians if we lack the intellectual aptitude to solve any given problem" (AT 10:367, CSM 1:13).

4.5 From Recreational Mathematics to *Mathesis Universalis*

In Chapter 3, Section 3.6, I showed how solving problems in the so-called "feminine arts" and recreational mathematics produces two intellectual virtues or *habitus* that perfect intuition and deduction: perspicacity and sagacity, respectively. In recreational mathematics (e.g., the problem of the Tantalus cup) one learns how to abstract problems from particular subject-matters and enumerate$_1$ all and only those conditions relevant to the solution of the problem. In *mathesis universalis*, the intellectual virtues or *habitus* acquired via practice in the so-called "feminine arts" and recreational mathematics are further perfected in three principal ways:

(1) Unlike recreational mathematics, *mathesis universalis* is a completely general science. The abstraction of problems from particular subject-matters learned in solving problems in recreational mathematics is *presupposed and integrated* in *mathesis universalis*; one must already know how to abstract problems from subject-matters in order to understand what the "*universalis*" in "*mathesis universalis*" means: the solution to all problems in mathematics requires abstracting problems from subject-matters and comparing pure magnitudes to one another in order deduce unknown magnitudes from known magnitudes.

(2) *Mathesis universalis* is the science in which the order required in the solution to any problem is most perspicuously exhibited and, therefore, most effectively learned. When I reflect on proportional series, I learn how to "distinguish the simplest things from those that are complicated and set them out in an orderly manner," so that I attend "to what is most simple in each series of things […] and observe how all the rest are more, or less, or equally removed from the simplest" (AT 10:381, CSM 1:21). For example,

in the proportional series 3:6 = x:12, the simplest relation in the series is the relation between the first two magnitudes (3:6), and the more complex relation (x:12) comes second because it is defined by the first, which it contains (i.e., $x = 6$, which doubles 3, and 12 doubles 6). The series must be ordered so that the place each member occupies in the series is determined exclusively by its relation to the other members, and the more complex members are always deducible from the simpler members (whatever they may be in any given case).

(3) Finally, in *mathesis universalis* I learn how to classify different types of problem and the procedures whereby they may be solved. Reflection on *mathesis universalis* yields what has aptly been described as a "logic of problems."[18] In short, the transition from recreational mathematics to *mathesis universalis* is a transition from *solving problems individually* to a more developed *theory of problems* via (1) a definite regime of abstraction; (2) a definite conception of serial order; and (3) a definite classification of types of problem and the procedures whereby they may be solved.

As I argued in Chapter 2, Section 2.1, what distinguishes the Cartesian scientific *habitus* from scholastic theories of scientific *habitus* is that the Cartesian scientific *habitus* is not restricted to reproducing the acts whereby it is acquired. By solving and reflecting on simple problems, *I perfect my ability to solve other, more complex problems*, "for knowledge of one truth does not, like skill in one art, hinder us from discovering another; on the contrary it helps us" (AT 10:360, CSM 1:9). It is precisely in this sense that reflection on *mathesis universalis* prepares the human *ingenium* for the "somewhat more advanced sciences," as Descartes puts it in Rule 4 (AT 10:378–9, CSM 1:20).

4.6 *Mathesis Universalis* and the Unity of Mathematics in Rules 13–21

The theory of proportions plays a foundational role in all mathematics, including the "analytic art" or algebraic analysis. As Viète had shown prior to Descartes, proportions constitute equations, and equations reduce to proportions.[19] Suppose the problem is to solve $7x = 35$. This problem is equivalent to the problem of finding one mean proportional: $1:x = 7:35$.

Suppose the problem is to solve $35/x = 5$. This problem is also equivalent to the problem of finding one mean proportional: $1:5 = x:35$. These simple cases illustrate that equations, no matter how complex they may be, express proportional relations and can, therefore, be reduced to problems about proportions. Solving an equation is equivalent to finding one or more mean proportionals. As I will argue in more detail in Chapter 9, in Rules 13–21 Descartes shows how algebraic equations can be employed in order to solve problems in geometry. This effectively means that any problem in mathematics can be reduced to a problem about proportions. To solve any problem in mathematics, one must determine the relation between the relevant magnitudes and order them in such a way that the unknown magnitudes can be deduced from the known magnitudes. One assigns letters to all known and unknown magnitudes, supposes that the problem is already solved, and regresses from the solution to its conditions in order to discover the value of the unknown magnitudes or variables on the basis of their relation to known magnitudes or values. *Mathesis universalis* contains the principle behind algebraic analysis because algebraic analysis is founded on the theory of proportions. Furthermore, since algebraic equations reduce to proportions, and since Descartes establishes in Rules 13–21 that such equations can be employed in arithmetic no less than geometry, *mathesis universalis* unifies mathematics. As I argued in Section 4.4, this is one of the things Descartes means when he writes: "This one point encompasses the essential core of the entire science of pure mathematics [*purae Mathematicae*]" (AT 10:384–385, CSM 1:23).

4.7 *Mathesis Universalis*, Cartesian Mathematics, and Method

I have intentionally postponed discussion of the much-debated relation (i.e., identity or lack thereof) between *mathesis universalis*, on the one hand, and Descartes's mathematics pre- and post-*Rules*, Descartes's method, and/or Cartesian science in general, on the other. As I indicated in Sections 4.1–4.2, since the nineteenth century, there has been a widespread tendency to interpret *mathesis universalis* as either identical to Cartesian mathematics as a whole or to Descartes's method.[20] Recent scholarship has revealed that neither of these interpretations is textually sound.[21] I will briefly address

each of these interpretations on the basis of the standard edition of $\textit{Rules}_{\mathrm{AT}}$ before citing recent evidence from the Cambridge manuscript as further confirmation of the interpretation of *mathesis universalis* provided in Sections 4.1–4.5. As we will see, while one does not need the Cambridge manuscript in order to contest standard interpretations of *mathesis universalis*, the manuscript does confirm the interpretation provided in Sections 4.1–4.5. Only on a habitual interpretation of Descartes's method does the proper function of *mathesis universalis* emerge in *Rules*.

The basic textual facts about *mathesis universalis* are not in dispute. Descartes only explicitly mentions *mathesis universalis* twice in his entire *oeuvre*, both times in Rule 4 (see Section 4.2 above). According to the evidence currently available, Descartes never mentions *mathesis universalis* anywhere else in his unpublished or published writings or correspondence. In the Amsterdam manuscript, Descartes's discussion of *mathesis universalis* is contained in Rule 4. (This is what has led some scholars to argue that Descartes regarded *mathesis universalis* as closely related to the method and perhaps as even identical to the method, since Descartes defines the method in Rule 4.) However, in the Hanover manuscript, Descartes's discussion of *mathesis universalis* is appended to the end of the treatise.[22] (This is what has led some scholars to argue that Descartes regarded *mathesis universalis* as distinct from the method.) In the Cambridge manuscript, Descartes does not refer to *mathesis universalis* at all, neither in Rule 4 nor anywhere else.

Since scholars have argued that *mathesis universalis* is identical to either (1) Descartes's mathematics pre–*Rules*, in Rules 13–21, and/or post-*Rules;* (2) Descartes's method as such (and, therefore, Cartesian science as a whole),[23] I will address each interpretation in turn.

(1) *Mathesis universalis* and Descartes's algebraic geometry in Rules 13–21 and the *Geometry*. In the seventeenth century, van Schooten and others identified *mathesis universalis* with Descartes's *Geometry*,[24] but in a letter to Ciermans, Descartes explicitly *distinguishes* between the two. He writes: "I did not explain any of the problems […] in which one considers order and measure" in the *Geometry* (March 23, 1638, AT 2:70). More recently, many historians have argued that *mathesis universalis* is a reference to Descartes's "reformed algebra," first developed in Rules 13–21 and later employed in the *Geometry*.[25] However, Descartes *never* identifies

mathesis universalis and the algebraic geometry developed in Rules 13–21. Furthermore, if *mathesis universalis* were identical to Descartes's algebraic geometry in Rules 13–21, then *mathesis universalis* would be included among "the somewhat more advanced sciences" (AT 10:379, CSM 1:20), but as we have seen in Section 4.2, in Rule 4 Descartes clearly asserts that *mathesis universalis* is *not* included among the more advanced sciences. Finally, regarding *mathesis universalis* Descartes explicitly claims that "everyone knows the name of this subject and without even studying it knows what its subject-matter is." This is certainly not the case when it comes to his algebraic geometry in Rules 13–21: Descartes's algebraic geometry in Rules 13–21 is historically novel. Consequently, *mathesis universalis* cannot be identified with Descartes's algebraic geometry in Rules 13–21 or the *Geometry*.

Others have distinguished between an earlier, purely programmatic *mathesis universalis* pre-*Rules* and a later, more developed *mathesis universalis* in *Rules*.[26] As evidence they cite Descartes's earlier, ambitious program to develop a "completely new science [*scientiam penitus novam*], which would provide a general solution of all possible equations involving any sort of quantity, whether continuous or discrete, each according to its nature," (letter to Beeckman, March 26, 1619, AT 10:156–7, CSMK 3:2). This evidence is then combined with (2) Descartes's research in "physico-mathematics" with Beeckman, which is interpreted to have consisted in solving problems in natural philosophy (e.g., hydrostatics) by means of mathematics alone and then physically interpreting these solutions according to a corpuscular-mechanical ontology.[27] It is then alleged that Descartes interpreted (1) and (2) through Proclus' concept of universal mathematics.[28] This earlier, purely programmatic *mathesis universalis* finally surfaces in Rule 4, and it pertains to the earliest strata of *Rules*.[29] It is then developed into "a general mathematical discipline, providing machinery for the analysis of all problems occurring in properly mathematical fields, including physico-mathematical ones, and putatively establishing the truth of its own procedures and the ontological reference of its objects – its legitimatory functions."[30] This more developed *mathesis universalis* can be found in Rules 12–21 and pertains to the later strata of Rules (1626–1628).

This interpretation of *mathesis universalis* fails for reasons similar to the previous interpretation. The “completely new science” Descartes refers to in his letter to Beeckman in 1619 is not a “venerable term with a well-established meaning,” nor is it a science “everyone knows the name of […] and without even studying it knows what its subject-matter is,” and it is hardly a science “no one bothers to learn” because “everyone thinks the subject too easy” (AT 10:378, CSM 1:19–20). On the contrary, Descartes explicitly describes the “completely new science” as a “gigantic task, […] hardly suitable for one person,” and an “incredibly ambitious project” (AT 10:157, CSMK 3:3). The problems this science is meant to deal with are far more advanced than anything one encounters in *mathesis universalis*. Finally, at no point in his discussion of *mathesis universalis* in Rule 4 does Descartes mention or even obliquely refer to “physico-mathematics.” His claim that arithmetic, geometry, astronomy, music, optics, and mechanics are branches of mathematics cannot be considered an oblique reference to “physico-mathematics,” but rather (by Descartes’s own reckoning) to a standard classification of the mathematical sciences. *Mathesis universalis* in Rule 4 is not an ambitious program that combines mathematics and natural philosophy, since in Rule 4 Descartes clearly regards it as an antecedently established, elementary, and purely mathematical science, and he provides no indication whatever that he intends to revise or expand this science in any way. Finally, as I argued above, on any interpretation, the mathematics developed in Rules 13–21 is historically novel, and cannot be described as a science “everyone knows.”

(2) *Mathesis universalis* and Descartes’s method. Others have maintained that *mathesis universalis* is identical to the method and, therefore, embraces all sciences, both the mathematical and the non-mathematical.[31] However, as we have seen, Descartes describes *mathesis universalis* as prior to “the somewhat more advanced sciences” in Rule 4. Consequently, it does not seem that *mathesis universalis* is identical to the method or that it embraces the more advanced sciences produced by the method.[32]

4.7.1 *Mathesis Universalis* and the Cambridge Manuscript

The Cambridge manuscript also rules out the identity of *mathesis universalis* and the method. In the Cambridge manuscript, Descartes discusses the method in detail without ever referring to *mathesis universalis*. It is, therefore, highly unlikely that the method originated in or grew out of *mathesis universalis*.[33] On the contrary, the Cambridge manuscript clearly indicates that *mathesis universalis* came on the scene *after* Descartes had already developed the method in some detail. Furthermore, Descartes developed the more advanced mathematics in Rules 13–16 (where the Cambridge manuscript ends) independently of *mathesis universalis*. Assuming that the Cambridge manuscript is an earlier draft of *Rules* (as it certainly seems to be; see Appendix), then both the method and the mathematics in the second part of *Rules* come chronologically *before*, not after *mathesis universalis*. *Mathesis universalis*, the method, and the mathematics developed in Rules 13–21 are all distinct from one another.

The Cambridge manuscript also provides additional evidence that by “order and measure” Descartes is essentially referring to the theory of proportions. Both *mathesis universalis* in Rule 4 and Descartes’s discussion of continuous and mean proportionals in Rule 6 appear together in later manuscripts, but they are both absent in the Cambridge manuscript. This suggests that the appearance of *mathesis universalis* in Rule 4 and the discussion of continuous and mean proportionals in Rule 6 are *parts of a single reflection and, therefore, constitute one textual unit*. When seen in this light, Descartes’s decision to place the discussion of continuous and mean proportionals in Rule 6 makes perfect sense. The theory of absolutes and relatives having already been developed in the Cambridge manuscript, relations between absolutes and relatives are nowhere more transparently exhibited than in proportional relations, i.e., *mathesis universalis*.[34]

This naturally raises the following question: why, after having developed the method in some detail in *Rules*, does Descartes decide to introduce *mathesis universalis* in later manuscripts? My thesis is that the question is misleading: *mathesis universalis* is present in all but name in the Cambridge manuscript, precisely as the theory of proportions.

(1) The relation between the theory of absolutes and relatives and the theory of proportions is very clearly articulated in the Cambridge manuscript. As in all other versions of Rule 11, in Rule 11_{CM} Descartes discusses the problem of finding continuous and mean proportionals in a manner that is

nearly identical to his discussion of these problems in the second part of Rule 6. He distinguishes between these problems by distinguishing between the number of operations required to solve them, and he notes that "by reflecting on the mutual dependence of simple propositions, we acquire the habit of distinguishing at a glance what is more, and what is less, relative, and by what steps the relative may be reduced to the absolute" (Rule 11_{CM} in CM14$^{\text{v}}$ = AT 10:409, CSM 1:38). This suggests that, even if *mathesis universalis* is not present in the Cambridge manuscript in name, examples of problems in *mathesis universalis* are present in Rule 11_{CM}, and the role they play there vis-à-vis the method is also clearly delineated. These examples play exactly the same role in Rule 11_{CM} (and all other versions of Rule 11) that they play in the versions of Rule 6 drafted after the Cambridge manuscript.

(2) In Rule 14_{CM} as in all other manuscripts, Descartes explicitly refers to order and measure—the object of *mathesis universalis*—without mentioning *mathesis universalis* by name. The immediate context is one in which his basic concern is to explain relations between magnitudes: "Moreover, if we are to explain which of all the available figures we are going to make use of here, we should know that all the relations which may possibly obtain between entities of the same kind should be placed under one or other of two headings: order or measure" (Rule 14_{CM} 25$^{\text{r}}$ = AT 10:451, CSM 1:64). This clearly reflects what Descartes would later explicitly assert in subsequent manuscripts vis-à-vis *mathesis universalis*, namely, that "the exclusive concern of mathematics is with questions of order or measure" (AT 10:378, CSM 1:19). The explicit appearance of *mathesis universalis* in subsequent manuscripts serves only to isolate the science of order and measure as such—independently of its embeddedness in the more advanced algebraic geometry developed in Rules 13–21—and assign it a definite place in the order of sciences that must be learned in order to master the method and acquire the Cartesian scientific *habitus*.

(3) While Descartes does not mention *mathesis universalis* in the Cambridge manuscript, he is clearly committed to the thesis that there is an order of sciences that must be learned in order to master the method and move on to the more advanced sciences. Simpler sciences must be learned first, more complex sciences later. He already regarded the mathematical sciences as the simplest sciences. Within the mathematical sciences, it is no

doubt *mathesis universalis* that is the least advanced. Consequently, *mathesis universalis* is the least advanced science in the order of the sciences and, therefore, the first that must be learned. However Descartes may have come across the concept of *mathesis universalis*, once he did, he immediately assigned it a definite place in the order of sciences that must be learned in order to master the method.[35] Thus, even if *mathesis universalis* is not explicitly mentioned in the Cambridge manuscript, its *place* is nevertheless clearly circumscribed there. This is not inconsequential. Without a clear definition of the science of order and measure, the clearest examples of the “the most insignificant and easiest of matters” Descartes discusses in the Cambridge manuscript are the so-called “feminine arts” and recreational mathematics in Rule 10_{CM} (see Chapter 3, Section 3.6 above). It is not clear in the Cambridge manuscript what one must do after sufficient practice in the so-called “feminine arts” and recreational mathematics. Descartes clearly assigns the problem of the limits of knowledge an important place in Rule 8_{CM}, but he does not explicitly state that it is “the first problem of all that should be examined by means of” the method, as he does in all subsequent manuscripts (AT 10:398, CSM 1:31). He only insists that it is a problem one should solve “once in life [*semel in vita*]” (CM 9^{v}–10^{r} = AT 10: 395, CSM 1: 30), and this recommendation provides no indication of *when* one should examine the problem. Thus, in the Cambridge manuscript it seems that one moves directly from recreational mathematics to advanced mathematics in Rules 13–16, without any clear isolation of *mathesis universalis* as an intermediary, let alone a clear identification of the problem of the limits of knowledge as the first problem that must be solved by the method after practice in *mathesis universalis*. Descartes’s subsequent introduction of *mathesis universalis* as well as his clarification of the place of the problem of the limits of knowledge in the order of sciences and problems reflects a maturation in his conception of the order of sciences that must be learned and the problems that must be solved in order acquire the Cartesian scientific *habitus*.

These three considerations indicate that a revision in standard accounts of the chronological genesis of Rules 4 is needed. The Cambridge manuscript contains Descartes’s discussion of how to deduce the anaclastic line in Rule 8, and it is known that Descartes conducted research in dioptrics with Mydorge in Paris in 1626/1627.[36] Anything in *Rules* that is

not contained in the Cambridge manuscript but that is contained in subsequent manuscripts most likely dates from after this period. Consequently, *mathesis universalis*, far from belonging to the earliest stratum of *Rules*, in fact belongs to the *latest* stratum. Descartes introduced it in *Rules* after 1626/1627, but probably not before. Since the theory of absolutes and relatives is already contained in the Cambridge manuscript, it constitutes a separate part of Rule 6 and could have been written any time between 1619–1626/1627. Descartes's discussion of continuous and mean proportionals is not contained in Rule 6_{CM} and must, therefore, be similarly dated after 1626/1627, but probably not before.[37] Since, moreover, a very similar discussion is contained in Rule 11_{CM}, it seems that later Descartes simply reproduced, with slight modification, the examples discussed in Rule 11_{CM} in versions of Rule 6 drafted after the Cambridge manuscript.

4.8 From *Mathesis Universalis* to the Problem of the Limits of Knowledge

I have argued that *mathesis universalis* is a propaedeutic science one must practice in order to graduate to problems in the more advanced sciences. But is the degree to which the Cartesian scientific *habitus* has developed thus far sufficient to solve problems in these sciences? It is at this point that a peculiar problem arises, one that is not, properly speaking, a problem in any one of the mathematical or natural sciences: the problem of the limits of knowledge. Why does this problem arise? The principle of proportionality requires that problems be reduced to such a form that they may be solvable by the natural operations of the human mind, but how does one know which problems *can* be so solved? How does one even know that intuition and deduction are the *only* operations that yield science? Perhaps there are others that have not been discussed. But even supposing that they are the only operations that yield science, the operator of the method must delineate the *limits* of these operations, and to do that, they must do two things. First, *they must determine what their cognitive powers are and how they must be employed in order to maximally extend the scope of intuition and deduction*. This has not been done yet. Second, *they must enumerate$_3$ the objects of intuition and deduction.* This too has not yet been done. The

problem of the limits of knowledge is a *practical-methodological* problem, not a *theoretical-epistemological* one. Without a solution to the problem of the limits of knowledge, the operator of the method will not be able to apply the method because they will never be in a position to determine whether any given problem is a problem they actually have the power to solve. The operator of the method must have some means of determining whether a given problem is solvable by the human mind *before* they set out to solve it. They must acquire a definite degree of *self-knowledge*, i.e., knowledge of their own cognitive constitution and abilities, in order to acquire knowledge of anything *else*. And they must acquire this knowledge no less methodically than they acquire any other knowledge.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0005

[1] Rabouin 2009 traces *mathesis universalis* as far back as Aristotle and Proclus.

[2] See, e.g., Natorp 1882; Cassirer 1902, 1937, and 1942, 9; Husserl [1954] 1976, 20–60 (esp. 45, 75, and 95), trans. Husserl 1970, 23–60 (45, 74, 92); Heidegger 1962, 111, trans. Heidegger 1967, 101; Foucault 1966, 13–16, trans. Foucault 1970, 79–86; Arndt 1971, 1, cited in Rabouin 2009, 9–33. For an exhaustive bibliographical essay on *mathesis universalis* in Descartes and Leibniz, see Rabouin 2009, 367–75. On the problems that the expression "mathematization of nature" produces as an historiographical category, see Ariew 2016.

[3] See Marion 1975 and van de Pitte 1979. See also Heimsoeth 1912 and Röd 1971.

[4] Sasaki 2003, 189–205, esp. 195, 197, and 200; Liard 1880, 1911; Boutroux 1900; and Klein [1934–1936] 1992 identify *mathesis universalis* and Descartes's algebraic geometry in *Rules* and *Geometry*. Schuster 1980 and 2013 argues that *mathesis universalis* can be traced back to Descartes's earliest research in mathematics and ultimately blossoms into "a general mathematical discipline, providing machinery for the analysis of all problems occurring in properly mathematical fields, including physico-mathematical ones, and putatively establishing the truth of its own procedures and the ontological reference of its objects – its legitimatory functions" (Schuster 2013, 230–5). I discuss these interpretations in more detail in Section 4.7.

[5] See Vuilleman 1960; Rabouin 2009; and de Buzon 2013.

[6] See van Roomen 1597, 23–4 and 25, def. 12–13, cited in Rabouin 2009, 265. Van Roomen equates *mathesis universalis*, *prima mathematica*, and *prima mathesis*, and he explicitly defines it as the science of order and measure. In the class of properties that are common to all magnitudes, van Roomen includes relations (*ratio*), which he then divides into number, order, and measure. Proportions are defined by the number of terms they contain (number), their relations of equality and inequality (order), and their commensurability or incommensurability (measure). See also Alsted 1613, lib. 1, cap. 11–12, 38–40, cited in Rabouin 2009, 265. Alsted regards "relation" and "proportion" as concepts that pertain to *mathematica generalis*, which he distinguishes from *mathematica specialis*. Both van Roomen and Alsted cite Benito Pereira, who refers to *scientia mathematica communis*. See Periera 1576, lib. 1, cap. 17, 57, who himself cites Piccolomini 1547. As

Rabouin notes, the relevant corpus of texts clearly indicates that the terms *mathematica communis*, *mathesis generalis*, *mathesis universalis* and *prima mathesis* were employed interchangeably during the sixteenth and seventeenth centuries. See also Crapulli 1969. On the relation between the theory of proportions and *mathesis universalis* in Descartes, see, in addition to Rabouin 2009, Vuilleman 1960; and de Buzon 2013.

7 Descartes does not employ algebraic notation in his discussion of these problems in Rule 6; he only employs numbers separated by commas. For convenience, I will occasionally employ algebraic notation when discussing these examples, but it is important to remember that nothing in *mathesis universalis* hinges on the use of such notation, and that *mathesis universalis* can be learned without even the most basic knowledge of algebraic notation. On Descartes's algebraic notation in *Rules*, see Chapter 9.

8 This point is repeatedly emphasized in Rabouin 2009.

9 See Rabouin 2009, 271ff.

10 For interpretations of Rule 6 and *Rules* more broadly as an epistemology, see Natorp 1882; Cassirer 1902; Heidegger 1962, trans. Heidegger 1967; Mahnke 1967; and Marion 1975.

11 See Aristotle 1984, 2:1612 (*Metaphysics* V.15 1021^{a}1–27). See also Marion 1975, 89.

12 Descartes came up with a substantial part of his theory of absolutes and relatives in Rule 6 before he ever introduced *mathesis universalis* in *Rules*. The Cambridge manuscript contains the core of the theory of absolutes and relatives in Rule 6, but not the thesis that sagaciously reflecting on proportional series better equips the mind to solve problems. This thesis only appears in subsequent manuscripts. The theory of order chronologically precedes *mathesis universalis*, but in the order of learning *mathesis universalis* precedes or initiates the mind into the theory of order.

13 See AT 10:156–157, CSMK 3:2 and Descartes 2001 and the discussion in Bos 2001, 285–399.

14 Descartes regards root extraction as a species of division in which one must divide a given magnitude by another given magnitude in order to discover the root. The root is the divisor. See AT 10:467, CSM 1:75 and Bos 2001, 266.

15 Descartes reproduces much of this material in the final paragraph of Rule 11 (AT 10:409–10, CSM 1:38–9; AT 10:439–40, CSM 1:57).

16 On direct and indirect (perfectly understood) deductions, see Dubouclez 2013, 207–19, whom I have relied on in this section.

17 On Descartes's understanding of the classification of problems and solutions after *Rules*, when he abandons the idea of providing a fully general classification of problems and solutions and instead opts for classifications that are local to specific sciences (such as geometry), see Chapter 11, Section 11.2.

18 Rabouin 2009, 278–85.

19 See Viète 1591.

20 See the references in Section 4.2.

21 See especially Crapulli 1969 and Rabouin 2009.

22 See AT 10:374. Since Rule 4 in the Hanover manuscript ends after AT 10: 374, line 15, Weber 1964, 4–5 distinguished between Rule 4A (=AT 10:371, line 14–374, line 15), which is devoted to the method, and Rule 4B (=AT 10:375, line 16–379, line 13), which is devoted to *mathesis universalis* (he believed that Rule 4B was written earlier, see Section 4.7.1 below). Weber's distinction between Rule 4A and Rule 4B can be maintained, but must be revised, since the principal

differences between Rule 4_{CM} and all other manuscripts begin at AT 10:373, line 24, considerably earlier than where Weber places the ending of Rule 4A (AT 10:374, line 15).

23 See references in Section 4.2.

24 See van Schooten 1651, an introduction to his Latin translation of Descartes's *Geometry*, titled *Principia matheseos universalis, seu introductio ad geometriae methodum Renati Des Cartes.*

25 Sasaki 2003, 189–205, esp. 195, 197, and 200. See also Liard 1880 and 1911; Boutroux 1900; and Klein [1934–1936] 1992; Israel 1998.

26 Schuster 1980 and 2013.

27 I dispute the characterizations of Descartes's early scientific research with Beeckman as "physico-mathematics" in Chapter 10.

28 See Schuster 2013, 230–5. Schuster assumes that Descartes either read or was already familiar with Proclus in 1619, but acknowledges that this is a speculation.

29 See Weber 1964, 16–17.

30 Schuster 2013, 329–30.

31 See Marion 1975: 64–71 and Van de Pitte 1979.

32 For a detailed development the arguments pursued in this section, see Rabouin 2009, 253–70.

33 Schuster 2013, 225–65 argues that Descartes's method grew out of *mathesis universalis*, but eventually extended beyond its purely mathematical or physico-mathematical boundaries.

34 It is therefore incorrect to regard the theory of absolutes and relatives in Rule 6 as a *byproduct* of reflection on solutions to problems in *mathesis universalis* (as Rabouin does in Rabouin 2009, 262, 269, 270–85). Instead, *mathesis universalis* should be regarded as the science in which the antecedently developed theory of absolutes and relatives in Rule 6 is most perspicuously *exhibited*. This could not have been appreciated prior to the discovery of the Cambridge manuscript.

35 For an intriguing suggestion that Descartes may have learned about *mathesis universalis* via Mersenne 1625, see Rabouin 2009, 246–9. As Rabouin notes, Mersenne 1625 refers to Hardy's translation of Euclid's *Data*, where one finds a direct reference to *mathesis universalis* very similar to Descartes's description of *mathesis universalis* in Rule 4. See Hardy's Latin translation in Hardy 1625, 14, cited in Rabouin 2009, 247.

36 Mydorge would later write a letter to Mersenne in 1631 about his collaboration with Descartes years earlier in 1626/1627. See Mersenne 1932–1988, 1:404–15. For an analysis of the letter, see Schuster 2013, 167–221, esp. 184–90. On the dating of the various manuscripts of *Rules*, see Appendix.

37 According to Weber 1964, 16–17 and Schuster 2013, 242–8, Descartes's interest in *mathesis universalis* dates to 1619.

PART III

THE FIRST PROBLEM OF THE METHOD

The “Noblest Example”

5

Defining the Problem of the Limits of Knowledge in *Rules*

5.1 The Noblest Example: Three Problems

The problem of the limits of knowledge arises directly out of a residual indeterminacy in Descartes's definition of the method in Rule 4. Descartes defines the method as a system of rules that will enable one to arrive "at a true understanding of everything within one's capacity" (AT 10:372, CSM 1:16). This definition leaves completely undetermined how broadly or narrowly "everything within one's capacity" should be construed; it does not determine what *is* "within one's capacity." Without criteria defining the class of problems the human mind has the capacity to solve, the application of the method to any particular problem becomes arbitrary and, therefore, blind. The method must determine its own limits, and so it must *solve the problem of the limits of knowledge in a manner no less rigorous than it would solve any other problem*. Any non-methodological determination of these limits would be little more than mere conjecture, which is not only excluded by the method, but would also exert no genuine constraint on the operator of the method. For Descartes, the operator of the method must know what problems they do and do not have the ability to solve, such that when they come upon a problem that they do not have the ability to solve, they will know not only that they have not in fact solved the problem, but also that the problem cannot be solved by anyone, and they will know this on the basis of definite criteria. The operator of the method must "be able to demonstrate that the thing he wants to know wholly exceeds the grasp of the human mind [*rem quaesitam omnem humani ingenij captum excedere demonstrabit*] – in which case he will not regard himself as more ignorant

on that account, for this discovery amounts to knowledge no less than any other" (AT 10:400, CSM 1:32–3). This explains why Descartes insists so strongly on the priority of the problem of the limits of human knowledge as "the first problem of all that should be examined" by the method (AT 10:397–8, CSM 1:31), a problem that must be solved "before […] acquiring knowledge of things in particular [*antequam ad res in particulari cognoscendas nos accingamus*]" (AT 10:396, CSM 1:30). Indeed, the problem of the limits of knowledge is so central to Descartes's broader enterprise in *Rules* that he describes it as the "noblest example" of the method (AT 10:395, CSM 1:29).

The scholarly consensus is that Descartes does not employ the method in his solution to the problem of the limits of knowledge in *Rules*, principally because his solution to the problem bears very little resemblance to his treatment of the problem of the shape of the anaclastic lens in the first part of Rule 8 (briefly discussed in Chapter 3 and fully reconstructed in Chapter 10).[1] Descartes's treatment of the problem of the shape of the anaclastic lens is almost universally regarded as the very paradigm of the application of the method in general. Clearly, any such interpretation presupposes the Uniformity Thesis, which I have argued Descartes rejects. In Chapter 3, Sections 3.4.1 and 3.7, I argued that *the parameters of the problem dictate how the method must be applied in every case, such that different problems require (or may require) different modes of application*. Consequently, *the method can be applied in a variety of rationally-constrained ways*. Descartes's method not only tolerates more than one mode of application; it *requires* it as a *principle*. This entails that *no one application of the method can serve as a paradigm for all the others*. Consequently, it does not seem to me that Descartes is obliged to reproduce (without any variation) the procedure employed in his discussion of the anaclastic in his solution to the problem of the limits of knowledge. As will be shown in more detail below, the problem of the limits of knowledge is simply too different a problem to be solved in quite the same way.

The real difficulties in Descartes's solution to the problem of the limits of knowledge run deeper and lie elsewhere. First, *there seems to be no way to determine whether the solution to the problem itself lies beyond the limits of human knowledge*. It seems that I must solve the problem of the limits of human knowledge in order to determine whether the problem can be solved. This difficulty threatens to upend Descartes's method before it has even

been applied. Second, Descartes seems to require that the principal operations of the method—intuition and deduction—be employed in the solution to the problem (he insists that Rules 1–7 be employed, and Rule 3 is devoted to intuition and deduction) (AT 10:397–8, CSM 1:31). However, these operations, it seems, can only be validated *after* the problem of the limits of knowledge has been solved. But how can operations whose validation depends on the solution to the problem of the limits of knowledge be employed in the solution to the problem? It seems that Descartes can neither determine whether the human mind has the capacity to solve the problem nor employ the basic operations needed to solve the problem without circularity.[2] Descartes stresses that the "true instruments of knowledge and the entire method are involved in the investigation of the problem" (AT 10:398, CSM 1:32), but if the method cannot be employed in order to solve the problem of the limits of knowledge, then it seems that the method cannot be employed at all.[3] Finally, Descartes's solution to the problem of the limits of knowledge depends wholly on "suppositions" and "assumptions" (*suppositiones*, AT 10:412, CSM 1:40; *assumenda*, AT 10:417, CSM 1:43) about the human cognitive faculties and their objects. How can a purely "hypothetical" solution to a problem of such importance be anything but arbitrary?

Before reconstructing Descartes's solution to the problem of the limits of knowledge in Chapters 6–7, these problems—problems that bear on the very possibility of a non-arbitrary, methodologically-controlled solution to the problem of the limits of knowledge—must be addressed. That is what I will do in this chapter. In Section 5.2, I argue that the problem of the limits of knowledge can be solved only by enumeration$_{1–3}$, and I reconstruct its role in the solution to the problem in detail. In Section 5.3, I examine Descartes's use of "suppositions" or "assumptions" in his solution to the problem of the limits of knowledge. In Section 5.4, I further isolate the concept of epistemic limit relevant to the problem of the limits of knowledge.

5.2 The "Method of Enumeration"

As I indicated in Chapter 3, Section 3.4.1 and will explain in more detail below, intuition and deduction cannot be employed in the solution to the problem of the limits of knowledge. Contrary to the suggestion entertained in Section 5.1, however, this is *not* because these operations must be "validated" by the solution to the problem. The word "validation" is misleading, insofar as it suggests that the solution to the problem of the limits of knowledge requires demonstrating that intuition and deduction yield science. Descartes's problem in Rules 8 and 12, however, is not to demonstrate that intuition and deduction yield science (he freely assumes that they do), but rather to demonstrate that they *alone* yield science, i.e., that there are no *other* operations of the human mind that satisfy the definition of science as certain and evident cognition. The solution to the problem is supposed to establish that "there are no paths to certain knowledge of the truth accessible to men *save* manifest intuition and necessary deduction" (AT 10:425, CSM 1:48; my emphasis). Thus, in principle, Descartes is perfectly free to employ intuition and deduction in his solution to the problem of the limits of knowledge, *if the problem can be solved by means of these operations*. But it cannot be solved by means of these operations: one cannot intuit that intuition and deduction are the only operations of the human mind that yield science, nor can one deduce it. To demonstrate that intuition and deduction are the only operations of the human mind that yield science, one must "make a precise enumeration of all the paths to truth which are open to men," so that they "may follow one which is reliable. There are not so many of these that [they] cannot easily discover them by means of a sufficient enumeration [*sufficientem enumerationem inveniat*]" (AT 10:396, CSM 1:30).Consequently, in Rule 8, Descartes insists that *enumeration* will be the principal operation employed in his solution to the problem of the limits of knowledge (he even argues that there is nearly no problem that cannot be solved by means of enumeration).[4] Since enumeration includes the reduction of problems to their simplest component parts (enumeration$_1$) and complex non-linear inference from discrete inferential chains (enumeration$_2$) sometimes based on the construction of classes (enumeration$_3$), enumeration is an operation uniquely suited to solve the problem of the limits of knowledge in *Rules*. Indeed, when Descartes first introduces enumeration in Rule 7, he begins by

describing its utility in direct relation to the problem of the limits of knowledge:

> In this context enumeration, or induction, consists in a thorough investigation of all the points relating to the problem at hand, an investigation which is so careful and accurate that we may conclude with manifest certainty that we have not inadvertently overlooked anything. So even though the object of our inquiry eludes us, provided we have made an enumeration we shall be wiser at least to the extent that we shall perceive with certainty that it could not possibly be discovered by any method known to us. If we have managed to examine all humanly accessible paths towards the object of our inquiry (which we often do), we shall be entitled confidently to assert that knowledge of it lies wholly beyond the reach of the human mind (AT 10:389, CSM 1:25–26).[5]

The fact that Descartes introduces enumeration by stressing its relation to the problem of the limits of knowledge is no coincidence. Later in Rule 8, Descartes once more insists that enumeration is uniquely suited to solve the problem of the limits of knowledge:

> It should not be regarded as an arduous or even difficult task to define the limits of the mental powers we are conscious of possessing, since we often have no hesitation in making judgments about things which are outside us and quite foreign to us. Nor is it an immeasurable task to seek to encompass in thought everything in the universe, with a view to learning in what way particular things may be susceptible of investigation by the human mind. For nothing can be so many-sided or diffuse that it cannot be encompassed under a few headings by means of the method of enumeration […] (AT 10:398, CSM 1:31).

What enumeration provides is the possibility of pursuing "every humanly accessible path which leads to knowledge of the truth" (AT 10:399, CSM 1:32). After everything relevant to the solution of the problem has been "encompassed within definite limits" and "placed under a few headings" (AT 10:398, CSM 1:31), all that remains is to examine each one of these headings within the limits required by the solution to the problem. In the case of the problem of the limits of knowledge, every faculty and every object must be evaluated relative to the definition of science as "certain and evident cognition" in Rule 2. Any operation or object that does not satisfy this definition must be excluded. What is distinctive about enumeration, then, is its ability to economize problems and its ability to evaluate faculties and objects relative to a definite criterion. Descartes regards enumeration as an operation specifically designed to simplify and control problems that would otherwise remain far too complex to permit of a solution proportioned to human cognitive capacity. Enumeration is an operation that even the most complex problems cannot escape. Because there are no other

operations available besides intuition, deduction, and enumeration, and because intuition and deduction cannot solve the problem of the limits of knowledge, Descartes prioritizes enumeration as the *only* operation capable of solving the problem of the limits of knowledge.[6] At the very beginning of Rule 12, he reduces the problem to a series of simpler problems via enumeration$_1$, and immediately after doing so, writes: “This seems to me to be a complete enumeration [*enumeratio…completa*] and to omit nothing which is within the range of human endeavor” (AT 10:411, CSM 1:39). Similarly, at the very end of Rule 12, he concludes his solution to the problem by reminding his readers that “we have explained distinctly and, I think, by a sufficient enumeration [*sufficientem enumerationem*] what at the outset we were able to present only in a confused and rough-and-ready way, viz., that there are no paths to certain knowledge of the truth accessible to men save manifest intuition and necessary deduction” (AT 10:425, CSM 1:48; translation modified).

Nowhere does Descartes indicate that he has made use of intuition or deduction in his solution to the problem of the limits of knowledge. As I have mentioned numerous times, Descartes regards it as part of his *task* in Rule 12 to *demonstrate* that intuition and deduction are the sole “paths to certain knowledge of the truth accessible to men.” He can only demonstrate that intuition and deduction alone produce knowledge by enumerating$_3$ all possible paths to certain knowledge and excluding those that do not yield “certain and evident cognition.” Intuition and deduction cannot by themselves establish their exclusive claim to epistemic credibility; their claim to such credibility must be shown via an enumeration$_3$ in which they are *compared to all other candidate paths to science*. Once all other candidate paths have been excluded, intuition and deduction remain as the only properly scientific operations. The demonstration that intuition and deduction alone yield science in *Rules* is, therefore, purely methodological, not metaphysical.[7] Descartes does not regard the cognitive faculties and their objects as requiring any further substantiation. The method must, by means of one of its operations (enumeration), establish the exclusive claim to epistemic credibility of its other operations (intuition and deduction), for in no case can intuition and deduction establish their own exclusive claim to epistemic credibility. Enumeration can lay claim to having examined “all the points relating to the problem at hand” in an “investigation so careful

and accurate that we may conclude with manifest certainty that we have not inadvertently overlooked anything" (AT 10:388, CSM 1:25–6). No other operation has the *scope* that enumeration does, and once all relevant alternatives have been evaluated according to definite criteria (in this case, whether or not an operation or an object yields certain and evident cognition), then the mind is *rationally compelled*—it has *no choice* but to *accept* intuition and deduction as the "only paths to certain knowledge of the truth" (AT 10:425, CSM 1:48).[8]

There is another reason why intuition and deduction cannot be employed in the solution to the problem of the limits of knowledge: the problem is *too complex* for these operations. Deduction is only possible when all of the propositions in the deduction are connected to one another in a *continuous series.* If "on the other hand we infer [*inferamus*] a proposition from many disconnected propositions [*multis et disjunctis*], our intellectual capacity is often insufficient [*non est tanta*] to enable us to encompass all of them in a single intuition [*unico intuitu*]; in which case the certainty of this operation [enumeration$_2$] must suffice" (AT 10:389, CSM 1:26; translation modified). Descartes's solution to the problem of the limits of knowledge consists of a series of inferences (conclusions) drawn from many disconnected propositions or discrete inferential chains via enumeration$_2$. This further explains why he chooses enumeration as the operation uniquely suited to solve the problem of the limits of knowledge. Inference via enumeration$_2$ is both methodologically prescribed under definite conditions and yet not identical to deduction, which can hardly be described as an inference from "many disconnected propositions" (ibid.).

There would be little reason for Descartes to expand his concept of inference in Rule 7 if all problems could be reduced to the same form or be solved in the same way. In the very first paragraph of Rule 8, immediately before discussing the problem of the anaclastic and the problem of the limits of knowledge, Descartes distinguishes between problems in which order is absolutely necessary and problems in which it is merely useful:

> The three preceding Rules [Rules 5–7] prescribe and explain order; the present Rule shows when order is absolutely necessary, and when it is merely useful. […] Of course if many things belong to a given step, as is often the case, it is useful to survey all of them in due order. But we are not forced to follow the order strictly and rigidly; generally, we may proceed further, even though we do not have a clear knowledge of all the terms of the series, but only a few or just one of them (AT 10:392, CSM 1:28; translation modified, my emphasis).

Descartes essentially enters a caveat about how Rules 5–7, all of which define and refine Descartes's concept of order in *Rules*, should be applied: not "strictly and rigidly," but rather *relative to the degree of complexity of the relevant problem*. He explicitly states that there are many cases in which we are not forced to observe order strictly and rigidly "if many things belong to a given step, as is often the case," and he states this in the very Rule in which he discusses both the problem of the shape of the anaclastic lens and the problem of the limits of knowledge. There is little doubt that, in the case of the problem of the limits of knowledge, "many things belong to a given step." This is not the case in the problem of the anaclastic; in the case of the anaclastic, the individual problems are complex, and solving them is by no means easy, but there is *one simplest problem to which all the others may be reduced*, while in the problem of the limits of knowledge this is not possible (see Chapter 3, Section 3.4.1 and Chapter 10). One of Descartes's principal points in Rule 8, then, is that the differences between these two problems require applying Rules 5–7 differently. In the case of the anaclastic, the problem will be solved by enumeration, intuition, and deduction. In the case of the noblest example, however, the problem must be solved by enumeration alone. It is therefore the nature of the problem of the limits of knowledge, together with the fact that intuition and deduction cannot be deployed in Descartes's solution to the problem, that makes enumeration uniquely suited to solve the problem of the limits of knowledge in *Rules*.

The fact that "we are not forced to observe order strictly and rigidly" conflicts with a number of interpretations of Descartes's concept of order in *Rules*, especially those interpretations that deny that Descartes applies his method to the problem of the limits of knowledge in *Rules*. Descartes's concept of order in *Rules* and beyond is usually interpreted as requiring a linear inference of many connected propositions from a single intuition (answers to the question of whether Descartes consistently satisfies such a demanding requirement vary).[9] This conception of order is certainly suggested by the example that has so frequently been singled out as the paradigm example of the application of Descartes's method in *Rules*: the problem of the shape of the anaclastic lens. However, that Descartes would explicitly recommend *not* following order "strictly and rigidly" in cases where "many things belong to a given step" indicates that Descartes always

intended his method to be applied in a manner that *takes into account the nature of the problem.*

Now that the role played by enumeration in Descartes's solution to the problem of the limits of knowledge has been examined, Descartes's actual use of the operation in his reduction of the problem can be reconstructed. Descartes divides the problem into two principal parts by enumerating$_1$ "whatever is relevant to the question; for the question ought to relate either to us, who have the capacity for knowledge, or to the actual things it is possible to know" (AT 10:398, CSM 1:31–2).

Regarding the first part, Descartes prescribes the following order in Rule 8:

> If someone sets himself the problem of investigating every truth for the knowledge of which human reason is adequate – and this, I think, is something everyone who earnestly strives after good sense should do once in his life – he will indeed discover by means of the Rules we have proposed [1] that nothing can be known prior to the intellect, since knowledge of everything else depends on the intellect, and not vice versa. Once he has [2] surveyed everything that follows immediately upon knowledge of the pure intellect, among what remains he will [3] enumerate whatever instruments of knowledge we possess in addition to the intellect; and there are only two of these, namely imagination and sense-perception. He will therefore [4] devote all his energies to distinguishing and examining these three modes of knowing [*cognoscendi modis*]. He will see that there can be no truth or falsity in the strict sense except in the intellect alone, although truth and falsity often originate from the other two modes of knowing; and he will pay careful heed to everything that might deceive him, in order to guard against it. He will make an exact enumeration [*enumerabit exacte*] of all the paths to truth which are open to men, so that he may follow one which is reliable. There are not so many of these that he cannot discover them all by means of a sufficient enumeration (AT 10:395–396, CSM 1:30; translation slightly modified).

Later in Rule 8, he writes:

> Within ourselves we are aware that, while it is the intellect alone that is capable of knowledge, it can be helped or hindered by three other faculties, viz., imagination, sense-perception, and memory. We must therefore look at each of these faculties in turn, to see in what respect each of them could be a hindrance, so that we may be on our guard, and in what respect an asset, so that we may make full use of their resources. We shall discuss this part of the question by way of a sufficient enumeration […] (AT 10:399, CSM 1:32).

Regarding the objects of knowledge, in Rule 8 Descartes prescribes the following order:

> We should then turn to the things themselves; and we should deal with these only insofar as they are within the reach of the intellect [*tantum spectandae sunt prout ab intellectu attinguntur*]. In that respect we divide them into absolutely simple natures and complex or composite natures (AT 10:399, CSM 1:32).

Finally, in Rule 12, Descartes proposes a similar order of inquiry:

> Where knowledge of things is concerned, only two factors need to be considered: ourselves, the knowing subjects, and the things which are the objects of knowledge. As for ourselves, there are only four faculties which we can use for this purpose: viz., intellect, imagination, sense-perception, and memory. It is of course only the intellect that is capable of perceiving the truth, but it has to be assisted by imagination, sense-perception, and memory if we are not to omit anything which lies within our power [*quod in nostra industria positum sit, omittamus*]. As for the objects of knowledge, it is sufficient [*sufficit*] if we examine the three following questions: What presents itself to us spontaneously? How can one thing be known on the basis of something else? What conclusions can be drawn from each of these? This seems to be a complete enumeration and to omit nothing which is within the range of human endeavor [*Atque haec enumeratio mihi videtur completa, nec ulla prorsus omittere, ad quae humana industria possit extendi*] (AT 10:411, CSM 1:39).

Descartes describes his reduction of the problem of the limits of knowledge as a "complete enumeration." This means that the problems that compose the problem of the limits of knowledge have been exhaustively enumerated (see Table 5.1).[10]

Table 5.1 Descartes's reduction of the problem of the limits of human knowledge in Rules 8 and 12

Q1 What is human knowledge and what is its scope?	
Q2 What is the subject of knowledge?	Q7 What is the object of knowledge?
Q3 What faculties do they have?	Q8 What presents itself to us spontaneously?
Q4 What are the operations of these faculties?	Q9 How can one thing be known on the basis of something else?
Q5 Which operations produce science?	
Q6 Which operations do not produce science?	
Q10 What conclusions can be drawn from each of these?	

We soon learn that while Descartes's enumeration$_1$ of the relevant problems is complete, his *solution* to these problems (via enumeration$_{2–3}$) is only "sufficient." He will *not* completely solve each problem. In Rule 8, he proposes to solve the first part of the problem of the limits of knowledge "by way of a sufficient enumeration" (AT 10:399, CSM 1:32). Similarly, immediately before he introduces his theory of the faculties in Rule 12, he indicates that he "lacks the space […] to include all the points which have

to be set out before the truth about these matters can be made clear to everyone," so "it will be sufficient [*sufficiet*] if I explain as briefly as possible what, for my purposes, is the most useful way of conceiving everything within us which contributes to our knowledge of things" (AT 10:411–12, CSM 1:40; my emphasis). He omits completely "explain[ing] what the human mind is, what the body is and how it is informed by the mind, what faculties within the composite whole promote knowledge of things, and what each particular faculty does" (ibid.). Toward the end of Rule 12, where Descartes draws conclusions from his solution to the problem of the limits of knowledge, he similarly writes that "…we have explained distinctly and, I think, by a sufficient enumeration [*sufficientem enumerationem*] […]" that intuition and deduction are the only "paths to certain knowledge of the truth" and "what the simple natures are which were mentioned in Rule 8" (AT 10:425, CSM 1:48; translation modified). Thus, while Descartes's reduction of the problem of the limits of knowledge via enumeration$_1$ is complete, his solution to each problem via enumeration$_{2–3}$ is only sufficient. Many "points which have to be set out" are not set out. Due to the sufficiency of the enumeration$_2$, the mechanism of the faculties is not discussed in any serious detail. Furthermore, the examination of each "mode of knowing" involves the use of suppositions, which are introduced in order to economize Descartes's mechanical explanation of the cognitive process.

For the remainder of Rule 12, Descartes provides a sufficient enumeration$_{2–3}$ of the faculties and their objects. Beginning with the intellect, the individuation and evaluation of each faculty proceeds on the basis of two criteria: cognitive function and utility vis-à-vis science, respectively. Since assessments of cognitive utility vis-à-vis science depend on what cognitive function each faculty performs, Descartes describes the function of each faculty before he assesses its utility vis-à-vis science. Descartes begins his solution to the second part of the problem as he did the first, by beginning with the intellect: "We should then turn to the things themselves; and we should deal with these only insofar as they are within the reach of the intellect" (AT 10:399, CSM 1:32). He then introduces the other faculties, by reference to which he enumerates$_3$ different classes of simple nature (intellectual, material, and common), with each class correlated to the faculty or faculties required in their intuition. As we will

see in more detail in Chapters 7–8, the intellectual simple natures must be intuited by the pure intellect alone; the material simple natures must be intuited by the intellect aided by the imagination; and the common simple natures can be intuited by either the pure intellect alone or by the intellect aided by the imagination (AT 10:419–20, CSM 1:44–5). Descartes's solution to the first part of the problem of the limits of knowledge yields his theory of the faculties in *Rules*, which in turn serves as the basis of his solution to the second part of the problem, which yields his ontology of the objects of knowledge. Together, these solutions establish *inter alia* that "there are no paths to certain knowledge of the truth accessible to men save manifest intuition and necessary deduction" (AT 10:425, CSM 1:48). Any problem that cannot be solved by following one of these avenues lies wholly beyond the reach of the human mind. In fact, any problem that cannot be solved by means of one of these avenues is not, strictly speaking, a problem at all, but rather a mystery.[11] Thus, Descartes's solution to the problem of the limits of knowledge in *Rules* furnishes, in a manner that is secured by the method, the tools needed to solve particular problems in the sciences: the faculties and how they should (and should not) be employed, and the simple natures, which compose every object, and which can be intuited by the intellect or the intellect aided by an auxiliary faculty. It is, therefore, in virtue of the "method of enumeration" that the method "resembles the procedures in the mechanical crafts, which have no need of methods other than their own, and which supply their own instructions for making their own tools" (Rule 8, AT 10:397, CSM 1:31), for it is by means of the "method of enumeration" that these tools can first be produced.

5.3 Sufficient Enumeration$_2$, Supposition, and Truth in Rule 12

As we have seen, Descartes introduces both his theory of the human cognitive faculties and his theory of the objects of knowledge via a series of suppositions or assumptions (*suppositiones*, AT 10:412, CSM 1:40; *assumenda*, AT 10:417, CSM 1:43). He employs suppositions because, as he puts it, he cannot "include all the points which have to be set out before the truth [*veritas*] about these matters can be made clear to everyone" (AT

10:411, CSM 1:40). He insists on making "no assertions on matters which are apt to give rise to controversy, without first setting out the reasons which led me to make them […]" (ibid.). There is a direct relation between Descartes's use of sufficient enumeration$_2$ and his use of suppositions in Rule 12. Because the enumeration$_2$ whereby Descartes solves the problem of the limits of knowledge is sufficient, not complete, he can exclude from consideration whatever he deems unnecessary to his basic purpose in Rule 12. Consequently, he does not provide a complete theory of "what the human mind is, what the body is and how it is informed by the mind, what faculties within the composite whole promote knowledge of things, and what each particular faculty does" (ibid.). In other words, he does not provide a complete theory of the faculties of the sort one finds in scholastic commentaries on Aristotle's *De anima*, where every disputed question about the soul and its faculties is dealt with in exhaustive detail. Descartes does, of course, discuss all of the subjects mentioned (what the human mind is, etc.), *but only within the limits required by his solution to the problem of the limits of knowledge via a series of interconnected suppositions*. The relevant contrast here is not (or not only) between a theory of the faculties based on suppositions and a theory of the faculties deduced from metaphysical first principles (whatever they might be). Deduction from metaphysical first principles is not a requirement Descartes lays down in *Rules*. The relevant contrast in Rule 12 is principally between methodologically sanctioned suppositions and dialectical demonstrations based on the resolution of disputed questions.[12] Descartes's decision to employ suppositions in his theory of the faculties in *Rules* need not be seen as a deficiency. On the contrary, since dialectical demonstrations are by their very nature merely probable without being clear or easy to understand, Descartes has every reason to regard his suppositions as *superior* to dialectical demonstrations. Relative to the criteria of the method, Descartes's suppositions are clear and easy to understand, since they are derived from a series of analogies and comparisons to macro-mechanical processes everyone is already perfectly familiar with. In this sense, while there is certainly *room* for doubt about the truth of Descartes's suppositions, there is not necessarily any *reason* for doubt, *at least not if the relevant reasons are drawn from theories whose epistemic credentials the method does not recognize*. Furthermore, as I briefly mentioned in Section 5.2, more radical reasons for doubt are simply not relevant in *Rules*.[13] Descartes

describes the human cognitive faculties only insofar as each faculty contributes (or not) to the production of science (AT 10:398–9, CSM 1:32). His ambitions are such that a complete enumeration$_2$ of the faculties would be entirely useless, even counterproductive. The suppositions Descartes employs in Rule 12 are sufficient because they are narrowly circumscribed, not accidentally, but rather because the problem he intends to solve requires nothing more.

Beyond the fact that it is no part of Descartes's intention to write a treatise on the soul in *Rules*, Descartes provides an interesting defense of the suppositions employed in his solution to the problem of the limits of knowledge by comparing them to the suppositions employed in geometry and astronomy. In the prefatory paragraph to his theory of the faculties in Rule 12, he writes:

> [I]t will be sufficient [*sufficiet*] if I explain as briefly as possible everything within us which contributes to our knowledge of things. Of course you are not obliged to believe that things are as I suggest [*Neque credetis, nisi lubet, rem ita se habere*]. But what is to prevent you from following these suppositions if it is obvious that they detract not a jot from the truth of things, but simply make everything much clearer [*nihil illas ex rerum veritate minuere, sed tantum reddere omnia longe clariora*]? This is just what you do in geometry when you make certain assumptions [*supponitis*] about quantity, which in no way weaken the force of the demonstrations, even though in physics you often take a different view of the nature of quantity (AT 10:411–12, CSM 1:39–40).

Similarly, in his prefatory paragraph to the theory of simple natures in Rule 12, Descartes writes:

> Let us now take up the second factor [the objects of science]. Our aim here is to distinguish carefully the notions [*notiones*] of simple things from those which are composed of them, and in both cases to try to see where falsity can come in, so that we may guard against it, and to see what can be known with certainty, so that we may concern ourselves exclusively with that. To this end, as before, certain assumptions must be made in this context which perhaps not everyone will accept. But even if they are thought to be no more real that the imaginary circles which the astronomers use to describe the phenomena they study, this matters little, provided that they help us to pick out the kind of apprehension of any given thing that may be true and to distinguish it from the kind that may be false (AT 10:417–18, CSM 1:43–4).[14]

In both of these prefatory paragraphs, Descartes compares his use of suppositions to the use of suppositions in geometry and astronomy. In both cases, the principal point is that neither Euclidean geometry nor Ptolemaic astronomy (which is a branch of mathematics, not physics, in Greek and scholastic science) is concerned with how things are in reality, and in

astronomy one is only concerned with “saving the appearances” via mathematical descriptions whose physical (and, therefore, ontological) foundations are not determined within either science. The suspension of ontological considerations about the real natures of physical continuous magnitudes or the real natures of celestial bodies does not in any way undermine either science. On the contrary, ontological neutrality is precisely what makes both sciences possible. So long as neither science is regarded as having ontological ambitions, there is no risk of falsity. Whatever Descartes’s ontological ambitions may be in his theory of the faculties and their objects in Rules 8 and 12, *they are not based on any description of how the human cognitive faculties and the objects of knowledge are in reality*. Descartes’s ontological ambitions (i.e., his desire to have his suppositions and demonstrations accepted as true) are based on the fact that his suppositions are *superior* to any competing theory on purely *epistemic* grounds, and *he invites his readers to accept his suppositions as true even though he does not demonstrate their truth* (I discuss Descartes’s epistemic grounds below). Thus, Descartes’s reference to the use of suppositions in geometry and astronomy is but his way of reminding readers that they should have no difficulty accepting his suppositions about the human cognitive faculties and the objects of knowledge, since they (his readers) already accept suppositions in other sciences without hesitation.

Nevertheless, once one descends into the details, one sees that suppositions function rather differently in geometry and astronomy, and that Descartes intends his readers to draw different lessons from each.

Descartes intends his readers to draw four related lessons from the use of suppositions in geometry. (1) When it comes to the nature of quantity, Aristotle claims that the principal difference between the geometer and the physicist is that “the mathematician, though he too treats of these things [magnitudes], nevertheless does not treat them as the limits of a natural body; nor does he consider the attributes indicated as the attributes of such bodies. That is why he separates them; for in thought they are separable from motion.”[15] Aristotle then points out that “it makes no difference, *nor does any falsity result*, if they [magnitudes] are separated.”[16] This is presumably what Descartes has in mind in the above-cited passage from Rule 12. As Descartes understands Aristotle, the definitions, axioms, and postulates about continuous magnitudes in geometry play no role in

physics, and sometimes they even contradict what physics has to say about continuous magnitudes, even though physics too deals with continuous magnitudes.[17] Just as geometrical demonstrations about continuous magnitudes are in no way undermined by the fact that physics often takes "a different view of the nature of quantity," so too in *Rules* Descartes's demonstrations about the human cognitive faculties and their objects are in no way undermined by the fact that other theories often taken a different view of the nature of the soul and its objects. Descartes is effectively asking his readers to accept his suppositions and demonstrations because even though they are about the same things that other sciences—sciences with well-established ontological credentials—deal with, he is considering them from a different point of view. *Descartes's descriptions of the human cognitive faculties and their objects cannot be falsified by how things are in reality, because they do not aim to describe how things are in reality.*[18] In short, Descartes's theory of the faculties and the objects of knowledge in *Rules* is to Aristotelian theories of the soul as Euclidean geometry is to Aristotelian physics.

Relatedly, (2) Euclidean definitions, axioms, and postulates must be *accepted by the learner who desires to become a geometer*, but they need not be accepted by anyone else. Similarly, Descartes's suppositions must be accepted by the learner who desires to become a Cartesian scientist by acquiring the Cartesian scientific *habitus*, but they need not be accepted by anyone else. In his commentary on Euclid's *Elements*, Clavius writes that postulates "merely demand the assent of the hearer, and neither is there any hesitation or difficulty in explaining them."[19] Descartes read Clavius's commentary on Euclid's *Elements* at La Flèche,[20] and Clavius's definition of a postulate perfectly expresses how Descartes intends his readers to understand his own suppositions in Rule 12. (3) Postulates "are very clear and perspicuous in the science that is under consideration, so that they need no confirmation,"[21] they do "not require proof or geometrical evidence but are taken as known and used as starting points in what follows."[22] Similarly, Descartes's suppositions do not require proof and are used as starting points from which a series of conclusions are drawn toward the end of Rule 12. Each conclusion further consolidates his solution to the problem of the limits of knowledge. Finally, (4) geometrical demonstrations can only be evaluated relative to the criteria of rationality that define geometry.

Similarly, Descartes's suppositions about the human cognitive faculties and their objects can only be evaluated relative to criteria that define the method.

Descartes's comparison of his suppositions to the "imaginary circles which the astronomers use to describe the phenomena they study" bears some resemblance to his previous comparison to the use of suppositions in geometry, but it also differs in revealing ways. Like the suppositions about continuous magnitudes in Euclidean geometry, Ptolemaic astronomy employs suppositions about the motions of celestial bodies that cannot be deduced from the principles of Aristotelian physics. In Aristotelian physics, motions in the heavens must be perfectly circular and uniform (they must describe a single circle), because the motions of celestial bodies are not disturbed by matter.[23] Ptolemaic astronomy posits the existence of motions in the heavens that violate these rules, but that do save the phenomena, such as the epicycles of the planets.[24] Ptolemaic astronomy also includes many imaginary points, such as the eccentric location of the earth and the equant. According to Ptolemy himself, these lines and points have no real basis in the heavens, but they do save the phenomena by accurately describing the course of celestial motions.[25] The "imaginary circles of the astronomers" Descartes refers to in Rule 12 are *suppositions that the astronomers themselves regard as imaginary or false*.[26] This clearly differs from suppositions in geometry; geometers do not regard their suppositions as false. The point here, rather, is that *astronomers nevertheless deduce many true conclusions from the very suppositions they themselves regard as false*. Even if Descartes's suppositions are false, it "matters little, provided they help us to pick out the kind of cognition [*cognitio*] of any given thing that may be true and to distinguish it from the kind that may be false" (AT 10:417, CSM 1:43–44; translation modified).[27]

At this point, however, Descartes's analogy between the suppositions in geometry and astronomy and suppositions in *Rules* comes to an end. Geometry cannot demonstrate its suppositions, and the imaginary circles used by the astronomers are known not to exist in nature, so the possibility of demonstrating their existence is also ruled out. Descartes, by contrast, believes that he *can* demonstrate his suppositions, even though he does not do so in *Rules*. This distinguishes Descartes's suppositions in Rule 12 from the suppositions of both the geometers and the astronomers, and as I will

show below, this has important consequences for how Descartes's theory of the faculties and their objects is interpreted in *Rules*.

By appealing to the use of suppositions in both geometry and astronomy, Descartes defends his theory of the faculties and the objects of knowledge by relying on the authority of a criterion other than that of truth. Descartes appeals to five interdependent criteria: (1) utility (measured by how effectively the suppositions help solve the problem of the limits of knowledge); (2) explanatory power (measured by how effectively the suppositions explain the cognitive process in a manner that is less complex, and so more distinct, than any competing theory); (3) simplicity or clarity (measured by how readily intelligible, or easy to understand, the suppositions are); (4) economy (measured by the number of suppositions required); and (5) ontological parsimony (measured by the exclusion of all occult, "useless entities," such as substantial forms or other entities not intuitable by the intellect).[28] These criteria mutually support one another: explanatory power depends on clarity, which in turn depends on ontological parsimony, which enables economy, etc. Descartes's suppositions in Rule 12 describe a *model* of the human cognitive faculties and their objects, and leave open whether the model describes anything in reality. Deduction from metaphysical first principles or the principles of natural philosophy plays no role here. Nor does empirical adequacy based on observation. Descartes does not demonstrate (and does not set out to demonstrate) that the human cognitive faculties and their objects are as he describes them. He only conditionally demonstrates that, if the human cognitive faculties and their objects are as he describes them, then "the attentive reader will have no difficulty in gathering what aids we should seek to obtain from each of these faculties" in acquiring "certain knowledge of all sciences" (AT 10:416, CSM 1:43). This, he insists, is "sufficient" for his purposes.

Descartes's use of suppositions in Rule 12 may seem to suggest that Descartes's solution to the problem of the limits of knowledge relies on what is oftentimes referred to as the "hypothetico-deductive method."[29] This is not so: the hypothetico-deductive method is a method in which hypotheses are confirmed by experience alone. Deduction from first principles plays no role. As I mentioned above, Descartes presents his suppositions as *conclusions* deduced from principles he *chooses* not to provide (sufficient enumeration$_2$). Descartes believes that his suppositions in Rule 12 can be demonstrated from first principles, much as he believes

that his suppositions in the *Essays* can be demonstrated from first principles: “I have called them ‘suppositions’ simply to make it known that I think I can deduce them from the primary truths” of metaphysics (*Discourse* VI, AT 6:76, CSM 1:150).[30] The difference is that while *Discourse* VI refers to the primary truths of a metaphysics briefly sketched in *Discourse* IV (and more fully elaborated in *Meditations* and *Principles* I), no clear decision is made in *Rules* on how the suppositions Descartes employs there should be demonstrated. Once Descartes establishes the priority of metaphysics over physics after *Rules*, these suppositions—especially his suppositions about the objects of knowledge: the simple natures—are deduced from the first principles of metaphysics. By and large, the content of Descartes’s suppositions in Rule 12 do not fundamentally change over the course of his career; only their epistemic status changes as they find their place in the system of science.[31]

5.4 Two Concepts of Epistemic Limit in Rule 8

In Sections 5.1–5.2, I argued that Descartes solves the problem of the limits of knowledge by enumeration$_{1–3}$, and that enumeration$_{1\text{-}3}$ yields a methodologically secure solution to the problem. This does not mean, however, that Descartes’s principal intention in Rule 8 is to illustrate the method. Descartes’s principal intention in Rule 8 is rather to *distinguish between two methodologically operative concepts of epistemic limit*: provisional (or surmountable) limits and absolute limits. In the case of the anaclastic, Descartes shows that those whose studies are confined to mathematics will not be able to deduce the anaclastic line, not because the anaclastic lies beyond the limits of human knowledge, but rather because the problem “has to do with physics rather than with mathematics” alone (AT 10:394, CSM 1:29). Error arises here from a failure to follow Descartes’s prescription in Rule 1 to learn all sciences together (in this case, mathematics and physics). Once they have correctly followed the method, the operator of the method will have all the resources needed to deduce the anaclastic line: “I can see nothing to prevent anyone who uses our method exactly from gaining a clear knowledge” of the anaclastic line (ibid.). Descartes’s discussion of the anaclastic in Rule 8 is intended, not (or not

only) to illustrate the application of the method to a particular problem, but rather to illustrate the type of limit one encounters when one has failed to define the problem so that intuition and deduction can be deployed. This type of limit is provisional; it can be overcome.

The concept of epistemic limit at work in the problem of the limits of knowledge is different, since the limits are in this case absolute, not provisional. Despite the fact that Descartes discusses the problem of the anaclastic before discussing the problem of the limits of knowledge, it is clear that he believes that the latter problem must be solved before any other problem. The order in which these problems must be solved is important because anybody who has solved the problem of the limits of knowledge will immediately be able to determine whether the problem of the anaclastic can be solved as well as how best to solve it. The reverse, however, is not true: one who has not determined the limits of human knowledge will not be able to determine whether the difficulties they encounter in their endeavor to discover the anaclastic line are surmountable or insurmountable. An absolute determination of the limits of human knowledge prevents merely provisional limits from seeming absolute; when provisional limits are encountered, the operator of the method will recognize them as such, and when absolute limits are encountered, the operator of the method will recognize them as such too. The relation between the problem of the anaclastic and the problem of the limits of knowledge in Rule 8 should not, therefore, be construed in terms of two competing examples of the application of the method, only one of which—the anaclastic—is credible. The unity of Rule 8 consists in the fact that each example illustrates a different sense of epistemic limit with which any prospective operator of the method must become conversant if they are to successfully apply the method at all.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0006

[1] Garber 1992, 42, Gaukroger 1995, 157, and Schuster 2013, 312 deny that Descartes employed the method in his solution to the problem of the limits of knowledge.

[2] In their discussion of the problem of the limits of knowledge, Beck 1952, 25–31, 65–83, 126–34 and Marion 1975, 113–55 do not address (or identify) these potential difficulties.

3 Garber has argued that it "is not at all clear whether" the noblest example "is essential in order for us to have any knowledge, or whether it is simply a practical suggestion about where we might begin" (Garber 1992, 42). Descartes insists that the problem of the limits of human knowledge must be solved "before […] acquiring knowledge of things in particular" (AT 10:396–7, CSM 1:30). This is not a practical suggestion about where we might begin, but rather a clear identification of where we must begin. It is also clear that solving the problem of the limits of knowledge is essential for us to have any reliable way of acquiring knowledge.

4 For the moment, I will simply employ the term "enumeration," without subscripts, since all three species of enumeration are in play in Descartes's solution to the problem of the limits of knowledge in *Rules*. When a determinate species is relevant, I indicate as much by means of the relevant subscript.

5 This passage is not contained in the Cambridge manuscript. While Descartes assigns pride of place to enumeration in his solution to the problem of the limits of knowledge in both Rules 8_{CM} and 12_{CM}, he does not explicitly link enumeration to the problem of the limits of knowledge in Rule 7_{CM} as he does in Rule 7_{AT}. As I will show in more detail in Chapter 8, Descartes's solution to the problem of the limits of knowledge in Rules $8_{CM}$and 12_{CM} is severely incomplete (Rule 8_{CM} ends at AT 10:396, line 25, CSM 1:30, and Rule 12_{CM} ends at AT 10:416, line 16, CSM 1:43, shortly after Descartes's definition of *ingenium*). Descartes's decision to emphasize this link between enumeration and the problem of the limits of knowledge, together with his decision to further clarify what enumeration is in Rule 7_{AT}, suggests that, as he completed his solution to the problem of the limits of knowledge, he further reflected on the role played by enumeration in his solution, and decided to develop the operation in more detail in Rule 7_{AT}, immediately before the reader first encounters the problem of the limits of knowledge in Rule 8_{AT}.

6 In the Cambridge manuscript, Descartes is explicit about the fact that "there are some problems in which the whole method consists solely in this operation [enumeration] [*Et nonnullae sunt difficultates, ad quas tota Methodus in hac sola Operatione consistit*][…]" (CM fo. 8[r]). See also AT 10:391, CSM 1:27.

7 In fact, as I argue in Chapter 11, Section 11.3, Descartes *never* doubts that intuition yields truth, not even in *Meditations*. In this sense, Descartes never undertook to "validate" intuition in the sense that such validation would establish a link between intuition and truth.

8 Not all alternatives are relevant in *Rules*. Skeptical hypotheses are not relevant, since Descartes's project in *Rules* is not to "devote myself sincerely and without reservation to the general demolition of my opinions" (*Meditations* I, AT 7:18, CSM 2:12).

9 The most famous proponent of deductivist concept of order in Descartes remains Gueroult's interpretation of Descartes's philosophy according "to the order of reasons" in Gueroult 1984–1985. See also Garber 2001, 34–9; Schuster 2013, 248–57; Clarke 1982, 165–80. I should emphasize that Garber, Schuster, and Clarke do *not* claim that Descartes always and everywhere adheres to a deductivist concept of order (Garber and Clarke make room in different ways for non-deduced/non-deducible hypotheses in Cartesian science, while Schuster denies that Descartes employed the method at all). However, they *do* interpret the order required in *Rules* as a purely deductive order in which problems are solved by inferring many connected propositions from a single intuition.

10 Garber 1992, 42 claims that after the reduction of the problem of the limits of knowledge into its two principal parts, "the treatment of the question becomes sketchier" because Descartes does not "continue the reduction and give a full answer." In fact, Descartes's solution to the problem of the limits of knowledge is far more complete than his merely proposed solution to the problem of the

anaclastic in Rule 8. Garber bases his interpretation of order in *Rules* principally on Rule 5, which he sees most clearly illustrated in Descartes's proposed deduction of the anaclastic line in Rule 8. He does not discuss enumeration or Descartes's insistence that "we are not forced to follow the order strictly and rigidly" in Rule 7. This partly accounts for why he denies that Descartes applies his method to the problem of the limits of knowledge. Schuster 2013, 312 argues that Descartes's solution to the problem of the limits of knowledge in Rule 12 is "less an example of the method of Rules 3–7 and more a program for the construction and legitimation of universal mathematics […]." I see no evidence for such a program in Rule 12 (see my discussion in Chapter 4, Section 4.7). Schuster 2013, 248–57 focuses principally on deductive order in his interpretation of Descartes's method, assigning enumeration a subsidiary role as a purely "heuristic" device. He does not regard enumeration as a fundamental operation of the method. Despite his well-known sensitivity to the various senses of deduction in Descartes, Clarke 1982, 165–80 interprets Descartes's method in *Rules* as a method of "analysis and synthesis," which he also sees most clearly illustrated in Descartes's deduction of the anaclastic line in Rule 8. At one point, he describes enumeration as an "anomalous" species of inference (67). (On the confusion that arises in describing Descartes's method in terms "analysis" and "synthesis," see Chapter 3, Section 3.4.1). Because they adhere to the Uniformity Thesis, Garber, Clarke, and Schuster regard Descartes's reduction of the problem of the anaclastic as a paradigm example of the application of the method, such that any cases that "deviate" from this one are either anomalous or simply not applications of the method at all.

11 On the difference between problems and mysteries in science, see Chomsky 1976.

12 For an example of such demonstrations, see Conimbricenses [1598] 1604. On seventeenth-century Jesuit commentaries on Aristotle's *De anima*, see Simmons 1999.

13 See n. 8 above.

14 Descartes consistently compares his use of suppositions to the use of suppositions in astronomy. See *Dioptrics* I, AT 6:84, CSM 1:152–3; letter to Morin, July 13, 1638 (AT 2:198–9, CSMK 3:107. For an analysis of Descartes's exchange with Morin, see Belgioioso 2009, 289–93); *Fifth Replies* (AT 7:349–50, CSM 2:242). See also *Principles* III. 15–19, 43–7 (AT 8A:84–6, 99–104, CSM 1:250–1, 255–8). On the relation between Descartes's use of suppositions and the use of suppositions in astronomy, see Martinet 1974. See also Clarke 1982, 108–33.

15 Aristotle 1984, 1:330–1 (*Physics* II.2 193^{b}21–35); my emphasis. On the relation between physics and geometry in Aristotle, see Pfeiffer 2018, 26–53 and White 2009, 265–71.

16 Aristotle, ibid. (my emphasis).

17 As White 2009, 268 points out, in Aristotelian physics, the extension of physical space is finite and bounded, while in Euclidean geometry, any line can be produced indefinitely or infinitely (*ep'apeiron*) (see Euclid's *Elements* I., post. 5 in Euclid 1908, 1:202).

18 See also Marion 1975, 113–16. Beck 1952, 25–31 does not discuss Descartes's use of suppositions in Rule 12. Schuster 2013, 316, n. 34 argues that Descartes intended his readers to accept his theory of the faculties and their objects as true, and that, consequently, the "hypothetical tone" of Descartes's theory should be dismissed. Schuster assumes that intending readers to accept a theory as true *conflicts* with the use of suppositions, when in fact the use of suppositions is precisely what intending readers to accept a theory as true amounts to in this case.

19 Clavius 1611–1612, 1:9, cited and discussed in Dear 1995, 218.

20 See Gaukroger 1995, 58; Sirven 1928, 35; AT 1:71; and AT 4:730–1.

21 Clavius 1611–1612, 1:9, cited and discussed in Dear 1995, 218.

22 Proclus 1970, 140. Clavius cites and follows Proclus' discussion of definitions, postulates, and axioms in his own commentary on Euclid's *Elements*. For more discussion, see Dear 1995, 216–20.

[23] See Aristotle 1984, 1:472–3 (*On the Heavens* I.3 286^a16–17).

[24] See Descartes's letter to Morin, July 13, 1638 (AT 2:198–199, CSMK 3:107).

[25] See Ptolemy 1984, 422–3. On the status of models and the concept of "saving the phenomena" in ancient and medieval astronomy, see Goldstein 1997 and 2008.

[26] In the case of Copernicus, see Goddu 2010, 368–9.

[27] As Martinet 1974 points out, suppositions can have two distinct types of epistemic status in Descartes: suppositions known to be false, and probable suppositions. In his letter to Morin, Descartes argues that true conclusions can be deduced from either type of supposition. Descartes regards his suppositions in Rule 12 as probable, for they are, he believes, demonstrable.

[28] See Martinet 1974, 327–30.

[29] See, e.g., Laudan 1968, 21; Buchdahl 1969, 118–26; Clarke 1982, 108–65; Hatfield 1988, 259.

[30] See also Garber 2001, 116–17 and Chapter 11, Sections 11.2–11.3.

[31] See also Fichant 1998, 29–59 and Chapter 11, Sections 11.4–11.6.

6

Descartes’s Theory of the Faculties in *Rules*

6.1 Mechanism, *Habitus*, and the Limits of Knowledge in *Rules*

To solve the problem of the limits of knowledge, Descartes must not only establish that intuition and deduction are the only operations of the mind that produce science, he must also determine their complete *extension* or the limits beyond which they cannot cross (i.e., what objects they can and cannot have). No purely mechanical theory of the human cognitive faculties can do either of these things. The mere mechanism of the faculties does not produce science, let alone determine the limits of science or knowledge. Otherwise, the human mind would automatically produce science merely by operating, and the method would be otiose. Only a mind directed by *rules*—i.e., a mind that has learned how to employ its faculties *correctly*—can produce science, and only a mind that has enumerated$_3$ and evaluated its faculties can determine the limits of its own cognitive capacity. The *limits of knowledge* are nothing but the *limits of human cognitive capacity*, and so it is in the end this capacity that is the principal object of inquiry in the problem of the limits of knowledge. Descartes’s primary goal in Rule 12 is *not* (or not only or even principally) to mechanize Aristotelian faculty psychology, but rather to ensure that “no part of human industry has been neglected” in the pursuit of science (AT 10:410, CSM 1:39; translation modified) and to establish the means to “cultivate our mental powers [*ingenia excolenda*]” and acquire the Cartesian scientific *habitus* (AT 10:429, CSM 1:51). The entire purpose of “look[ing] at these faculties” is precisely “to see in what respect each of them could be a hindrance, so that

we may be on our guard, and in what respect an asset, so that we may make full use of their resources" (AT 10:398, CSM 1:32). Descartes needs a mechanical theory of the faculties only because he needs to describe how the faculties mechanically operate in order to describe how best to *exploit* the mechanism.[1]

In Sections 6.2–6.4, I reconstruct Descartes's theory of the faculties in *Rules*, including sense-perception (*sensus*); the common sense; the phantasy; and what he terms the "force" or "power" of knowing (*vis cognoscens*). In Section 6.4, I also show how *vis cognoscens* can freely intervene in the cognitive mechanism and modify the operation of the other cognitive organs and faculties (above all, the phantasy and, therefore, the imagination) such that the intuition of the simple natures by the intellect becomes possible. Throughout Sections 6.2–6.4, I compare Descartes's theory of the faculties in *Rules* to late Aristotelian alternatives in order to better isolate what is distinctive about Descartes's theory of the faculties in *Rules* and related texts. In the remainder of this section, I further elucidate the relation between mechanism and method in Descartes's solution to the problem of the limits of knowledge.

The clearest indication that Descartes needs a mechanical theory of the faculties in order to describe how best to exploit the mechanism is that the entire mechanism is based on what Descartes terms "figure," which denotes both the modified shape of the sensory organs when affected by objects (e.g., the dilation of the retina in the eye) and the objects of sense themselves, which Descartes reduces to differences between measurable, extended magnitudes (shapes). Sensory representation occurs when the objects of sense modify the shape of the sensory organs, which transmit these figures to the brain (via the nerves) and produce the corresponding sensory representations in the mind or "knowing force" (*vis cognoscens*). Once Descartes's description reaches *vis cognoscens*, he shows how the latter can reverse the mechanism by actively modifying the shape of the phantasy (a corporeal organ located in the brain), which in turn produces the representation of the corresponding figure (e.g., a line or a rectangle) in *vis cognoscens*, in which case *vis cognoscens* is termed *ingenium*, "the proper term for [*vis cognoscens*] when it forms new ideas [i.e., figures] in the phantasy" (AT 10:416, CSM 1:42).[2] The production of figures by *ingenium* plays a pivotal role in mathematics and, therefore, in the solution to perfectly understood problems, to which Descartes devotes the entire

second part of *Rules*, beginning in Rule 13. In Rule 13, Descartes requires that all perfectly understood problems in mathematics be "re-expressed in terms of the real extension of bodies" and "pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect" (AT 10:438, CSM 1:56). Descartes prioritizes figure in his description of the cognitive mechanism in order to secure the possibility of an operation needed in solutions to problems in mathematics: the production of figures that can be distinctly perceived and manipulated by the intellect in geometrical or *spatial intuition*. The intuition of the material simple natures (extension, shape, and motion), the solution to problems in mathematics, and the very possibility of mathematics as a science (i.e., certain and evident cognition based on intuition) require the cooperation of the intellect and the imagination and is based entirely on *ingenium*'s mechanical capacity to produce figures in the phantasy and, therefore, the imagination.[3] Descartes devotes the remainder of the treatise (Rules 13–21) to describing how the intellect and the imagination should be employed in mathematics. Here as elsewhere, the mechanism is not the *terminus ad quem*; it is described so that it may be maximally exploited.

Descartes's use of figure does not constitute his solution to the problem of the limits of knowledge, but rather its most immediate application to problems in *one* science: mathematics. The broader solution consists in the cognitive regime Descartes prescribes immediately after he concludes his description of the cognitive mechanism. This cognitive regime is based on two fundamental "faculty configurations," as I will henceforth refer to them: the use of the intellect alone (bracketing all other cognitive faculties), which enables the intellect to intuit the intellectual simple natures (the objects of metaphysics), and the use of the intellect aided by the imagination, which enables the intellect to intuit the material simple natures (the objects of mathematics and physics). (The common simple natures can be intuited by either faculty configuration.) These two faculty configurations are the foundation of the Cartesian scientific *habitus*, since they define the employment of the faculties that ultimately yields the simple natures and, therefore, Descartes's solution to the problem of the limits of knowledge in *Rules*. Nothing that is not either a simple nature or composed of (correctly conjoined) simple natures can be an object of science or knowledge.

6.2 Sensation and Figure

Descartes describes "sense-perception" as "merely passive […]; sense-perception occurs in the same way in which wax takes on an impression from a seal" (AT 10:412, CSM 1:40). Descartes insists that "[i]t should not be thought that I have a mere analogy [*analogiam*] in mind here: we must think of the external shape [*figuram*] of the sentient body [*corporis sentientis*] as being really changed [*realiter mutari*] by the object in exactly the same way [*sicut*] as the shape of the surface of the wax is altered by the seal" (AT 10:412, CSM 1:40).[4] Here, "figure" refers, not (or not yet) to the shape of the object of sense, but rather to the modified shape of the sensory organs when they are affected by objects. Descartes focuses exclusively on how the "external shape of the sentient body" is physiologically or "really changed" when affected by objects: "[I]n the eye, the first opaque membrane receives the shape impressed upon it by multi-colored light; and in the ears, the nose and the tongue, the first membrane which is impervious to the passage of the object takes on a new shape from the sound, the smell and the flavor respectively" (AT 10:412–13, CSM 1:40). The shape of the sensory organs can change in a variety of ways. "[T]he first membrane which is impervious to the passage of the object," such as the surface of the eye, is plastic. The *plasticity* of the sensory organs—their ability to suffer modifications in shape—is a purely mechanical property of the human body. These modifications produce motions that are transmitted (via the nerves) to the common sense and the phantasy (corporeal organs in the brain) (see Section 6.3), where they cause sensations and sensory representations in the mind (*vis cognoscens*) (see Section 6.4).

Having described how the sensory organs change shape when affected by objects, Descartes turns to describe the objects themselves, and he reduces all qualitative differences between the objects of sense (e.g., differences in color, sound, taste, smell, and tactility) to differences in shape alone:

> Take color, for example: whatever you may suppose color to be, you will not deny that it is extended and consequently has shape. So what troublesome consequences could there be if – while avoiding the useless assumption and pointless invention of some new entity, and without denying what others have preferred to think on the subject – we simply make an abstraction, setting aside every feature of color apart from its possessing the character of shape, and conceive of the difference between white, blue, red, etc., as being like that of the difference between the following figures or similar ones [see Figure 6.1]. The same can be

said about everything perceivable by the senses, since it is certain that the infinite multiplicity of figures is sufficient for the expression of all the differences in perceptible things (AT 10:413, CSM 1:41).

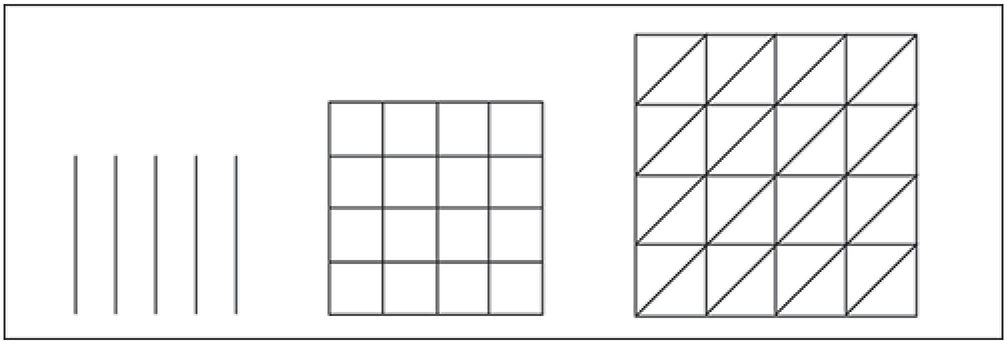


Figure 6.1 Descartes's geometrical representation of color in Rule 12 (AT 10:413)

By reducing all qualitative differences between the objects of sense to differences between extended magnitudes alone, Descartes ensures that these differences can be exactly measured and, therefore, intuited by the mind. This is the principle behind what has been described as Descartes's geometrical "codification" of the sensible.[5] The code operates at two levels: qualitative differences between the objects of sense are "encoded" as differences between figures, and these differences are then "decoded" by the mind when it is affected by the figures transmitted from the common sense and the phantasy, thereby producing sensory representations that do not resemble their causes. What the geometrical code provides is an exact representation of differences between sensibilia as differences in *proportion*: "One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 1 to 3 unless we have recourse to a certain analogy with the extension of a body that has shape" (Rule 14, AT 10:441, CSM 1:58). To establish the code, Descartes first "simply make[s] an abstraction [*abstrahamus*], setting aside every feature of color apart from its possessing the character of shape." Descartes then encodes differences between colors (and other sensible qualities) by differentiating the figures themselves. In *Rules*, the geometrical codification of differences between sensibilia is purely conventional and, therefore, arbitrary. Whatever class of figures one

chooses, the codification of these differences is possible because “the infinite multiplicity of figures is sufficient for the expression of all the differences in perceptible things” (AT 10:413, CSM 1:41). Indeed, the infinite multiplicity of figures exceeds the finite multiplicity of figures the sense organs can assume, and so ensures that no differences between sensibilia escape exact representation by means of Descartes’s geometrical code.

Descartes notes that everybody—both he and the Aristotelians—“will not deny” that qualities such as color *inhere* in extension and, therefore, have shape. Descartes here appeals to the thesis held by his Aristotelian contemporaries that quantity (specifically, extension) has *ontological priority over all other accidents* (specifically, qualities such as color and heat) in order to motivate his own reduction of qualitative differences between the objects of sense to differences between extended magnitudes alone. For Suárez, “[a]ll [accidents] inhere in substance by way of quantity, as all the philosophers teach us; and experience seems to prove this, since white is extended over the surface [of a thing]; similarly heat, when it spreads through a body, is extended in the quantity of the body.”[6] Descartes parts ways with his Aristotelian contemporaries when he *reduces* these qualities themselves, together with the body in which they inhere, to the material simple natures of extension and figure alone. This is historically very radical. In Aristotelian ontologies, “figure” is but a *quality of quantity* in virtue of which material substances have this or that shape; body is in no way *reducible* to figure and extension, for there is ontologically far more to body than figure and extension *alone*.[7] Figure, moreover, ranks *ontologically lower than other qualities*, such as color or heat. Color is a quality of the substance in which it inheres, and while it inheres in substances by way of quantity, it is not a mode of extension as figure is. Figure, by contrast, is a mode of extension; it is a quality, not of the substance, but rather of the quantity of the substance. In short, *figure is an accident of an accident*; only the quantity of a substance inheres in the substance, while the shape of the quantity is but an accidental mode of the quantity (which may assume any number of shapes).[8] When seen in historical context, in Rule 12 *Descartes effectively reduces the objects of sense to the species of quality most removed from the category of substance in Aristotelian ontologies*. His decision is hardly accidental. While figure is *ontologically lower* in rank than substance, quantity, and qualities such as

heat or color, in *Rules* it is *epistemically higher* in rank than all three. Descartes regarded substantial forms and real qualities as confused ideas, and he regarded the Aristotelian category of quantity as similarly confused, since many Aristotelians distinguished between quantity, matter, and extension.[9] Figure, by contrast, is wholly transparent to intuition. This is an excellent example of how Cartesian epistemic considerations overturn Aristotelian ontological considerations in *Rules*.

A further important implication of Descartes's reduction of qualitative differences between the objects of sense to differences between figures alone is that Descartes *eliminates Aristotelian proper sensibles* from his theory of sense in *Rules*, and so *does not individuate any one sensory faculty by reference to its proper sensible object.* For Aristotle, each sensory faculty must be individuated by "what is special" or proper to it, as "color is the special object of sight, sound of hearing, flavor of taste," and so on.[10] Figure is not a proper sensible, but rather a *common sensible*, since it "is not special to any one sense," and as Descartes points out in Rule 12, "nothing is more readily perceivable by the senses, for it [figure] can be touched as well as seen" (AT 10:413, CSM 1:40). In Rule 12, Descartes replaces the proper sensibles with a common sensible,[11] the most geometrical (and, therefore, the most intuitable) common sensible. In so doing, he eliminates Aristotle's criterion for individuating the sensory faculties, for there are no more proper sensibles by which to individuate each one. The object of every sensory faculty is one and the same: figure and figure alone. Henceforth, each sensory faculty and, indeed, all cognitive faculties (with the exception of the pure intellect) will be distinguished from one another by reference to the function each performs in how it receives, transmits, or produces figure alone.

Nevertheless, despite these differences, Descartes insists that he is not "denying what others have preferred to think on the subject" of color or sensible qualities more broadly (AT 10:413, CSM 1:41). The existence of color as a quality inhering in hylomorphic substances is not denied. Indeed, Descartes *cannot* deny that color inheres in hylomorphic substances; abstraction and codification are purely intellectual acts that have no necessary basis in how things are in reality.[12] Descartes excludes sensible qualities from his description of the objects of sense in Rule 12 because, as he puts it in the passage cited above, he deems them a "useless assumption and pointless invention of some new entity." Descartes does not

demonstrate (and does not purport to demonstrate) that real qualities do not *exist* in nature—not in *Rules*, at any rate. The reduction of body to extension in *Rules* is purely methodological, not metaphysical.

6.3 From Figure to Representation: The Common Sense, the Phantasy, and the Passivity of *Vis Cognoscens*

The modifications in the shape of the sensory organs produce motions in the nerves that are instantaneously transmitted to the common sense (*sensus communis*):

> Secondly, it must be conceived that [*concipiendum est*] when an external sense organ is stimulated by an object, the figure which it receives is conveyed at one and the same moment to another part of the body known as the 'common' sense [*quae vocatur sensus communis*], without any entity really [*entis reali*] passing from the one to the other. […] Thirdly, the common sense functions like a seal, fashioning in the phantasy or imagination [*phantasia vel imaginatione*], as if in wax, the same figures or ideas [*figuras vel ideas*] which come, pure and without body [*puras et sine corpore*], from the external senses (AT 10:414, CSM 1:41; translation modified).

What does the common sense do? Before the figures received from the sensory organs can affect *vis cognoscens*, they must be *synthesized* in the common sense and delivered as a bundle to *vis cognoscens* via the phantasy. The synthesis of figures in the common sense occurs *mechanically* and *passively*; the common sense can synthesize the figures received from the external senses because it is *physically located in the part of the brain where the figures transmitted from the external senses converge* and *because it* "*is large enough to allow different parts of it to take on many different figures*" (ibid., my emphasis). In *Rules*, the *mechanical properties of an organ* (such as location and size) explain its *cognitive function*: the common sense can exercise its function only because it is located where it is and has the size that it does.[13] However, Descartes appeals to no empirical observations in order to demonstrate his supposition that figures are instantaneously transmitted from the five external senses to the common sense or from the common sense to the phantasy. Instead, he introduces a second comparison: "In exactly the same way I understand that while I am writing, at the very moment when individual letters are traced on the paper, not only does the point of the pen move, but the slightest motion of this part

cannot but be transmitted simultaneously to the whole pen" (AT 10:414, CSM 1:41). He concludes: "Who then would think that the connection between the parts of the human body is less close than that between the parts of a pen? What simpler way of portraying the matter can be devised [*simplicius excogitari*]?" (ibid; translation modified). Descartes appeals to the epistemic criterion of *simplicity* in order to defend his supposition (see Chapter 5, Section 5.3).

Unlike the Conimbricenses, Descartes does not describe the common sense as the cognitive faculty responsible for synthesizing and distinguishing between the proper sensibles of the five external senses.[14] As I argued in Section 6.2, Descartes eliminates proper sensibles from his description of the cognitive process in *Rules* and replaces them with figure (a common sensible). This has led some commentators to argue that Descartes's theory of sense in *Rules* removes the need for the synthetic function Aristotelians typically assign to the common sense. According to this interpretation, Descartes eliminates the diversity of proper sensibles whose phenomenological unity in perception requires explanation via the synthetic function of the common sense. He eliminates the *explanandum* for which the synthetic function of the common sense is the *explanans*.[15] However, even though Descartes eliminates the proper sensibles, the relevant phenomenological *explanandum* is *not* thereby eliminated. The figures my sensory organs receive are diverse in at least two respects. (1) The figures received from my eyes are different than the figures received from my ears, nose, tongue, and flesh. (2) Most of my sensory organs are double, not single (I have two ears, two eyes, two nostrils, and every part of my flesh is sensitive), and each of these organs produces two or more figures at any given time.[16] These figures may be the same shape and size (my two eyes are typically affected by objects in the same way), but it is no less true that they are *numerically distinct* from one another. And yet I do not perceive two objects, but only one. Figures received from a variety of sensory organs eventually produce the sensory perception of a single object. Thus, before the figures received from the five external senses affect *vis cognoscens*, the common sense must synthesize them.

The synthetic function is the only function of the common sense that Descartes retains from Aristotelian psychologies in *Rules*. For Descartes, the common sense cannot yield any sensory representations without *vis cognoscens*. This is decidedly not the case in Aristotelian psychologies. For

Aristotle, the common sense is a freestanding representational faculty common to both human and non-human animals. Its representational functions include: (1) simultaneous perception (e.g., sensing many objects via many senses at once); (2) perceptual discrimination (e.g., distinguishing between many objects via many senses at once or between different sensory properties in one and the same object); (3) synthesis or cross-modal binding (perceiving many properties via many senses in one object); (4) the perception of common sensibles (e.g., the perception of shape by the eyes and the hands); and (5) perceptual self-consciousness (knowing that I am sensing an object).[17] In their commentary on *De anima*, the Conimbricenses add that the common sense is responsible for (6) *making judgments* about what is beneficial or harmful in the environment.[18] For Aristotle and the Conimbricenses, these functions do not depend on the possession of higher cognitive faculties such as the intellect, since non-human animals also perform these functions.[19] For Descartes, animals are reducible to extension, shape, and motion alone, and so do not enjoy sensory representations of any sort. There is, therefore, no need to explain the possibility of such representations in animals by reference to a non-intellectual cognitive faculty such as the common sense. Animals *do* have a common sense where the figures transmitted by their sensory organs converge, but since animals do not have *minds*, *these figures produce no sensory representations*. They only produce a mechanical response in the phantasy, which then causes movements in the nerves and muscles of the animal: “This enables us to understand how all the movements of other animals can come about, even though we refuse to allow that they have any awareness of things [*rerum cognitio*], but merely grant them a purely corporeal imagination” (Rule 12, AT 10:415, CSM 1:42). Only *vis cognoscens* can perform the functions Aristotle assigns to the common sense;[20] all of the functions Aristotelian psychologies assign to the common sense Descartes outsources to *vis cognoscens* alone.

As many commentators have observed, Descartes’s claim that “the common sense functions like a seal, fashioning in the phantasy or imagination, as if in wax, the same figures or ideas [*figuras vel ideas*] which come, pure and without body [*puras et sine corpore*], from the external senses” suggests that the common sense somehow *spiritualizes* figures, so that they become incorporeal ideas. Curiously, it also seems to suggest the exact opposite: Descartes equates figures and ideas, which

suggests that *ideas, like figures, are themselves corporeal* in *Rules*.[21] When Descartes describes the figures received from the five external senses as "pure and without body," however, he only means to insist that no body passes from the sensory organs to the common sense. Figure is not a body, but rather a *mode* of body: the *human* body. All that happens in the transmission of figure is that a modification in the shape of one part of the body produces a modification in the shape of another part. There is no need to interpret "pure and without body" as denoting a process of spiritualization. How could a corporeal part of the body spiritualize anything anyway?

As for the identification of figures and ideas, in Rule 14 Descartes clearly states that in Rule 12 "we conceived of the phantasy [*phantasiam*], *along with the ideas existing in it*, as being nothing but a real body with a real extension and shape" (AT 10:441, CSM 1:58; my emphasis). With one exception, Descartes consistently employs the term "idea" in *Rules* to refer to *figures in the phantasy*, which he also variously terms "species," "images," "corporeal ideas," and "phantasms," sometimes vacillating between physical and mental (representational) senses of these terms.[22] This *in no way entails that the pure intellect cannot understand incorporeal objects such as the intellectual simple natures*. The possibility of purely intellectual intuition does not depend on there being incorporeal ideas in *Rules*. For the simple natures, Descartes reserves the broader term "notions" (AT 10:417, CSM 1:43), employing "idea" in all but one case to refer to figures in the phantasy.[23]

Returning to the role the common sense plays in the cognitive mechanism, the transmission of figures from the sensory organs to the common sense is instantaneous because the nerves that connect them to the common sense are continuous. Relative to the five external senses, the common sense corresponds to the wax in Descartes's wax-and-seal comparison, and is wholly passive. Relative to the phantasy, however, the common sense corresponds to the seal, and is active (activity and passivity are relative terms; see *Passions* I.1, AT 11:327–328, CSM 1:328). It is here that the phantasy causes representations in *vis cognoscens.* How a corporeal organ can affect a spiritual force is no more obscure here than it is elsewhere in Descartes's corpus, but even on the assumption that this problem can be solved,[24] there remains yet another: the problem of the *object* of representation in *Rules*. Is the direct object of representation a

figure in the phantasy, or is it rather the extra-mental object itself? Some commentators have argued that the direct object of sensory representation in *Rules* is the figure in the phantasy itself, not the extra-mental object that causes it.[25] Others have argued that the direct object of sensory representation in *Rules* is not the figure in the phantasy, but rather the extra-mental object that causes the figure in the phantasy. The figure merely causes a sensory representation of an extra-mental object.[26] Which interpretation is correct?

I regard the latter interpretation as more likely. In *Fifth Replies*, Descartes writes that "[in] the case of the imagination [*imaginationem*] […] which can have only corporeal things as its object, we do indeed require a semblance [*specie*] which is a real body: the mind applies itself to this semblance but does not receive it [*mens se applicet sed non quae in mente recipiatur*]" (AT 7:387, CSM 2:265). Here, the mind *applies itself* to a figure in the phantasy, but does not *receive* the figure *as it exists in the phantasy*. Similarly, in *Comments on a Certain Broadsheet*, Descartes writes that "neither the motions themselves nor the figures arising from them are conceived by us exactly as they occur in the sense organs" (AT 8B:304, CSM 1:304). The mind never "sees" anything in the body, but is only affected by the body in the manner which is peculiar to it: the body causes it to have sensory representations, but there is no resemblance between these sensory representations and the figures in the phantasy that cause them. This is clearly the case in Rule 12: the figures that cause the sensory representations of color in no way resemble the colors perceived. As a purely spiritual force, *vis cognoscens* can *causally interact* with the body, but *cannot "see" or receive anything corporeal in the body*. Descartes insists that the wax-seal comparison can only be taken literally when applied to interactions between sensory organs (AT 10:415, CSM 1:42). This means that the figures *vis cognoscens* receives from the phantasy and the common sense are not physically inscribed on its surface (*vis cognoscens* is not a body, and so it has no surface). To directly "see" these figures, it would have to do so in the phantasy. But it can only interact with or be affected by these figures *in the manner that is ontologically peculiar to it as a purely spiritual representational faculty*. The interaction between *vis cognoscens* and the brain is *purely causal*. Even in the case of a figure produced by *vis cognoscens* in the phantasy, any figure so produced must in turn affect *vis cognoscens* in order to cause the relevant act of

imagining in *vis cognoscens*. *Vis cognoscens* is active in its production of figures in the phantasy, but *passive in its affection by the very figures it actively produces in the phantasy*. As I have been arguing, this affection produces a mental representation; *vis cognoscens* never directly inspects figures in the phantasy.

Other commentators have argued that the direct objects of sensory representation in *Rules* are incorporeal ideas caused by figures in the phantasy. On this interpretation what *vis cognoscens* directly perceives is not the object represented by the idea (e.g., the sun in the sky), but rather the idea itself (e.g., the idea of the sun in the sky, by the immediate perception of which I "indirectly perceive" the sun in the sky); ideas mediate between the mind and extra-mental objects as a *tertium quid*.[27] This sort of interpretation draws encouragement from Descartes's apparently interchangeable use of the terms "figure," "idea," "picture," and "image" in Rule 12 (see AT 10:413–17, CSM 1:139). The latter two words especially suggest that in *Rules* the direct object of consciousness in sensory representation is a picture or image of an object, and that this picture or image mediates between the mind and the object whose picture or image it is, effectively constituting a "veil" separating the mind from extra-mental reality, such that the mind only ever has indirect access to extra-mental reality via the intermediary of ideas.[28]

Descartes's theory in *Rules* neither needs nor leaves any room for a *tertium quid* mediating between the mind and extra-mental reality. To show that sensory representations direct the mind to extra-mental objects, all that needs to be shown is that there are *systematic causal correlations* between the extra-mental object, the figure it causes in the phantasy, and the representation the figure in the phantasy causes in *vis cognoscens*.[29] That such a correlation obtains is clear. The figure in the phantasy is only the *terminus* of a causal sequence initiated by the extra-mental object, *which is also a figure*. The figure in the phantasy and the figure of the object, moreover, are distinct *only in terms of location and size, but not in terms of the ordered relation their parts have to one another.* This relation is a relation of *proportionality*, as a drawing of a house differs in scale from the house drawn. Consequently, the figures received in the phantasy are causally connected to the extra-mental object, and their shapes and sizes systematically covary with it according to a definite proportion. Since any variation in the extra-mental object entails a correlative variation in the

shape and size of the figure it causes in the phantasy, there is a systematic correlation between the extra-mental object, the figure it causes in the phantasy, and the representation the figure in the phantasy causes in *vis cognoscens*. Any variation in the figure in the phantasy is caused by the extra-mental object and *ipso facto* causes a correlative variation in the sensory representation. The required systematic correlation obtains. This means that when an extra-mental object causes a sensory representation in *vis cognoscens* via a figure in the phantasy, the object becomes what Suárez terms the "objective concept," i.e., "the singular and individual thing, insofar as it can be given to the mind [*quatenus menti objici potest*] and conceived by a formal act […]."[30]

6.4 The Activity of *Vis Cognoscens* and Descartes's Habitual Theory of the Faculties in *Rules*

Descartes describes *vis cognoscens* in Rule 12 as follows:

> Fifthly, and lastly, the force through which we know things in the strict sense [*vim illam, per quam res proprie cognoscimus*] is purely spiritual, and is no less distinct from the whole body than blood is distinct from bone, or the hand from the eye.[31] It is one single force, whether it receives [*accipit*] figures from the common sense at the same time as does the phantasy, or applies itself [*se applicat*] to those which are preserved in the memory, or forms new ones […]. In all these functions the cognitive force [*vis cognoscens*] is sometimes passive, sometimes active [*interdum patitur, interdum agit*]; sometimes resembling the seal, sometimes the wax. But this should be understood merely as an analogy [*per analogiam tantum*], for nothing quite like this force is to be found in corporeal things. It is one and the same force [*una et eadem*]: when applying itself along with the imagination to the common sense, it is said to see, touch, etc.; when addressing itself to the imagination alone, insofar as the latter is invested with various figures, it is said to remember; when applying itself to the imagination in order to form new figures, it is said to imagine or conceive; and lastly, when it acts on its own [*sola agat*], it is said to understand [*intelligere*]. […] According to its different functions [*functiones diversas*], then, the same force is either called pure intellect, or imagination, or memory, or sense-perception. But when it forms new ideas in the corporeal imagination, or concentrates on those already formed, the proper term for it is *ingenium* (AT 10: 415–16, CSM 1: 42; translation slightly modified).[32]

Vis cognoscens is the subject of sensing, remembering, imagining, and understanding. Because it is "sometimes passive, sometimes active," *vis cognoscens* is also the subject of will. *Vis cognoscens*'s active function as will constitutes the possibility of cognitive self-determination in *Rules*; *it*

enables the operator of the method to intervene in and regulate the mechanical operation of the brain.[33] Descartes's habitual theory of the faculties in *Rules*—his theory of how the cognitive faculties should be employed in order to produce science—depends on the causal agency of *vis cognoscens*:

> The intellect can be stimulated by the imagination or act upon it [*vel contra agere in illam*] […]. So we can conclude with certainty that when the intellect is concerned with matters in which there is nothing corporeal or similar to the corporeal, it cannot receive any help from these faculties; on the contrary, if it is not to be hampered by them, the senses must be kept back [*arcendos*] and the imagination [*imaginationem*] must, as far as possible, be divested of every distinct impression [*omni impressione distincta exuendam*]. If, however, the intellect proposes to examine something which can be referred to the body, the idea of that thing must be formed as distinctly as possible in the imagination [*imaginatione*] (AT 10:416–17, CSM 1:43).

Vis cognoscens can and must exploit the mechanical operation of the brain and configure its faculties in different ways in relation to different classes of knowable object. When the intellect "is concerned with matters in which there is nothing corporeal," *vis cognoscens* must bracket the senses and the imagination. The senses must be "kept back," and the "imagination must be divested of every distinct impression." The intellectual simple natures cannot be intuited unless the intellect is operating entirely on its own. When this rule is violated, the intellectual simple natures and the material simple natures become confused with one another. The possibility of intuiting the intellectual simple natures depends, therefore, on *vis cognoscens*'s *active intervention in the mechanism of the brain and, therefore, the human body*. The pure intellect is not the only faculty that plays a positive role in Cartesian science in *Rules*. The imagination plays an irreducible role in the intuition of the material simple natures, and so in both natural philosophy and mathematics. The imagination must aid the intellect whenever the latter "proposes to examine something which can be referred to body." Qua *ingenium*, *vis cognoscens* can freely initiate motion in the phantasy in order to "form new figures" in the imagination (AT 10:416, CSM 1:42). When it comes to the simple natures (and the sciences they define), properly employing the intellect or the intellect aided by the imagination is equivalent to actively controlling the human body, either by bracketing those cognitive functions that depend on the brain or by employing them as an aid to the intellect. *The cognitive self-discipline required by the method is also corporeal self-discipline; to correctly employ the pure intellect or the*

intellect aided by the imagination, one must control how the body affects the mind. (This explains why purely intellectual intuition requires effort; the phantasy typically affects the intellect in a manner that renders the very distinction between these two modes of cognition difficult to discern.) Cognitive and corporeal self-discipline are two parts of one and the same methodological practice. It is only via repeated acts of such self-discipline that I can acquire and perfect the *habitus* that disposes me to employ my cognitive faculties correctly, such that I may intuit the relevant class of simple natures and solve problems by their means. This is how I become a *bona fide* subject of science who has the "intellectual aptitude to solve any given problem" (AT 10:367, CSM 1:13). As we will see in more detail in Chapter 7, the employment of the pure intellect and the intellect aided by the imagination is the foundation of Descartes's solution to the second part of the problem of the limits of knowledge: the things it is possible to know, or the objects of knowledge—the simple natures.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0007

[1] Because they interpret Descartes's theory of the faculties in *Rules* independently of his solution to the problem of the limits of knowledge, most commentators see the mechanization of Aristotelian faculty psychology as Descartes's principal ambition in Rule 12. See, e.g., Beck 1952, 15–30; Marion 1975, 113–31; Clarke 2003, 16–50, 81–9; Gaukroger 1995, 152–86.

[2] See Fichant 1998, 4–5. With one exception (in Rule 4, AT 10:376, CSM 1:18), "idea" and "figure" are employed interchangeably in *Rules*. I discuss this in more detail below in Section 6.3.

[3] While Descartes employs the terms *phantasia* and *imaginatione* interchangeably in *Rules*, sometimes referring to the organ in the brain that causes representations of figure(s) in the mind, sometimes referring to these representations themselves, I will employ "phantasy" to refer exclusively to the organ in the brain, and "imagination" to refer exclusively to the act of representation and/or the representations themselves (e.g., the act of imagining a square; an imagined square, etc.). (Clarke 2003, 82, n. 6 maintains that in *Rules* Descartes does employ these terms in the way I have decided to employ them here, but the evidence indicates that this is not so.) Similarly, I will employ "faculty" to refer to the relevant cognitive function of *vis cognoscens* (e.g., the faculty of imagination), and "organ" to refer to the part of the brain that causally underwrites it (e.g., the phantasy as the organ that causes representations in the imagination).

[4] Aristotle compares sensation to the impression of a seal on wax in *De anima* II.12 424^{a}17–23 in Aristotle 1984, 1: 674. Descartes mechanizes and literalizes the comparison here. See also Marion 1975, 116–22.

[5] See Marion 1975, 116–22 and Marion 1981, 231–64.

[6] See Suárez 1856–1878, 25:495 (*DM*, disp. 14, sec. 4.7), cited in Des Chene 1996, 97, n. 28.

[7] See Des Chene 1996, 109.

[8] See Des Chene 1996, 112.

[9] See Rule 14, AT 10:447, CSM 1:62: “In this context, then, we are concerned with an extended object, thinking of it exclusively in terms of its extension, and deliberately refraining from using the term ‘quantity’; for there are some philosophers so subtle that they have even distinguished quantity from extension.” See also *Principles* II. 9–15, AT 8A:45–9, CSM 1:226–9. For examples of the distinction between quantity, matter, and extension in Aristotelianism, see Fonseca 1599, 2:647–8 (lib. 5, cap. 13, q. 2, sec. 2); Suárez 1856–1878, 25:535 (*DM*, disp. 40, sec. 4.2) and 538 (disp. 40, sec. 2.20), cited in Des Chene 1996, 103. On late Aristotelian conceptions of quantity, see Des Chene 1996, 97–109. On medieval conceptions of quantity more broadly in relation to Descartes, see Biard 1997.

[10] Aristotle 1984, 1:665 (*De anima* II. 6 418[a]7–418[a]19).

[11] See Marion 1975, 116–22.

[12] For more discussion, see Chapter 2, Section 2.4.

[13] Descartes consistently accounts for the function of the pineal gland in terms of its location. See letter to Meyssonier, January 29, 1640 (AT 3:19–20, CSMK 3:143). Note also that, for Descartes, the common sense is located in the brain, whereas for Aristotle the common sense is located in the heart. The location of the common sense in the brain is Galenic in origin, and is taken up by the Conimbricenses. See Conimbricenses [1598] 1604, 375 (*In De anima*, lib. 3, q. 2, art. 2): “*Sensum communem non in corde, sed in cerebro residere*.”

[14] See Conimbricenses [1598] 1604, 370–6 (*In De anima*, lib. 3, q. 2). On the common sense in Aristotle, see Gregoric 2007 and Marmodoro 2014. On the history of the concept of the common sense from Aristotle to Reid, see Azouvi and Kambouchner 1991.

[15] See Marion 1975, 122–6: “[L]e rôle synthétique du sens commun doive disparaître; il ne reste donc plus […] qu’à lui attribuer une autre fonction, la simple transmission des informations, et leur stockage […]” (124). In fact, it is the phantasy, not the common sense, that is responsible for stocking figures. See AT 10:414, CSM 1:41–2: “The phantasy is a genuine part of the body, and is large enough […] to retain [figures] for some time; in which case it is to be identified with memory.”

[16] See also *Passions* I. 32 (AT 11:352–3, CSM 1:340) and letter to Meyssonier, January 29, 1640: “…it must necessarily be the case that the impressions which enter by the two eyes or by the two ears, and so on, unite with each other in some part of the body before being considered by the soul” (AT 3:19–20, CSMK 3:143).

[17] See Marmadoro 2014, 156–89 and Gregoric 2007, 129–63. Regarding perceptual self-consciousness, Aristotle sometimes seems to assign this function to the special senses themselves (see *De anima* 425[b]12–17 in Aristotle 1984, 1:677), sometimes to the common sense (see *De somno* 455[a]15–17 in Aristotle 1984, 1:723 and Marmadoro 2014, 237–62, esp. 257–8).

[18] These judgments can be made by non-rational animals about prey. See Conimbricenses [1598] 1604, 369 (*In De anima*, lib. 3, q.2, art. 1). Many sixteenth- and seventeenth-century philosophers (Toletus, Dupleix, Eustachius) affirmed that animals can judge, while others (Rubio) denied that animals can judge. For references and discussion, see Hatfield 2008 and 2012, 165, n. 42.

[19] As Marmadoro 2014, 118, n. 4, points out, for Aristotle “the faculty of perception is common between humans and animals,” which explains why “Aristotle develops an account of complex perceptual content such that its complexity is not the result of the intervention of a higher cognitive faculty (which animals don’t possess).”

[20] See also *Dioptrics* IV: "[It] is the mind which senses, not the body [...]. And we know that it is not properly because the mind is in the parts serving as organs to the exterior senses that it experiences sensation, even though it is in contact with various objects, but because it is in the brain, where it exercises that faculty which is called the common sense" (AT 6:109; Descartes 2001, 87). There is some debate about whether Descartes believed that animals have no sentience whatever. See Cottingham 1998; Gaukroger 2002, 180–215; and Hatfield 2008, 404–26.

[21] Cousin (Descartes 1824–1826, 11:265); Sirven (Descartes 1951, 76); Brunschwicg (Descartes [1963] 1997, 135); and Marion (Descartes 1977, 42) all translate "*sine corpore*" by "incorporeal." G. Le Roy (Descartes 1933, 113); CSM; and Beyssade and Kambouchner (Descartes 2016, 413) have preferred to translate it more literally as "*sans corps*" or "without body." Clarke 2003, 50 accepts the translation of "*sine corpore*" by "incorporeal," and then rather curiously denies that "incorporeal" means "non-material." He could have made his point more easily by translating "*sine corpore*" as "without body." Roy argues that "l'imagination en tant que fantasie, c'est-à-dire corporelle, se modifie dans sa matière pour transmettre à l'esprit une figure qui n'est pas cette matière" (Roy 1944, 18–19). It remains unclear how, on Roy's account, a corporeal organ can spiritualize anything at all. See also Yolton 1984, 20, who argues that "the physical forms in the external sense organs have to be incorporealized," although he admits that it is "not clear" why this is so or how it is even possible. With the exception of Marion 1999, 48, most commentators argue that in *Rules* ideas are corporeal. See McRae 1965, 178; Reed 1982, 733; Costa 1983; Michael and Michael 1989; Gaukroger 1992; Sepper 1996, 97; Clarke 2003, 45–78; Olivio 2005, 275–81; Kambouchner 2006.

[22] The exception occurs in Rule 4, where Descartes writes that the "primary seeds of truth naturally implanted in human minds" enabled the ancients to "grasp true ideas in philosophy and mathematics" (AT 10:376, CSM 1:18). That this is a distinctively Cartesian use of the term "idea" seems clear from the fact that these ideas are in the human mind (not in the divine mind, as most pre-Cartesian philosophers believed; see Ariew and Grene 1995). These ideas seem to be non-corporeal.

[23] In *Treatise on Man* (AT 11:176–7, CSM 1:106) and *Second Replies* (AT 7:160–1, CSM 2:113), Descartes also refers to figures as ideas, but he expressly indicates that they are ideas only insofar as they cause sensory representations in the mind, never in their own right. He is not as explicit in *Rules*.

[24] For an interesting interpretation, see Garber 2001, 133–68.

[25] For Smith 1966, 54 and Schuster 2013, 314–20, Descartes regards the figure traced out in the phantasy as the direct object of sensory representation. See also McRae 1965, 178; Reed 1982, 733; Arbini 1983, 335; Larmore 1980; and Ott 2017, 30–63.

[26] See Alanen 2003, 138–47. My argument in this section about representation in *Rules* builds on hers.

[27] "Veil-of-ideas" interpretations of Descartes in *Rules* and elsewhere can be found in Beck 1965, 150–61; Kenny 1968, 114–16; Berger 1972, 110; Rorty 1979, 50–1; Hacking 1975, 33; Secada 2000, 79 and 283, n. 1; Newman 2009, 134–54; Dreyfus and Taylor 2015, 1–27.

[28] Variants of this interpretation are cited in n. 27 above. This interpretation of Descartes's theory of ideas has come under fire by a number of commentators. See, e.g., O'Neil 1974; Yolton 1984; Cronin 1966; Lennon 1974; Costa 1983; Normore 1986; Cook 1987; Ariew and Grene 1995; Alanen 2003; and Adriaenssen 2017.

[29] By "representation," I mean the intentional act whereby the object is presented or delivered to *vis cognoscens*. In *Rules*, "representation" cannot mean "idea" (in the sense of a *tertium quid* between act and extra-mental object), since the latter is identified with figure, and I have already shown that figures are not the direct object of sensory representation in *Rules*.

30 See Suárez 1856–1878, 25:65 (*DM*, disp. 2, sec. 1.1). Descartes uses similar language in Rule 12: “We should note that the intellect can never be deceived by any experience, provided that it limits itself to intuiting the exact thing that is its object [*intellectum a nullo unquam experimento decipi posse*, *si praecise tantum intueatur rem sibi objectam*], either within itself or in the imagination” (AT 10: 423, CSM 1: 47). The term “*objectam*” reflects scholastic usage, and it is here distinguished from “*rem*” because “*rem*” denotes the extra-mental object or *thing* independently of its relation to the mind, and “*objectam*” denotes *the very same thing insofar as it is given to the mind*. See also Beyssade and Kambouchner in Descartes 2016: 712, n. 264 and Cronin 1966.

31 In the Cambridge manuscript, Descartes’s language is less explicitly dualistic: “…the force by which we know things should be conceived as something in us no less distinct from the phantasy than is the eye, or the hand” (CM fo. 16^{v}). I discuss Descartes’s dualism in *Rules* in more detail in Dika 2020 and Chapter 8.

32 See also *Meditations* VI: “As for the faculties of willing, of understanding, of sensory perception and so on, these cannot be termed parts of the mind, since it is one and the same mind that wills, and understands and has sensory perceptions” (AT 7:86, CSM 2:59).

33 See Fichant 1998, 4–5.

7

Descartes's Theory of the Objects of Knowledge in *Rules*

7.1 The Simple Natures

On the basis of his theory of the faculties in Rule 12, Descartes proceeds to solve the second part of the problem of the limits of knowledge: "the actual things it is possible to know [*res ipsas*, *quae cognosci possunt*]" (AT 10:398, CSM 1:32) or the objects of knowledge, which Descartes terms "pure and simple natures" or "simple natures" (*naturam puram et simplicem*, AT 10:381, CSM 1:21; *naturas simplices*, AT 10:419, CSM 1:44). In this chapter, I reconstruct Descartes's theory of simple natures in *Rules* and explain its role in his solution to the problem of the limits of knowledge in Rules 8 and 12. In Section 7.2, I provide an interpretation of the rarely discussed, albeit central criteria of "cognitive indivisibility" and "univocity" whereby Descartes enumerates$_3$ the simple natures in Rule 12 and demarcates them as a class of notions from all other notions and ideas. In Sections 7.3–7.6, I show how Descartes draws on his theory of the faculties discussed in Chapter 6 in order to divide the class of simple natures into separate subclasses, and I discuss each subclass separately: the intellectual simple natures (Section 7.3); the material simple natures (Section 7.4); and the common simple natures and their corresponding privations and negations (Sections 7.5–7.6). In Sections 7.7–7.8, I reconstruct Descartes's theory of conjunction, which lays down rules that determine how simple natures may be combined with one another in intuition and deduction. In Section 7.7, I show how Descartes's theory of conjunction includes his earliest "theory of distinctions," which he would later take up more explicitly in *First Replies* and *Principles* I. 60–2. Section

7.8 discusses the structure of complex intuitions and the difference between complex intuitions and confusion. In Sections 7.9–7.10, I discuss the relation between intuition and judgment as well as Descartes's theory of error in *Rules*, both of which are grounded in the theory of simple natures. Finally, in Section 7.11–7.12, I show how Descartes's theory of the simple natures solves the problem of the limits of knowledge and I reassess the longstanding debate over whether the theory amounts to a species of "idealism" or "realism."

The importance of the simple natures in Descartes's solution to the problem of the limits of knowledge cannot be overestimated. First and most importantly, the limits of knowledge are defined by the simple natures: nothing can be understood (intuited) beyond the simple natures and their combinations (AT 10:422, CSM 1:46). Second, the theory of simple natures establishes that intuition and deduction are the only operations that produce science (besides enumeration) (AT 10:425, CSM 1:48; AT 10:427, CSM 1:49); only these operations deal with the simple natures in a manner that reliably produces certain and evident cognition. Third, the theory of simple natures extends intuition, deduction, and enumeration to every possible object of science, such that every science enjoys the same epistemic status (AT 10:427–8, CSM 1:50). Recall that in Rule 3, Descartes provisionally regarded mathematics as "alone […] concerned with an object so pure and simple" that it is always certain (AT 10:365, CSM 1:12). In Rule 12, he extends the certainty previously restricted to mathematics alone to every science, including metaphysics and natural philosophy. Finally, the unity of science asserted in Rule 1 is also established in Descartes's solution to the second part of the problem of the limits of knowledge: because the objects of all sciences, however complex, reduce to simple natures, all problems in the sciences can be solved by one method (*habitus*) and the sciences are themselves connected to one another via the simple natures and the connections that obtain between them. By the end of Rule 12, Descartes has sufficiently enumerated$_3$ the objects of science, such that the extension, limits, and exclusive claim to epistemic credibility of intuition and deduction are thereby also established, together with the unity of science.

7.2 The Enumerative Criteria: Cognitive Indivisibility, Self-Evidence, and Univocity

The theory of simple natures replaces the Aristotelian ontological division of being into ten (or however many) categories with Descartes's own epistemic division of things into intellectual, material, and common simple natures. As I have discussed numerous times (Chapter 2, Section 2.4; Chapter 3, Sections 3.2–3.3), the intellectual simple natures include knowledge, doubt, ignorance, volition, and in general any nature that can be intuited "without the aid of any corporeal image."[1] The material simple natures include extension, shape, and motion, and in general any nature the intellect must intuit with the aid of the imagination. The material simple natures constitute the object of Descartes's mathematics and physics (or, more broadly, natural philosophy). The common simple natures include what I term "epistemic transcendentals," such as "existence," "unity," and "duration," which "are to be ascribed indifferently, now to corporeal things, now to spirits" or intellectual things (AT 10:419, CSM 1:45). The common simple natures can be known "either by the pure intellect or by the intellect as it intuits the images of material things" (AT 10:419–20, CSM 1:45). The common simple natures also include common notions, "which are, as it were, links which connect other simple natures together, and whose self-evidence is the basis for all the rational inferences we make," such as "things that are the same as a third thing are the same as each other" (ibid.). Descartes also includes "negations and privations" in his enumeration$_3$ of the simple natures. These include "nothing," "instant," "rest," and other such natures, which I will discuss in more detail in subsequent sections.

What are the criteria whereby Descartes enumerates$_3$ the simple natures? Descartes invokes three interrelated criteria: cognitive indivisibility, self-evidence, and univocity. As I will show below, cognitive indivisibility is the central criterion, and it entails or constitutes both self-evidence and univocity. In order to get a handle on these issues, it is necessary to review and discuss in more detail what Descartes means by "simple" and "complex" in *Rules*. As I argued in Chapter 2, Section 2.4, Descartes's theory of simple natures is based on a strategy of "extrinsic denomination." The "simplicity" of a simple nature is defined, not relative to how it is in reality (*a parte rei*), but rather "extrinsically," relative to how it is to the

human intellect (*respectu vero intellectus nostri; quantum ab intellectu percipiuntur*). The following passage is so central that it must be cited in full:

> [W]hen we consider things in the order that corresponds to our knowledge of them [*spectandas esse res singulas in ordine ad cognitionem nostram*], our view of them must be different from what it would be if we were speaking of them in accordance with how they exist in reality [*prout revera existunt*]. If, for example, we consider some body which has extension and shape, we shall indeed admit that, with respect to the thing itself, it is one single and simple entity [*a parte rei, esse quid unum et simplex*]. For, viewed in that way, it cannot be said to be a composite made up of corporeal nature, extension and shape, since these constituents have never existed in isolation from one another. Yet with respect to our intellect [*respectu vero intellectus nostri*] we call it a composite made up of these three natures, because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject. That is why, since we are concerned here with things only insofar as they are perceived by the intellect [*rebus… quantum ab intellectu percipiuntur*], we term "simple" only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known. Shape, extension and motion, etc. are of this sort; all the rest we conceive to be in a sense composed out of these (AT 10:418, CSM 1:44).

Here, Descartes effectively replaces the Latinized Aristotelian formula *ens in quantum ens* with his own *ens in quantum ab intellectu percipitur*, where "perceive" is understood to refer exclusively to intuition. Relative to the intellect, he argues, material things are complex because the intuition of material things is composed of more than one simple nature. These natures are simple because they are cognitively indivisible, or known "so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known" (I will come back to this below). Regarded as notions, the simple natures of extension, shape, and motion are not proper parts of bodies, but rather *distinguishable parts of the intuition of a body in motion*. These intellectual or *intuitional* parts are not *real* parts. The mereology here is clearly not ontological, but rather epistemic. Ontologically, a material thing "cannot be said to be a composite made up of corporeal nature, extension, and shape, since these constituents have never existed in isolation from each other." To be sure, all bodies are composite, since all bodies are divisible into parts, but according to Aristotle, individual material things are substances, and all substances are qua substances a "unit" (*Categories* 5, 3^{a}10–12).[2] One and the same material thing is both a hylomorphic or *substantial* unit and yet *divisible* into parts in virtue of its *quantity*. It is one in substance but many in

quantity. The taxonomy of distinctions between simple/complex relative to the intellect and in reality is provided in Table 7.1.

Table 7.1 Simplicity and complexity in Descartes's theory of simple natures

	Relative to the intellect (*respectu intellectus nostri*)	**In reality (*a parte rei*)**
Simple	Cognitively indivisible	Ontologically indivisible (i.e., a substance)
Complex	Cognitively divisible into simple natures	Physically divisible into parts

As I indicated above, a notion is simple, not if it cannot be divided by the mind into yet other notions, but rather only if it cannot be divided into notions *which are more distinctly known than it*. This is what the criterion of "cognitive indivisibility" means. To divide a notion is to *define* it according to notions ostensibly more distinctly known and, therefore, more basic than it. When this cannot be done, one has reached a simple nature. For example, as I argued in Chapter 3, Section 3.2, extension can be exhibited in spatial intuition (in the imagination), where I see a body extended in length, width, and breadth. As a simple nature, extension is supposed to be notionally basic or primitive. How can this be detected? First, because not only are all bodies extended, but no corporeal property (e.g., motion or shape) can exclude extension, while extension can be intuited entirely on its own. Second, because when the intellect divides extension into other notions, these notions are mere abstractions that turn out to *presuppose* extension. Extension cannot be divided into length, width, and breadth, since the length of a body cannot be imagined unless it is extended in space, and the width and depth of a body also cannot be imagined unless they too are extended in space. Consequently, extension is not "composed" of these notions, and so it cannot be defined by them. Indeed, because the simple natures are notionally indivisible, they cannot be defined at all. They can only be *intuited*.

The reason why the definition of a simple nature undermines the distinctness of the intuition is that definition presupposes the possibility of a *definens* more basic than the *definiendum*, but simple natures such as extension are *notionally basic* in intuition. *Cognitive indivisibility* entails *notional basicness*. Simple natures cannot be defined because they are

notionally prior to any and all *definiens*. *Notional basicness* entails *notional priority over any and all* definens. This is why simple natures can *only* be *directly exhibited* in intuition. Recourse to intuition is required because *discursive apprehension* is *in no way* appropriate to the simple natures. Discursive apprehension is excluded by the cognitive indivisibility of the simple natures and the notional basicness and priority it entails. To take another example, Aristotle defines motion as "the actuality of a potential being, insofar as it is potential."[3] With definitions like these, Descartes writes that the learned "give the impression of uttering magic words which have a hidden meaning beyond the grasp of the human mind" (AT 10:426, CSM 1:49).[4] The definition further divides the simple nature, but the result is a proposition that is less distinct than the direct exhibition of motion in spatial intuition by the intellect aided by the imagination. Descartes recommends exhibiting the local motion of a body in spatial intuition without the intermediary of definitions. The intellect aided by the imagination "sees" *motion itself* in a way that no definition could possibly compete with. "[We] should never explain [simple natures] by definitions," Descartes writes, "in case we take hold of composite things instead of simple ones" (AT 10:426–7, CSM 1:49).[5] "Each of us," he continues, "according to the light of his own mind [*pro lumine ingenij sui esse intuendas*], must attentively intuit only those things which are distinguished from all others" (AT 10:427, CSM 1:49). Descartes's habitual theory of the faculties yields results far superior to the ancient art of definition, because it yields the things themselves.[6]

Why insist so strongly on cognitive indivisibility when a much more obvious criterion whereby the class of simple natures may be enumerated$_3$ is available: self-evidence?[7] Because self-evidence, while important, is *derivative*: the simple natures are self-evident *because* they are cognitively indivisible and, therefore, notionally basic. When Descartes insists that the "simple natures are all self-evident and never contain any falsity," he explains that this is so because "if we have the slightest grasp of it in our mind […] it must follow that we have complete knowledge of it. Otherwise it could not be said to be simple, but a composite made up of that which we perceive in it and that of which we judge we are ignorant" (AT 10:420–1, CSM 1:45). As this passage makes clear, the self-evidence of the simple natures stems directly from their simplicity (cognitive indivisibility).

Incomplete knowledge is knowledge of one part of a thing, but not another. Simple natures *have no parts*. Their simplicity therefore excludes *the possibility of incomplete knowledge* a priori, making them *wholly self-evident*. Thus, if the intellect has *any* knowledge of a simple nature, then it *ipso facto* has *complete knowledge* of it. This, it seems to me, helps explain why Descartes only introduces the criterion of self-evidence *after* he introduces the criterion of cognitive indivisibility and *after* he enumerates$_3$ the three subclasses of simple nature (AT 10:420, CSM 1:45), i.e., in the *conclusion* of the theory. Self-evidence and simplicity are co-extensional, but simplicity consists in indivisibility, upon which the possibility of self-evidence depends. Self-evidence, by contrast, only *indicates* simplicity or cognitive indivisibility.

Cognitive indivisibility is not the only criterion that determines whether a notion is a simple nature. The second criterion is *univocity*. Above, I mentioned that simple natures are not terms because terms are universals and universals have definitions and are abstracted from other things. I should add that *anything abstracted from a simple nature is not a simple nature*, and is neither cognitively indivisible nor univocal. For example, the term "limit" can be abstracted from the material simple nature of shape, the material simple nature of motion, and the common simple nature of duration; any term that can be applied to many different natures "must have been abstracted from these as well" (AT 10:419, CSM 1:44). The logical extension of the term includes all three simple natures. It is, therefore, "compounded of many quite different natures" (ibid.), and so is necessarily *composite*. Abstract terms or universals are *divisible into the natures from which they have been abstracted*. Consequently, *these natures must be known more distinctly than they are*. "Limit" cannot be intuited by itself; a limit is always the limit of a shape, a motion, or a duration, which can be intuited by the intellect aided by the imagination. Furthermore, the sense of the term "limit" is clearly not identical in each of these cases, since the relevant natures included in its logical extension are different from one another; the limit of a duration is not the same as the limit of a shape or a motion. Terms violate the criterion of cognitive indivisibility, and this is why they also violate the criterion of univocity. Failure to satisfy these two criteria violates the conditions of membership in any class of simple natures.[8] Consider the contrasting case of the material simple nature of shape. Different bodies have different shapes, but the sense in which one

body has shape is no different than the sense in which any other body has shape. Simple natures are univocal. *Cognitive indivisibility entails univocity, and univocity indicates indivisibility*. In the case of terms or universals, by contrast, composition entails equivocation, and equivocation indicates composition. Thus, while abstract terms or universals such as "limit" are *more general* than simple natures, they are not *simpler*. On the contrary, they are more *complex*, and so more *obscure*. Generality and simplicity are radically distinct from one another.

Thus far, I have only discussed the enumerative criteria that determine whether any one notion is or is not a simple nature. What remains to be seen is how the class of simple natures can be further divided into separate subclasses: the intellectual, material, and common simple natures. In Section 7.3–7.6, I show how Descartes draws on his theory of the faculties discussed in Chapter 6 in order to divide the class of simple natures into separate subclasses, and I discuss each subclass separately: the intellectual simple natures (Section 7.3); the material simple natures (Section 7.4); the common simple natures and their corresponding privations and negations (Sections 7.5–7.6).

7.3 The Intellectual Simple Natures and the Use of the Pure Intellect

According to Descartes's habitual theory of the faculties, the intellectual simple natures cannot be intuited unless the pure intellect intuits them entirely on its own, without relying on the imagination and the senses: "[When] the intellect is concerned with matters in which there is nothing corporeal or similar to the corporeal, it cannot receive any help" from these two faculties (AT 10:416, CSM 1:43). The intellect cannot understand incorporeal things by turning toward images in the imagination. It is the impossibility of forming any corporeal idea of knowledge, doubt, ignorance, and volition, together with the fact these notions cannot be further divided into yet other, more distinct notions (cognitive indivisibility), that places them squarely in the subclass of intellectual simple natures:

> Those simple natures which the intellect recognizes by means of a sort of innate light, without the aid of any corporeal image, are purely intellectual. That there is a number of such things is certain: it is impossible to form any corporeal idea which represents for us [*idea corporea quae nobis repraesentet*] what knowledge or doubt or ignorance is, or the action of the will, which may be called "volition," and the like; and yet we have real knowledge of all of these, knowledge so easy that in order to possess it all we need is some degree of rationality (AT 10:419, CSM 1:44–45).

Each one of these acts can be directly exhibited in intuition simply by performing the relevant act: "[I]n order to know what doubt and thought are, all one need do is to doubt or to think. That tells us all it is possible to know about them, and explains more about them than even the most precise definitions" (*Search after Truth*, AT 10:524, CSM 2:418). To doubt, know that one doubts, and know what doubt is are one and the same; one cannot know that one doubts without knowing what doubt is.[9] The class of intellectual simple natures includes at least as many members as there are operations of the pure intellect (and in general any object whose intuition depends on the intellect alone). As acts of *vis cognoscens*, they are one and all its modes (Descartes would later refer to them as modes of thought). Descartes's enumeration$_3$ of the intellectual simple natures is not *complete*, but rather *sufficient*; any act of the pure intellect can be included in the class of intellectual simple natures. There is no need to enumerate$_3$ them all. The "*et similia*" and "*etc.*" appended to the end of his enumerations$_3$ should be read an open invitation to extend the class as needed.[10]

7.4 The Material Simple Natures and the Use of the Intellect and the Imagination

According to Descartes's habitual theory of the faculties, the intellect must intuit the material simple natures of extension, shape, and motion with the aid of the imagination. This leads Descartes to redefine the object of mathematics in Rule 14 as extension, and to the reduction of the Aristotelian category of quantity to extension alone. Indeed, from Descartes's point of view in *Rules*, the Aristotelian category of quantity arises from a failure to employ the cognitive faculties correctly. Descartes warns that when the intellect conceives extension without the aid of the imagination, it erroneously distinguishes between extension per se (void

space) and extended things; discrete and continuous quantity; point, line, surface, and solid; and extension and quantity:

> For although someone may convince himself that it is not self-contradictory for extension *per se* to exist all on its own even if everything extended in the universe were annihilated, he would not be employing a corporeal idea in conceiving this, but merely an incorrect judgment of the intellect alone. He will admit this to himself if he carefully reflects on the image of extension which he tries to form in his imagination. He will realize that he does not perceive it in isolation from every subject, and that his imagination of it is quite different from his judgment about it. Consequently, whatever our intellect believes about the truth of the matter, these abstract entities are never formed in the imagination in isolation from their subjects (AT 10:442–443, CSM 1:59).
>
> Thus, when the problem concerns number, we imagine some [extended] subject which is measurable in terms of a set of units. The intellect of course may for the moment confine its attention to this set; nevertheless we must see to it that, in doing so, it does not draw a conclusion which implies that the thing numbered has been excluded from our conception. Those who attribute wonderful and mysterious properties to numbers do just that. They would surely not believe so firmly in such sheer nonsense, if they did not think that number is something distinct from things numbered (AT 10:445–6, CSM 1:61).
>
> Likewise, when we are concerned with a figure, we should bear in mind that we are dealing with an extended subject, conceived simply with respect to its having a shape. When we are concerned with a body, we should bear in mind that it is the same thing we are dealing with, in that it is something which has length, breadth, and depth. In the case of a surface, we should conceive of the same thing, as being something with length and breadth – this time leaving out depth, but not denying it. In the case of a line, let us think of it as having just length; and in the case of a point, the same will apply, though this time we should leave out every other property save its being an entity (AT 10:446, CSM 1:61).
>
> In this context, then, we are concerned with an extended object, thinking of it exclusively in terms of its extension, and deliberately refraining from using the term "quantity"; for there are some philosophers so subtle that they have even distinguished quantity from extension (AT 10:447, CSM 1:62).

Descartes effectively rejects the Aristotelian category of quantity (conceived as a real accident) in its entirety in Rule 14. The definition of extension as "whatever has length, width, and breadth" (AT 10:442, CSM 1:59) is perfectly dispensable: once what the term denotes has been placed directly before the mind in spatial intuition, there is no need to debate whether extension per se and extended things are distinguishable. Intuition reveals that there is no distinction between extension and body. The intellect, aided by the imagination, sees an extended body in intuition, and *restricts its judgments about extension to what is revealed in intuition alone*. The definition of extension, by contrast, does not enable one to decide whether extension is a real body or a void space. On the contrary, it invites purely speculative, dialectical disputations that do not rest on the evidence

uniquely provided by intuition. Descartes's "intuitionism" in *Rules* is such that the *definition* of extension is replaced by the "*real idea of the thing*" (my emphasis), i.e., the spatial intuition of an extended body. Once the operator of the method has learned how to employ their cognitive faculties correctly, they can correct erroneous judgments about the simple natures by consistently referring them to the tribunal of intuition. Similarly, once the imagination anchors the intuition of extension by the intellect, the Aristotelian category of quantity is eliminated as equivocal between discrete and continuous quantity, so that discrete quantity (number) can no longer be regarded as "something distinct from things numbered" (AT 10:446, CSM 1:61). On the contrary, "when the problem concerns number, we imagine some [extended] subject which is measurable in terms of a set of units" (AT 10:445, CSM 1:61). Number measures extension and *has no being apart from extension*. (This explains why "extension" replaces "quantity" as the object of mathematics in Descartes.) Spatial intuition also reveals that the point, line, surface, and solid are only "nominally distinct" (AT 10:449, CSM 1:63). For mathematical purposes, one may, of course, focus exclusively on the surface or the solid in order to solve a definite problem, but these are abstractions, not real denials or exclusions.[11] In the imagination, all of these dimensions are given together in one extended subject, and one isolates one or more of them without denying that they belong to the extended subject from which they have been abstracted.

In Descartes's habitual theory of the faculties, the imagination functions as the arbiter of all propositions about quantity, and any proposition not certified by the imagination is simply excluded.[12] The proper use of the imagination enables the operator of the method to avoid the false judgments of the intellect when it conceives objects it has no business conceiving on its own. It is only when the imagination is disregarded that the intellect further divides simple natures into notions less distinct than they themselves are. The proper employment of the cognitive faculties—here, the intellect aided by the imagination—yields both the rejection of the Aristotelian category of quantity and a redefinition of the object of mathematics. As we will see in more detail in Chapter 9, the intellect aided by the imagination is the foundation of Descartes's reform of mathematics in Rules 15–21.[13]

7.5 Epistemic Transcendentals: The Common Simple Natures

The common simple natures "can be known either by the pure intellect or by the intellect as it intuits the images of material things" (AT 10:419–420, CSM 1:45). This is because the common simple natures are "those which are ascribed indifferently, now to corporeal things, now to spirits – for instance, existence, unity, duration and the like" (AT 10:419, CSM 1:45). They also include "those common notions which are, as it were, links which connect other simple natures together, and whose self-evidence is the basis for all the rational inferences we make," such as "'things that are the same as a third thing are the same as each other'" and "'things that cannot be related in the same way to a third thing are different in some respect'" (AT 10:45, CSM 1:419). We already know that the criteria Descartes employs to enumerate$_3$ the simple natures are cognitive indivisibility and univocity. We also know that the satisfaction of these criteria, while necessary, is insufficient to determine which subclass of simple natures any one simple nature belongs to. To determine which subclass a simple nature belongs to, we also need to know whether it should be conceived by the pure intellect alone, the intellect aided by the imagination, or both. When a simple nature can be intuited by both the intellect alone and the intellect aided by the imagination, then it enjoys a type of universality that the simple natures in the other two subclasses—the intellectual and material simple natures—do not. This universality is epistemic in nature; common simple natures like "existence" are part of the intuition of anything whatever. Just as, with respect to our intellect, we call a body "a composite made up of three natures, because we understood them separately before [*prius*] we were in a position to judge that the three of them are encountered at the same time in one and the same subject," so too we call anything whatever a composite made up, not only of either material or intellectual simple natures, but also of common simple natures. All things are composed of either intellectual and common simple natures or material and common simple natures. Thus, the common simple natures (excluding the common notions for the moment) are genuine "epistemic transcendentals," "epistemic" because the sense in which they "transcend" the intellectual and material simple natures consists in their *intuitional universality*.

That being said, there are a number of problems the common simple natures raise in Descartes's theory of simple natures in *Rules*—problems

that are central but rarely, if ever, discussed in the literature. What role do the common simple natures play in any given intuition, exactly? What does Descartes mean when he asserts that existence, unity, and duration compose the content of any intuition whatever? How, precisely, do these simple natures satisfy the criteria of cognitive indivisibility and univocity so central to Descartes's theory of simple natures in *Rules*? Finally, what about the common notions? They are *propositions*, and I earlier argued that simple natures are *not* propositions. How can a *proposition* satisfy the criterion of cognitive indivisibility when it is *composed of notions*? These problems threaten to undermine Descartes's theory of simple natures and, therefore, his solution to the problem of the limits of knowledge. I address each of these problems below.

The common simple nature of "existence" is part of the intuition of anything whatever, for "existence is contained in the idea or concept of every single thing, since we cannot conceive of anything except as existing. Possible or contingent existence is contained in the concept of a limited thing, whereas necessary and perfect existence is contained in the concept of a supremely perfect being" (*Second Replies*, AT 7:166, CSM 2:117). In order to intuit any simple nature or combination of simple natures, I must already have the common simple nature of existence on hand. Existence, moreover, is necessarily conjoined to two other common simple natures: unity and duration. Like existence, "unity" is an epistemic transcendental that is part of the intuition of both the intellectual and material simple natures. Whatever I intuit, I intuit as (possibly or necessarily) existent, and I intuit it as *one*, either as a simple unity (a simple nature) or as a composite unity (an ordered multiplicity of simple natures in any one thing, such as a material thing).[14] Furthermore, if I conceive its existence, I also conceive its *endurance*. The common simple nature of "duration" is simply the "mode under which we conceive the thing insofar as it continues to exist" (*Principles* I. 55, AT 8A:26, CSM 1:211). Duration may be known either by my own self-reflexive contemplation (the successivity of my acts, whereby I become aware of my own continuous existence as the subject in which they inhere), or by intuiting the images of material things, be they in motion (in which case motion entails duration) or at rest (in which case the stationary, but continuous existence of any body entails duration no less).[15] As common simple natures, existence, unity, and duration may be ascribed indifferently to both intellectual and material simple natures. Furthermore,

the intellectual and material simple natures cannot be intuited independently of the common simple natures and vice versa. Expressed in terms of Descartes's later metaphysics, the common simple natures are only *rationally distinct* from the intellectual and material simple natures, and they are only rationally distinct *from one another* (I come back to this point in Section 7.7 below, where I go into more detail about the types of relations that obtain between simple natures in all three classes).

Do the common simple natures satisfy Descartes's criterion of cognitive indivisibility? They do. Just as the material simple natures are contained in the intuition of any *material* thing whatever, so too the common simple natures are contained in the intuition of *anything* whatever. They are necessary constituents of any intuition or, more broadly, any intellectual act (be it clear and distinct or obscure and confused) directed toward anything whatever. Since every intellectual act presupposes them, they are notionally basic. They cannot be defined by any prior notion more distinct than they themselves are, and since they are not abstracted from many different natures, they do not suffer from the equivocation that terms which are so abstracted suffer from. The only difference in this respect is that the common simple natures are more universal than either of the other two subclasses of simple nature.

Besides the common simple natures already discussed, Descartes also includes common notions or axioms, i.e. *propositions* "which are, as it were, links which connect other simple natures together, and whose self-evidence is the basis for all the rational inferences we make" (AT 10:419, CSM 1:45). The common simple natures are more heterogeneous than the intellectual and material simple natures, since they include both *notions* and *propositions* (axioms). This suggests that not all common simple natures are equally simple, and that some of them are more complex than others. Perhaps even more paradoxical is the fact that Descartes's examples of common notions are *mathematical* (Euclidean). These common notions seem to pertain, not to both intellectual and material simple natures, but rather only to the material simple natures. This suggests that *they are not common at all*. How can these common notions serve as "links […] whose evidence is the basis for all the rational inferences we make" when they are restricted to the material simple natures? Descartes's inclusion of these common notions in his theory of simple natures threatens to undermine both the criteria whereby he enumerates$_3$ the simple natures as well as the

possibility of inferential principles sufficiently common to function as the basis of “all the rational inferences we make,” and not only those inferences we make in mathematics.

In what sense, then, are the common notions *simple*? They are simple *relative to all other propositions*. Descartes’s inclusion of common notions in his enumeration$_3$ of the common simple natures effectively introduces a second level of simplicity: *propositional simplicity*. Just as the analysis of complex notions ultimately terminates in simple natures, so too the analysis of propositions ultimately terminates in common notions, *which are only reducible to the simple natures, not other propositions*. The common notions Descartes introduces as examples in Rule 12 are composed of the simple natures Descartes enumerates in Rule 6: equality and similarity on the one hand, and inequality and dissimilarity on the other (see AT 10:381–2, CSM 1:21). The common notion, “Things that are the same as a third thing are the same as each other” (the transitivity of equality) mobilizes the simple natures of equality and similarity, while the common notion, “Things that cannot be related in the same way to a third thing are different in some respect,” mobilizes the relative simple natures of inequality and dissimilarity (“relative” because equality is notionally prior to inequality; see AT 10:381–2, CSM 1:21). Thus, the common notions are propositionally (not notionally) indivisible and, therefore, univocal, since cognitive indivisibility entails univocity.

The more difficult question is what makes these common notions *common*. Unlike unity or duration, Descartes’s examples of common notions seem to be restricted to magnitudes and, therefore, to the material simple natures alone. As Descartes puts it in Rule 14: “Nothing can be reduced to […] an equality except what admits of differences of degree, and everything covered by the term ‘magnitude’” (AT 10:440, CSM 1:58). Whereas Aristotle and medieval Aristotelians identified principles such as the principle of non-contradiction, the principle of identity, and the principle of the excluded middle as common notions, these principles belong to a syllogistics that Descartes rejects as scientifically unserviceable in *Rules*. This explains why Descartes does not include such principles in his enumeration$_3$ of the common notions in Rule 12. In Rule 14, where he introduces logical generalizations of the Euclidean common notions enumerated$_3$ in Rule 12, he reminds the reader that these logical

generalizations have no place in his method. He briefly discusses the principle, “If A is B, and B is C, then A is C,” a logical generalization of the first Euclidean common notion enumerated in Rule 12 (the transitivity of equality), and rejects it. Some have argued that Descartes’s negative assessment of the scientific utility of Aristotelian logic constrains him to introduce Euclidean common notions as examples in Rule 12,[16] but this does not answer the question of why Descartes does not simply introduce properly universal common notions like “what is done cannot be undone,” as he does in *Principles* I. 49 (AT 8A:24, CSM 1:209). The best (although by no means ideal) solution here seems to be to argue that *Descartes’s examples are intended as analogues, within mathematics, of common notions that are properly universal*. Just as Euclidean common notions are the basis of rational inference *in mathematics*, so too the common notions are the basis of rational inference *in any science whatever*. The problem with this “solution” is that it does not really solve the problem. Why didn’t Descartes save himself the trouble and simply introduce properly universal common notions instead of Euclidean common notions? The best explanation I can come up with is that Descartes hadn’t yet made any firm decision on the status of metaphysical principles in *Rules*, and was not yet sure whether to include them or regard them as properly scientific. The more important point, however, is that there is nothing in principle that prohibits regarding principles such as “what is done cannot be undone” (and, indeed, the entire class of eternal truths) as common notions (*Principles* I. 49, AT 8A:24, CSM 1:209).

7.6 Negations, Privations, and the Compositionality of Thought

After enumerating$_3$ the common simple natures, Descartes writes:

> It is as well to count among the simple natures the corresponding negations and privations. For when I intuit what nothing is, or an instant, or rest, my apprehension is as much genuine knowledge as my understanding of what existence is, or duration, or motion. This way of conceiving things will be helpful later on in enabling us to say that all the rest of what we know is put together out of these simple natures. Thus, if I judge that a certain shape is not moving, I shall say that my thought is in some way composed of shape and rest; and similarly in other cases (AT 10:420, CSM 1:45).

Why does Descartes include negations and privations in his enumeration$_3$ of the simple natures, and what concrete role do they play in his solution to the problem of the limits of knowledge in Rule 12? Descartes includes negations and privations in his enumeration$_3$ of simple natures in order to consolidate his thesis that human knowledge is composed *exclusively* of simple natures, each of which may be intuited either by the pure intellect or the intellect aided by the imagination. Had Descartes excluded negations and privations from his enumeration of the simple natures, there would be propositions (e.g., "This body is at rest") that have *contents that could not be intuited by the intellect*. There would be an "intuitional hole" in every proposition that contains negations and/or privations. The intuition requirement imposes the necessity that negations and privations have natures that can be intuited no less than the corresponding positive simple natures. Only in this way can every propositional component—positive and negative—become an object of intuition and, therefore, achieve *givenness* or complete evidence in intuition. Thus, by including negations and privations, Descartes effectively ensures that no propositions expressing sanctioned relations between simple natures, whatever their content, remain opaque to intuition. Descartes's decision to include privations and negations therefore plays an indispensable role in his solution to the problem of the limits of knowledge. He himself says as much when he writes that their inclusion "will be helpful later on in saying that all the rest of what we know is put together out of these simple natures" (ibid.).

That being said, the negations and privations Descartes enumerates$_3$ in Rule 12 raise special problems of their own, first and foremost the fact that negations and privations are *not* identical, and failing to distinguish between them can be disastrous. For Aristotle, a privation is the *lack* of something that a thing ought to have according to its nature (e.g., a human being that cannot see), while a negation is the mere absence of something that a thing either happens not to have (e.g., a human being who is not pale; a body that is not in motion) or cannot have according to its nature anyway (e.g., a tree that cannot see). Aristotle frequently employs the term "privation" (*steresis*) to refer to either of these.[17] For Descartes, the privative or negative simple natures are *not* the lack or absence of something else. Indeed, neither nothingness, nor instant, nor rest are privations. They are one and all negations. And yet they are *natures that can be intuited in their own right.*

Descartes does not regard rest as a privation of motion, but rather as a *positive nature in its own right* no less real than motion is (he regards the relation between duration and instant in the same way). For Aristotelians, rest is not a positive quality or mode of a body, but rather simply the *privation* of motion.[18] The privation, moreover, is usually regarded as the natural *terminus ad quem* of motion; once the relevant substantial or accidental form has been fully actualized in the relevant substance, the motion naturally comes to an end and *ceases of its own accord*. For Descartes, Aristotelian physics fails to distinguish between what are in fact two completely distinct and *equally positive simple natures of motion and rest*. A body in motion will remain in motion unless some other body arrests it, and a body at rest will remain at rest unless some other body moves it. The intuition of motion in no way contains rest and vice versa. Aristotelian physics erroneously conjoins these two natures: "[The] motion of which they speak has a very strange nature; for whereas all other things have their perfection as an end and strive only to preserve themselves, it has no other end and no other goal than rest and, contrary to all the laws of nature, it strives of its own accord to destroy itself" (AT 11:40, CSM 1:94). Thus, while Descartes's characterization of "rest" as a negation or privation seems to endorse the Aristotelian thesis that the being of motion is "much more solid and real" than that of rest, "which they [the philosophers] say is nothing but the privation of motion," in fact Descartes regards rest "as a quality too, which should be attributed to matter while it remains in one place," for "no more action is needed for motion than for rest" (*The World*, AT 11:40, CSM 1:94; *Principles* II. 26, AT 8A:54, CSM 1:234).[19]

Nor does Descartes regard "nothingness" or "non-being" (*nihil*) as a privation; as "that which is farthest removed from all perfection" (*Meditations* IV, AT 7:54, CSM 2:38), nothingness is not a privation, since there is no perfection it *should* have. Privation consists, not only in not having some perfection(s), but rather in not having those perfection(s) that *should* be had (AT 7:55, CSM 2:38). And yet, as a simple nature, "nothing" must also in some sense be a nature in its own right. But what does one intuit when one intuits "nothing"? In *Conversation with Burman*, Descartes relates that the idea of nothing "is purely negative, and can hardly be called an idea," and he adds when the word "idea" is employed here, it is "a rather extended use of the word 'idea'" (AT 5:153, CSM 1:338). Certainly, the idea of nothing is not an "'idea' in its strict and narrow sense" (ibid.), i.e.,

"thoughts [that] are as it were [*tanquam*] the images of things" (*Meditations* III, AT 7:37, CSM 2:25). What, then, does the intuition of "nothing" consist in? As a simple nature, it must be intuitable, and yet it seems to have no intuitable content. To represent "nothing" is certainly not to represent it as *beyond* representation, since it is not an ineffable "something" beyond the finite representational capacities of the human mind. It is an *empty* representation, i.e., a noetic act that directs the mind to no determinate being. This empty representation is the foundation of the common notion "nothingness possesses no attributes, that is to say, no properties or qualities" (*Principles* I. 52, AT 8A:25, CSM 1:210). Thus, "if we perceive the presence of some attribute, […] there must also be present an existing thing or substance to which it may be attributed" (AT 8A:25, CSM 1:210). The inference from the existence of a property to the existence of a thing or a substance (finite or infinite) that has the property depends on a common notion based entirely on the simple nature "nothing." Without the common notion "nothingness possesses no properties," there would be no basis for this inference, and so no place for substances in Descartes's metaphysics.

Indeed, nothingness plays a special role in the case of the intuition of *finite beings* because they *do not have any number of perfections*. The common simple nature of existence, far from excluding nothingness, includes it in finite existence, to such an extent that existence and nothingness are necessarily conjoined (but not confused) in the intuition of any finite being. This is one of the lessons of *Meditations* III, where the relation between existence and nothingness becomes clear in the paradigmatic case of my own existence, which I cannot intuit without employing the "negative idea of nothingness." When I reflect on my errors, "I realize that I am, as it were, something intermediate between God and nothingness, or between supreme being and non-being […]." "I participate in nothingness or non-being, that is, insofar as I am not myself the supreme being and am lacking in countless respects" (AT 7:54, CSM 2:38). The "negative idea of nothingness" *is* the simple nature "nothing," and by intuiting this simple nature, I immediately recognize that "nothing" and "existence" are co-constitutive of my own existence and of any finite being whatever. Thus, the relation between "nothing" and "existence" is not the same as the relation between motion and rest. In the latter case, the two simple natures are mutually exclusive, whereas in the former case, they are not.

The simple nature "nothing" also plays a central role in common notions about *causality*. In *Principles* I. 49, "nothing" constitutes the common notion "nothing comes from nothing [*ex nihilo nihil fit*]" (AT 8A:23, CSM 1:209).[20] In Rule 6, Descartes explicitly includes "cause" and "effect" among the simple natures, and he argues that "cause" is epistemically "absolute" relative to "effect," since "if we want to know what the effect is, we must know the cause first, and not vice versa" (AT 10:383, CSM 1:22). Indeed, the effect(s) is (are) epistemically relative to the cause(s), not only because *the cause(s) must be known first*, but also because *the cause(s) must yield knowledge of everything in the effect*. There can be nothing in the effect that cannot be deduced from the cause(s). Otherwise, no deduction of "effects from causes" is possible (AT 10:433, CSM 1:53). Were there something in the effect that could not be deduced from the cause(s), then the common notion "nothing comes from nothing" would be false; there would be something in the effect that came from nothing in the cause. The common notion "nothing comes from nothing" ensures *epistemic and causal closure* in natural philosophy and metaphysics.[21]

7.7 The Theory of Conjunction: Descartes's First Theory of Distinctions

As we have seen, Descartes's theory of simple natures in Rule 12 depends on his habitual theory of the faculties. The pure intellect yields intuitions of the intellectual simple natures; the intellect aided by the imagination yields intuitions of the material simple natures; and either of these faculty configurations yields intuitions of the common simple natures. The two parts of the problem of the limits of knowledge—the faculties of knowledge and the objects of knowledge—are so intimately connected that the solution to the second part depends entirely on the solution to the first part. The solution to the second part, however, remains incomplete. Descartes has enumerated$_3$ the simple natures according to definite criteria, and he has divided them into subclasses by exploiting the faculty configurations required by his habitual theory of the faculties, but he has not yet described how the simple natures may be combined with one another in non-arbitrary compositions certified by intuition. Simple natures have different relations

to one another, and these relations must be codified because in science solving problems consists in combining simple natures. In Rule 12, Descartes distinguishes between two types of combination or “conjunction”: necessary and contingent. He writes:

> The conjunction [*conjunctionem*] between simple things [*rerum simplicium*] is either necessary or contingent. The conjunction is necessary when one of them is somehow implied (albeit confusedly) in the concept of the other [*una in alterius conceptu confusa quadam ratione ita implicatur*] so that we cannot conceive either of them distinctly if we judge them to be separate from each other [*non possimus alterutram distincte concipere, si ab invicem sejunctas esse judicemus*]. It is in this way that shape is conjoined with extension, motion with duration or time, etc., because we cannot conceive of a shape which is completely lacking in extension, or a motion wholly lacking in duration. […] The union [*unio*] between such things, however, is contingent when the relation conjoining them is not an inseparable one [*nulla inseparabili relatione*] (AT 10:421, CSM 1:45–6).

If one separates two simple natures from one another in a judgment, and finds that neither can be intuited as so separated, then the relevant simple natures are necessarily conjoined. One or more of them is “somehow implied (albeit confusedly) in the concept of the other” (I will explain what this means shortly). If, by contrast, one separates two simple natures from one another in a judgment, and finds that they can be intuited as so separated, then the relevant simple natures are either contingently conjoined or not conjoined at all.

Clear examples of contingent conjunction include “The body is animate” or “The man is dressed” (AT 10:421, CSM 1:46). I can negate these propositions, since I can easily conceive an inanimate body or a naked man; I am in no way constrained to conceive all bodies as animate or all men as clothed. Conversely, I cannot both negate the proposition “This shape is extended” and intuit shape; any shape I imagine in spatial intuition must have some extension, and so the proposition “This shape is not extended” logically entails the proposition “This shape is not a shape,” which is a formal contradiction. For the same reasons, I cannot negate the proposition “This motion has duration” without contradiction; any motion I imagine in spatial intuition must have some duration. In the latter two cases, then, the conjunction between shape and extension, on the one hand, and motion and duration, on the other, is necessary.

As these examples illustrate, *negative judgments* play an important role in Descartes’s theory of conjunction in *Rules*. Whether any negative judgment about the conjunction of two or more simple natures is true or

false (contradictory) can only be determined if I employ my cognitive faculties correctly. *Formal principles such as the principle of non-contradiction can only play a role under the appropriate gnoseological conditions.* Discovering the logical contradiction in the proposition "Extension is not a body" depends on whether I have employed the imagination in addition to the intellect in the intuition of extension; it is only when I imagine extension in spatial intuition that I intuit it *as* a body, and so place my intellect in a position to see that the proposition "Extension is not a body" is equivalent to the proposition "Extension is not extension." Otherwise, the formal contradiction in the proposition "Extension is not a body" remains concealed. From a purely logical point of view, the proposition "Extension is not a body" is *not* a formal contradiction, and on some definitions of extension as void space—definitions based on the intellect unaided by the imagination—the proposition is true (see Sections 7.2–7.3 above). For Descartes, determining whether a proposition contains a formal contradiction depends on how its *matter* is understood, and the only way to intuit its matter is by employing one's cognitive faculties correctly. In *Rules*, there can be no separation between the *logical* and the *gnoseological*. Only the proper employment of the cognitive faculties enables one to establish relations of contingent and necessary conjunction between simple natures in intuition.

As we have seen, Descartes's theory of conjunction expresses relations of *epistemic dependency* between simple natures in intuition. The precise nature of the dependency that obtains in these cases, however, is not initially clear. The examples of shape and extension, on the one hand, and motion and duration, on the other, are examples of necessary conjunctions between simple natures, but Descartes's definition of necessary conjunctions as conjunctions in which *neither* simple nature can be intuited *separately from the other* suggests that extension and shape are *mutually inseparable*, in which case they are only *rationally*, not *modally* distinct, according to Descartes's definition of these distinctions in *Principles* I.61–2 (AT 8A:28–31, CSM 1:212–15). In both *Meditations* VI and *Principles* I. 61, Descartes affirms that extension and shape are modally distinct: extension can be separated from shape in intuition, but not vice versa ("one-way separability"). In *Rules*, by contrast, Descartes seems to affirm that extension and shape cannot be separated from one another in intuition, in which case they are only rationally distinct ("two-way inseparability"). This

is further suggested by the fact that Descartes seems to assimilate the distinction that obtains between extension and shape to the distinction that obtains between motion and duration. In *Principles* I. 56 and 62, the distinction between motion and duration is a *rational distinction:* duration and motion cannot be separated from one another in intuition ("two-way inseparability"). A second complication arises from the fact that Descartes seems to regard the relation between extension and body as a relation of inherence in Rule 14, where he writes that extension is an entity that "exists only in something else, and which can never be conceived apart from a subject [*in alio tantum sunt, nec unquam sine subjecto concipi possunt*]" (AT 10:444, CSM 1:60). This suggests that Descartes regarded extension as a *mode* of body, and not as the rationally distinct principal attribute or essence of body, as he later would in *Principles*. Attributes do not *inhere* in substances; they *constitute their essence*.

Putting these two problems together, it seems that in Rule 12, Descartes draws a rational distinction between extension and shape where he would later draw a modal distinction, while in Rule 14 he seems to draw a modal distinction between extension and body where he would later draw only a rational distinction. It seems that Descartes's theory of conjunction in *Rules* cannot be mapped onto his later theory of distinctions. Indeed, it seems that Descartes *must* have been confused about these distinctions in *Rules*, since he was confused about them well beyond *Rules*. In *First Replies*, he describes the distinction between shape and extension and the distinction between God's justice and God's mercy as modal distinctions, whereas only the former is a modal distinction. The distinction between God's justice and God's mercy is a rational distinction.[22]

Descartes's definition and examples of necessary conjunction in Rule 12 are indeed promiscuous between the modal and rational distinctions as these distinctions are defined in *Principles*. Nevertheless, if one focuses on the examples themselves, and provisionally sets aside the definition of necessary conjunction, it is clear that the necessary conjunction between extension and shape in Rule 12 is, in fact, a case of what Descartes would later call a modal distinction, and that his distinction between body and extension in Rule 14 is, in fact, a case of what he would later call a rational distinction, so that extension does indeed have epistemic priority over the other material simple natures.This is clear from the following considerations. First, regarding the relation between extension and motion,

there is no sense in which the material simple nature of extension epistemically depends on that of motion. Clearly, not all extended objects are in motion, nor do they have to be imagined as such in spatial intuition. The imagination does not forbid the intellect from intuiting the epistemic priority of extension over motion. On the contrary, it confirms this priority. Second, in the case of extension and shape, Descartes only asserts the epistemic dependency of shape on extension, not vice versa: "[We] cannot conceive of a shape which is completely lacking in extension" (AT 10:421, CSM 1:46). He does not add that we cannot conceive an extension which is completely lacking in shape, neither here nor anywhere else in *Rules*. Later in Rule 12, he similarly asserts that the necessary conjunction between extension and shape consists in the fact that "nothing which lacks extension can have a shape" (AT 10:425, CSM 1:48), but he does not assert the converse. And in Rule 14, Descartes writes: "When we are concerned with a body, we should bear in mind that it is" an "extended subject," i.e., "something which has length, breadth, and depth," but he does not add that it is something that necessarily has shape. This means that extension can be intuited independently of shape. Finally, regarding the relation between extension and body, Descartes insists in Rule 14 that "we do not form two distinct ideas in our imagination, one of extension, the other of body, but rather just the single idea of extended body" (AT 10:444, CSM 1:60). This clearly distinguishes the relation between extension and body from the relation between extension and motion. In the case of extension and motion, I *do* form two distinct ideas in the imagination, one of extension, the other of motion, and I can do so simply by imagining an extended body that is not in motion.[23] All of these considerations indicate that Descartes did indeed distinguish between the real, modal, and rational distinctions in *Rules*, but that he did not fully reflect these differences in his definition of necessary conjunction in Rule 12 (see Table 7.2).[24]

Table 7.2 Taxonomy of Descartes's theory of distinctions in Rules 12 and 14

Simple natures	**Type of distinction (*Rules*)**	**Type of distinction (*Principles*)**
Extension and body	Necessary conjunction between material simple nature and its subject (Rule 14)	Rational distinction between principal attribute and substance (*Principles* I. 62)
Extension and motion	Motion is necessarily conjoined to extension, but not vice versa (Rule 12)	Modal distinction between principal attribute and mode (*Principles* I. 61)
Extension and shape	Shape is necessarily conjoined to extension, but not vice versa (Rule 12)	Modal distinction between principal attribute and mode (*Principles* I. 61)
Motion and duration	Duration necessarily conjoined to both intellectual and material simple natures, and vice versa (Rule 12)	Rational distinction between attribute and substance (*Principles* I. 62)
Mind and body	No necessary conjunction between intellectual and material simple natures (Rule 12)	Real distinction between two substances (Meditations VI; *Principles* I. 60)
Doubt, knowledge of the difference between truth and falsity	Necessary conjunction between intellectual simple natures (Rule 12)	Modal distinction between two modes of mind (*Principles* I. 61)

As we have seen, some simple natures cannot be intuited by themselves, but are instead necessarily conjoined to (and so in some sense epistemically dependent on) others. For example, the intuition of shape depends on extension, but not vice versa. Extension has *epistemic priority* over all other material simple natures, and as such it is *absolutely simple*, while the other material simple natures are only *relatively simple*. Descartes's distinction between absolutely simple natures and relatively simple natures in Rule 6 becomes particularly salient here:

> [We] should note that there are very few pure and simple natures which we can intuit straight off and *per se* (independently of any others) […]. We should […] attend carefully to the simple natures which can be intuited in this way, *for these are the ones which in each series we term simple in the highest degree*. As for all the other natures, we can apprehend them only by deducing them from those which are simple in the highest degree […] (AT 10:383, CSM 1:22; my emphasis).

Relatively simple natures contain necessary relations to other simple natures. Absolutely simple natures contain no such relations (other than relations to the common simple natures, which themselves cannot be

intuited independently of either an intellectual simple nature or a material simple nature). It is therefore not the case that all simple natures are equally simple. Only those simple natures that can be intuited entirely by themselves are absolutely simple, while those that cannot be intuited by themselves are relatively simple, for they can only be intuited on the basis of other, simpler natures. However, this does not mean that a relative simple nature such as motion can be reduced to an absolute simple nature such as extension. Motion is a distinct simple nature, for when I imagine an extended body in spatial intuition, I do not necessarily imagine it as in motion. To intuit a body in motion is to intuit two irreducibly distinct, but necessarily conjoined simple natures. Descartes's distinction between absolute and relative simple natures reveals that the simple natures are *hierarchically ordered*, since some are epistemically prior to others. Any complex intuition has an *internal structure* that is constituted by the hierarchical relations codified in Descartes's theory of conjunction. This hierarchical order between simple natures is the foundation of Descartes's theory of distinctions in *Rules*.

7.8 Complexity and Confusion: Permissible and Impermissible Varieties

Descartes claims that in necessary conjunctions one simple nature is "somehow implied (albeit confusedly) in the concept of the other" (AT 10:421, CSM 1:45). The intuition of necessary conjunctions can in some sense be *confused*. How can this be? What distinguishes the confusion in which one simple nature is "somehow implied in the concept of the other" from the confusion that undermines intuition altogether? Without some distinction between these two types of confusion, both Descartes's theory of simple natures and his concept of intuition are threatened in their very foundations. Indeed, the problem is even more acute because Descartes clearly states that, at least initially, *confusion affects all cases of necessary conjunction*, not only some. He writes: "*It is in this way* [*hoc pacto*] [i.e., confusedly] that shape is conjoined with extension, motion with duration or time […]" (ibid.). These examples clearly indicate that in *every* intuition of

a necessary conjunction between simple natures the relations between the simple natures conjoined are in some sense confused. In what sense?

It turns out that there are important differences between types of confusion. In necessary conjunctions, the relevant simple natures are never intuited separately from one another, so much so that the nature of the relation between them is not always intuited. When I intuit shape, extension is necessarily included in my intuition, but *I do not necessarily intuit that shape is epistemically dependent on extension*, even though this epistemic dependence is *there to be intuited.* I simply intuit an extended shape, neither excluding its extension nor intuiting its epistemic dependence on extension. In no way do I confuse shape and extension *for one another*. That would require an *act of judgment*, and intuition is an *exhibition* of an object, not a *judgment* (see Chapter 3, Sections 3.2 and Section 7.9 below). Instead, I simply *do not notice* the epistemic dependency that determines the *relation* between shape and extension in intuition. The relation is *given*, and can perhaps be made more perspicuous by varying the content of the intuition (e.g., retaining the extension, but altering the shape in spatial intuition, say from a square to a rhombus), but it is not initially *noticed*. No error can arise here, since no judgment has been made, and only judgments, not intuitions, can be erroneous.

Error arises when simple natures that *exclude one another in intuition* are *combined in judgment*. Were I to combine the intellectual simple nature of volition and the material simple nature of motion in this way (i.e., were I to assert that the motion of bodies is a species of volition), then *I would be confusing two simple natures that are not given in intuition as having the relation asserted in the judgment.* The intellectual simple nature of volition is nowhere to be found in the intuition of motion; the only way to combine volition and motion is via judgment. Here, *judgment alone is responsible for adding contents that are not contained in the intuition* (neither explicitly nor implicitly). This type of confusion is problematic because it violates Descartes’s habitual theory of the faculties by mixing two distinct modes of intuitive cognition: purely intellectual intuition (required in the intuition of volition) and intellectual intuition aided by the imagination (required in the intuition of motion). When I intuit shape or motion, extension is confusedly contained *in the intuition*. No judgment connects them to one another. I simply cannot imagine a motion without imagining an extended thing that moves. This type of confusion is not problematic. No judgment combining

simple natures that exclude one another has been made. When I confuse motion for a species of volition, however, such a judgment is made. Strictly speaking, then, Descartes does not regard all confusion as problematic. In *Rules*, he distinguishes between two types of confusion and he distributes them between two different levels of cognition: apprehension (intuition), on the one hand, and judgment, on the other. He then locates confusions that lead to error exclusively at the second level. These confusions result from judgments that violate his habitual theory of the faculties and, therefore, transcend the bounds of intuition. By locating error exclusively at the level of judgment, Descartes immunizes even the most complex intuitions against the possibility of error.

As if to further emphasize this latter point, later in Rule 12 Descartes adds that "it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others. […] [What] requires effort [*operam*] is distinguishing one from another, and intuiting each one separately with steadfast mental gaze" (AT 10:422–5, CSM 1:46–8). Typically, the mind intuits simple natures in composites, not in isolation. Intuiting them on their own "requires effort." Descartes illustrates what he means by way of an important example: "For example, I can have knowledge of a triangle, even though it has never occurred to me that this knowledge involves knowledge also of the angle, the line, the number three, shape, extension, etc." (AT 10:422, CSM 1:46). This does not "preclude our saying that the nature of a triangle is composed of these other natures and that they are better known than the triangle, for it is just these natures that we understand to be present in it" (ibid.). I already know what these other natures are, and I would not be able to know what a triangle is without first having known these other natures and their ordered relations to one another (knowledge of the angle depends on knowledge of the line, which in turn depends on knowledge of extension, etc.). This example indicates that intuition frequently embraces many "mutually conjoined natures" at once, and that these natures are oftentimes not distinguished from one another in intuition. These natures can, upon reflection, be intuited by themselves, but the important point is that the intuition of a triangle (or any other composite object) does not depend on separately intuiting each of the natures of which it is composed.

Indeed, Descartes insists that there may be "many additional natures implicitly contained [*involuntur*] in the triangle which escape our notice

[*quae nos latent*], such as the size of the angles being equal to two right angles, the innumerable relations between the sides and the angles, the size of its surface area, etc." (ibid.). Descartes's constraints on intuition are minimal: intuiting a triangle requires that I know each of the mutually conjoined natures of which triangles are composed (angle, line, etc.), but not that I discern the epistemic dependency relations that obtain between them whenever I intuit a triangle or even that I intuit each one separately. There are, moreover, some natures that I only implicitly intuit, and others that I have not yet intuited at all, such as the fact that the angles of a triangle add up to two right angles. This fact can only be intuited via deduction,[25] but it is not required in order to intuit triangles, whereas natures like angles and lines are required. This example reveals an important distinction between what is *implicitly contained* in intuition and a *potential* intuition. That all triangles have angles that add up to two right angles can potentially be intuited (via deduction), but it is not implicitly contained in the intuition of the triangle because *I do not need to know this fact about triangles in order to intuit them.* The implicit/explicit distinction pertains to what must be known in order to intuit the relevant object. "Implicit" means that what must be so known is known, but not noticed, while "explicit" means that it is noticed. The potential/actual distinction, by contrast, pertains to what *can* be intuited, but *is not required in order to intuit the relevant object*.[26]

There are countless complex intuitions. For example, when I doubt, it follows that I know the difference between truth and falsity (see AT 10:421, CSM 1:46). However, I need not attend to this fact when I am absorbed in doubt. The difference between truth and falsity is implicitly known without having to be noticed. I know the difference so well that *I do not have to notice it*. In *Conversation with Burman*, Descartes makes a related point regarding the internal complexity of the *cogito*:

> Before this inference, "I am thinking, therefore I exist," the major "whatever thinks exists" can be known; for it is in reality prior to my inference, and my inference depends on it. That is why the author says in *Principles* that the major premise comes first, namely because implicitly it is always presupposed and prior. But it does not follow that I am always expressly and explicitly aware of its priority, or that I know it before my inference. This is because I am attending only to what I experience within myself – for example, "I am thinking, therefore I exist." I do not pay attention in the same way to the general notion "whatever thinks exists." (AT 5:147, CSMK 3:333; my emphasis. See also *Principles* I. 10 (AT 8A:8, CSM 1:196)).

In this case, I make the inference "I am thinking, therefore I exist," and upon making this inference, I concomitantly know that it depends on the general notion "whatever thinks exists," but only implicitly, since "I do not pay attention" to this fact. These examples illustrate that intuitions (including simple inferences) composed of many natures and relations can be "confused" in a perfectly harmless sense, for here as before, I do not combine via an act of judgment simple natures that do not belong together, but rather I do not explicitly notice simple natures that *are* given together in intuition. Not only are complex intuitions like the intuition of a triangle and the *cogito* possible—they are even the norm. What requires effort is distinguishing between what is already given in intuition and intuiting each simple nature or relation separately "with steadfast mental gaze."

7.9 Intuition and Judgment

As we have seen in Section 7.8 above, *the difference between intuition and judgment* plays a paramount role in Descartes's theory of simple natures in *Rules*. While there are a variety of causes of erroneous judgment (see Section 7.10 below), *error always consists in the fact that the content of a judgment in one way or another is not based exclusively on the content of intuition.* No intuition, however complex, is subject to error (see Chapter 3, Section 3.2). This is partly why Descartes insists that we must "distinguish between the faculty by which our intellect intuits and knows things and the faculty by which it makes affirmative or negative judgments" (AT 10:420, CSM 1:45).[27] Descartes's distinction between intuition and judgment is incredibly important because *without it there would no longer be a tribunal beyond judgment whereby erroneous judgments could be corrected.* Intuition is a species of "experience,"[28] and *the distinction between intuition and judgment is a distinction between experience as a non-discursive, non-doxastic act that always discloses truth, and judgment as a discursive, doxastic act that can be true or false.* Were all knowledge to consist in judgment alone, no "experience" could ever *correct* judgment, and Descartes would be forced to endorse a "coherentism" in which the logical consistency of a system of judgments, rather than intuition, alone constitutes truth.

If I directly exhibit a body in motion in my imagination or even see one via my senses (e.g., a magnetic attraction), and refrain from making any judgments about it, all I intuit is a body moving from one part of space to another (local displacement).[29] No error arises here, and none can, so long as I restrict myself to the manner in which the object is given in spatial intuition according to the material simple natures. If, however, I judge that there is something I am not seeing here, something beyond what I reach in intuition, then I am indeed mistaken, for there is *nothing in the intuition that suggests this*. For example, if, in addition to imagining or seeing a body in motion, I also judge that what moves the body is an immaterial soul located somewhere within it, then I have combined the intuition of motion with a judgment that is not based on the intuition, and it is only here that error can arise. This immaterial soul is not given in intuition. It is “some kind of entity the like of which our intellect has never perceived,” so that “we should need to be endowed with some new sense, or with a divine mind” in order to intuit it (AT 10:439, CSM 1:57). The content of judgment must be restricted to what is given in intuition alone, and should never transcend the bounds of intuition by means of probable conjecture, which opens the door to error. As Descartes would later put it in *Meditations* IV, error arises when judgment transcends the limits of what is given in clear and distinct perception or intuition.

7.10 Judgment, Composition, and Error

Descartes’s theory of error in Rule 12 is the photographic negative of his theory of conjunction. Descartes’s habitual theory of the faculties prescribes how the faculties must be employed in order to intuit the simple natures. The theory of error describes how error arises when the prescribed faculty configurations are violated. As we have seen, Descartes locates the source of error in judgments that exceed the bounds of intuition. Intuition does not err. Similarly, in the simplest necessary conjunctions, I cannot fail to intuit the simple natures, even if I do not typically intuit each one individually, but only the composite whole (see Section 7.8 above). It is only “when we ourselves compose in some way the objects of our belief” that we “can…go wrong” (AT 10:424, CSM 1:47). To fully immunize the method against the

possibility of error, Descartes must provide a complete enumeration$_3$ of the ways in which composition occurs, and he must distinguish between compositions that yield science and those that do not. Composition "can come about in three ways: through impulse [*impulsum*], through conjecture or through deduction" (AT 10:424, CSM 1:47). In complete enumerations$_3$, one deduces a conclusion disjunctively, by identifying and then eliminating relevant alternatives (see discussion in Chapter 3, Section 3.4.3). Descartes's strategy is to eliminate the first two types of composition (impulse and conjecture) so that deduction remains as the only methodologically-sanctioned type of composition.

Descartes subdivides composition through impulse into three types, each one produced by a unique non-rational cause:

> It is a case of composition through impulse when, in forming judgments about things, our mind leads us to believe something, not because any reasons convince us of it [*nulla ratione persuasi*], but simply because we are caused [*determinati*] to believe it, either by some superior power [*potentia aliqua superiori*], or by our own freedom [*propria libertate*], or by a disposition of the phantasy [*phantasiae dispositione*]. The first is never a source of error, the second rarely, the third almost always (AT 10:424, CSM 1:47; translation modified; my emphasis).

Descartes enumerates$_3$ three causes of composition through impulse here: one external (superior power, presumably God) and two internal (our own freedom; the disposition of the phantasy). These three types of composition via impulsion are ordered on a descending epistemic scale from most credible to least credible. Supernaturally caused beliefs are "never a source of error," my own freedom only "rarely," and the disposition of the phantasy "almost always." Below, I discuss each one in order.

Not all cases of composition through impulse are a source of error. God may dispose "my inmost thoughts" (*Meditations* IV, AT 7:58, CSM 2:40), and when he does, both the contents of the thoughts themselves and the judgments that express these contents are necessarily true. In this case, I do not compose the object of my belief; God does, and on pain of imperfection God is not a cause that can deceive. This explains why supernaturally caused beliefs are "never a source of error," but not why they are "irrelevant in this context" and "do not fall under the scope of the method [*sub artem non cadit*]" (Rule 12, AT 10:424, CSM 1:47). The reason is that supernaturally caused beliefs are free from error and yet *obscure*. In Rule 3, Descartes insists that although intuition and deduction are "the most certain

routes to knowledge that we have," "this does not preclude our believing that what has been revealed by God is more certain [*certiora*] than any knowledge, since faith in these matters, as in anything obscure [*obscuris*], is an act of the will [*voluntatis*] rather than an act of the understanding" (AT 10:370, CSM 1:15).[30] *Supernaturally caused beliefs remain opaque to intuition*. Consequently, even though they are never a source of error, they do not fall under the method because they cannot be rationally evaluated by the principal operation of the method. Nevertheless, Descartes conditionally leaves open the possibility of a knowledge of revealed truths that is not obscure, but acquired via natural reason (i.e., intuition and deduction): "And if our faith has a basis in our intellect, revealed truths above all can and should be discovered by one or other of the two ways we have just described [intuition and deduction], as we may show one day" (AT 10:370, CSM 1:15). Descartes would later pursue this possibility in *Meditations* (see especially AT 7:2, CSM 2:3).

After his brief mention of supernaturally caused beliefs, Descartes asserts, without explanation, that our own freedom is "rarely a source of error." It may seem that the freedom Descartes has in mind here is the freedom of judgment discussed in *Meditations* IV, where he argues that error arises when the will freely judges that something is the case even though the intellect has not clearly and distinctly perceived it to be so. That he does not have this sort of freedom in mind in Rule 12 is clear because the freedom under discussion in Rule 12 is (1) impulsive, and (2) rarely a source of error, whereas the freedom of judgment discussed in *Meditations* IV is so frequently misused that it would be false to say that it is rarely a source of error.[31] Nor can the impulsive beliefs under discussion in Rule 12 caused by our own freedom be assimilated to beliefs that arise from a natural impulse, since they are freely caused by our will. Thus, the impulse in this case comes neither from reason nor from nature, and yet it is rarely a source of error! This makes it very difficult to understand what Descartes even has in mind here. Alquié has usefully suggested that Descartes's source may be Montaigne, who describes the "daemon of Socrates" as "a certain impulsion of the will that came to him without awaiting the advice of his reason. In a well-purified soul such as his, prepared by a continual exercise of wisdom and virtue, it is likely that these inclinations, although instinctive and undigested, were always important and worth following."[32] Here, the impulsion is an "impulsion of the will" that is nevertheless

"important and worthy to be followed" because the impulsions are spontaneously informed by wisdom and virtue even when wisdom and virtue do not cause them via rational deliberation. Further evidence that Descartes has this sort of impulsion in mind is that Descartes intended to write a treatise on Socrates's daemon, *De deo Socratis*.[33] But even if all this were true, why would Descartes include *this* species of impulsively caused belief *here*? Because he is offering a *complete enumeration*$_3$ of composition through impulse, both practical and theoretical. Beliefs caused by my own freedom (in the relevant sense under consideration here) are a bona fide species of impulsively caused belief, one with which Descartes was familiar, and about which he had even intended to write. Even if both supernaturally caused beliefs and beliefs caused by our own freedom are not scientifically relevant and are either never a source of error or only rarely so, Descartes must include them in his enumeration$_3$ simply in order to set them aside and move on to his real concern: beliefs impulsively caused by the disposition of the phantasy, which is "almost always" a source of error.

Errors caused by the disposition of the phantasy are errors in which the phantasy induces the intellect to err *in its judgments about the objects of sense*. The phantasy can deceive in one of two ways here. (1) The phantasy can deceive anybody—wise or unwise—by inducing them to believe that "its disordered images represent real things," an error that arises when the phantasy is impaired by melancholia and the experience is to some degree disordered. Here, the phantasy does not transmit figures from the common sense, but exercises a pathological causality of its own.[34] The phantasy can also (2) deceive the intellect of an unwise—i.e., improperly habituated—person by causing them to believe that "whatever comes to him from his imagination [*imaginatione*] […] passes, complete and unaltered, from the external world to his senses, and from his senses to the phantasy [*phantasiam*]" (AT 10:423, CSM 1:47), an error that can arise even when the phantasy is in no way impaired and the experience is perfectly coherent. Here, the phantasy deceives only because the intellect is not on guard in its judgments. "The understanding of the wise man," however, "will not be deceived in such cases: while he will judge that whatever comes to him from his imagination really is depicted in it, he will never assert that it passes, complete and unaltered, from the external world to his senses, and

from his senses to his phantasy, unless he already has some other grounds for claiming to know this" (AT 10:423, CSM 1:47). If I judge that my imagination represents an object as green, then I have made no error; my judgment is not about the object but rather about the manner in which it is represented. If, however, I judge that the object is green in reality, then I have made an error. The judgment adds a belief to the experience, and the belief is erroneous. Similarly, "when someone who has jaundice is convinced that the things he sees are yellow, this thought of his will be composite, consisting partly of what his phantasy represents to him and partly of the assumption he is making on his own account: viz., that the color looks yellow not owing to any defect of vision but because the things he sees really are yellow" (AT 10:423, CSM 1:47).[35]

After impulsively caused beliefs or judgments, Descartes discusses the second type of composition: composition by conjecture. Descartes explicitly opposes conjecture (or "probable conjecture") to intuition and deduction in Rule 3. Like impulsively caused belief, conjecture is a species of judgment that exceeds the bounds of intuition, but unlike impulsively caused belief, probable conjecture is based on reason insofar as the latter traffics in propositions that are not certified by intuition:

> The main reason why we can find nothing in ordinary philosophy [*vulgari Philosophia*] which is so evident and certain as to be beyond dispute is that students of the subject first of all are not content to acknowledge what is clear and certain, but on the basis of merely probable conjectures [*probabilibus tantum conjecturis*] venture also to make assertions on obscure matters about which nothing is known (AT 10:367–8, CSM 1:14).

In Aristotelian dialectic and scholastic disputations, one begins by enumerating the probable conjectures of the wise (i.e., other philosophers) on any given subject and revising them as needed in order to eliminate the problems they produce.[36] Dialectic is the method employed when one is searching for the first principles of a science. For Descartes, however, first principles discovered by dialectic are not first principles at all, but rather probable conjectures that have not been certified by intuition. Any inquiry that begins with probable conjectures only produces more probable conjectures. There is no epistemic bootstrapping from probability to certainty. Composition through conjecture is a source of error when it "palm[s] off doubtful matters as true [*dubia pro veris obtruderent*]" (AT 10:367, CSM 1:13). Dialecticians "gradually come to have complete faith in these assertions, indiscriminately mixing them up with others that are

true and evident," with the "result […] that the only conclusions" one can "draw are ones which rest on some such obscure proposition, and which are accordingly uncertain" (AT 10:368, CSM 1:14). Probable conjectures are (1) obscure to intuition (i.e., purely discursive, with no foundation in anything that can be intuited by the intellect or the intellect aided by the imagination); (2) dissemble an epistemic status they do not have; and (3) have deeply significant downstream effects, since their probability contaminates any alleged knowledge that incorporates them as either principles or conclusions.[37] Composition by conjecture is the source of the steady accumulation of error in science over time. However, it should be noted that *Descartes does not regard conjecture as a source of error unless one confuses probable conjecture for truth*: "Nothing that we put together in this way really deceives us, so long as we judge it to be merely probable, and never assert it to be true" (AT 10:424, CSM 1:47–8). "Nor for that matter," he adds, "does it make us any wiser." Having eliminated both impulsively caused belief and probable conjecture, Descartes concludes: "Deduction, therefore, remains our sole means of compounding things in a way that enables us to be certain of their truth" (AT 10:424, CSM 1:48). As he put it in Rule 3: "Concerning the objects proposed for study, we ought to investigate what we can clearly and evidently intuit or deduce with certainty, and not what other people have thought or what we ourselves conjecture [*suspicemur*]" (AT 10:366, CSM 1:13). As I argued in Chapter 5, Section 5.2, in Descartes's solution to the problem of the limits of knowledge, intuition and deduction are shown to be the only operations that yield science by enumeration$_3$.

7.11 Descartes's Conclusions

Now that Descartes has fully developed his theory of simple natures and shown, via enumeration$_3$, that *intuition and deduction are the only operations of the intellect that yield science*, he can confidently assert that "we have explained distinctly and, I think, by a sufficient enumeration[$_2$] [*sufficientem enumerationem*], what at the outset we were able to present only in a confused and rough-and-ready way, viz., that there are no paths to certain knowledge of the truth accessible to men save manifest intuition and

necessary deduction" (AT 10:425, CSM 1:48; translation modified). Furthermore, since "it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another" (AT 10:422, CSM 1:46), and since "intuition extends to all [of the] simple natures and to our knowledge of the necessary connections between them, and in short to everything else which the intellect finds to be present exactly within itself or in the phantasy" (AT 10:425, CSM 1:48), it follows that the "whole of human knowledge consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things" (AT 10:427, CSM 1:49). Descartes has defined the *limits of intuition and deduction*; anything not composed of simple natures is not an object of human knowledge. Third, it also follows that "we should not regard some branches of our knowledge of things as more obscure than others, since they are all of the same nature and consist simply in the putting together of self-evident facts" (AT 10:427, CSM 1:50). Descartes has established the *uniform certainty of all sciences*, irrespective of object. Finally, Descartes has established the *unity of science*. The unity of science consists in the fact that the objects of the sciences are all simple natures, such that all problems in the sciences can be solved by means of the three principal operations of the method (intuition, deduction, and enumeration). As I argued in Chapter 3, in *Rules* the unity of science operates only at the level of object and method, not at the level of propositions: the intellectual and material simple natures are irreducibly distinct, and can only be examined by distinct sciences (metaphysics and physics, respectively). Deductions that contain intellectual simple natures and deductions that contain material simple natures remain discrete deductive series, not continuous. Descartes does not define the relation between these sciences in *Rules*. Indeed, since one of the outcomes of Rule 12 is that all simple natures and, therefore, all sciences are on an epistemic par with one another, no science enjoys more certainty than any other science, and so no science can found any another, at least not if the foundation of the sciences must in some sense enjoy more certainty than the sciences it founds. Descartes's theory of simple natures in *Rules* rigorously *excludes* the possibility of there being metaphysical foundations of physics. The most important achievement in Rule 12 from Descartes's point of view is that the objects over which the operations of the method can extend have been enumerated$_3$, and the exclusive claim to epistemic credibility of the

operations of the method has been fully established, so that any problem that cannot be solved by means of these operations and objects cannot be solved by the human mind at all, not even the most perfect. The Cartesian scientific *habitus* extends to objects and, therefore, all problems in all sciences.

7.12 The Theory of Simple Natures: Neither Realism Nor Idealism

A longstanding debate about Descartes's theory of simple natures in *Rules* is about whether it is a species of "realism" or "idealism." Descartes never employed either of these categories, neither in *Rules* nor elsewhere. In the literature on *Rules*, these categories are not clearly defined; they mean different things to different commentators. The least controversial way of stating what the debate is about is perhaps as follows. For some commentators, the simple natures are "ontological elements," which *constitute* reality. For others, the simple natures are only notions in the intellect, which determine how reality must be *epistemically represented*, but have little or no ontological foundation *in* reality. The debate is not about whether an intellectual simple nature such as doubt or a material simple nature such as extension or a common simple nature such as duration *exists* in reality. All three of these simple natures exist in reality, and Descartes nowhere denies that they exist in reality. The debate, rather, is about whether the simple natures *constitute* reality, i.e., whether reality is *wholly reducible* to the simple natures. More simply, the debate is about whether mind and body are reducible to the intellectual and material simple natures, respectively.[38]

My intervention here will be very simple: the debate about "idealism" and "realism" in *Rules* simply cannot be decided on the basis of *Rules*, and it is a mistake to believe that it can be so decided. As I argued in Chapter 2, Section 2.4, Descartes's theory of simple natures in *Rules* rests on what I term a strategy of "extrinsic denomination." This means that the real natures or essence of things are methodologically suspended, such that all things are denominated as "simple" or "composed" *relative to the intellect alone*. The methodological suspension of the real essences of things means

that no determinate thesis about the real essences of things is or can be made in *Rules*. Thus, *relative to the intellect*, body reduces wholly to the material simple natures (extension, shape, and motion). *In reality*, however, body *may* be an Aristotelian hylomorphic compound, in which case body certainly *possesses* extension, shape, and motion, but is not *constituted* by or *reducible* to them. Whether body is in reality what it is relative to the intellect Descartes simply does not decide in *Rules*. He does assert that there are *methodological grounds* for denying that body is composed of form and matter. Nevertheless, this does not establish that body is *not* composed of form and matter. It only motivates the *supposition* that body is exclusively composed of the material simple natures. Similarly, the intellectual simple natures, as purely intellectual acts of *vis cognoscens*, exist in reality, but this does not mean that in reality the soul is reducible to the pure intellect or the mind. In reality, *vis cognoscens* may be one part—the *intellectual* part—of the Aristotelian soul, which has other parts (vegetative, sensitive, appetitive, and locomotive) that inhere in and are causally responsible for the various functions of the body. Descartes cannot establish the ontological thesis that the soul and body are reducible to the intellectual and material simple natures based on the resources available to him in *Rules* alone. Extrinsically denominated relative to the intellect, the soul and the body are reducible to the intellectual and material simple natures, respectively. What they are in reality, as described by *any* given ontology, simply remains undecided in *Rules*. The intellectual and material simple natures are, as Descartes puts it in Rule 12, a "most helpful way of conceiving" things, since they "make everything much clearer" (AT 10:412–13, CSM 1:40), but again these methodological and pragmatic considerations have zero ontological import. As Descartes openly admits: "[You] are not obliged to believe that things are as I suggest" (AT 10:412, CSM 1:40). The simple natures are not "ontological elements"—not in *Rules*, at any rate. That the intellectual and material simple natures exist in reality Descartes does not deny, but the existence of simple natures in reality does not make them "ontological elements." Elements do not merely *exist* in reality, they *constitute* reality. That is what makes them "elements." Nor, moreover, are the simple natures merely *notions* (or concepts, as Mahnke describes them), for they do indeed *exist* in reality.

In *Rules* it is *one and the same reality* that is either (1) extrinsically denominated relative to the intellect as a simple nature or as composed of

simple natures, or (2) described as possibly having a nature described by an ontology that cannot satisfy the epistemic requirements of the method. Interpreting Descartes's theory of simple natures in terms of the distinction between "realism" and "idealism" is inappropriate. What *is* clear is that Descartes focuses exclusively on how things are relative to the intellect, and repeatedly reminds his readers that he is not in the least bit interested in how things are "*a parte rei*" (AT 10:418, CSM 1:44). He is only interested in how things are "in the order that corresponds to our knowledge of them [*spectandas esse res singulas in ordine ad cognitionem nostram*]" (AT 10:418, CSM 1:44); "with respect to our intellect [*respectu vero intellectus nostri*]" (ibid.); "only insofar as they are within the reach of the intellect [*tantum spectandae sunt prout ab intellectu attinguntur*]" (AT 10:399, CSM 1:32); "insofar as some things can be known on the basis of others" (AT 10:381, CSM 1:21). Descartes solves the problem of the limits of knowledge by extrinsically denominating all things relative to the intellect alone as either a simple nature or as composed of simple natures. Anything more ontologically robust requires a metaphysics, which Descartes does not have in *Rules*, and which he would only pursue in order to solve problems that arise *in Rules* (see Chapter 11).

There remains one residual problem. The debate over whether Descartes embraces "realism" or "idealism" in *Rules* is partly motivated by differences between his enumeration$_3$ of the simple natures in Rule 6 (AT 10:381–2, CSM 1:21) and his enumeration$_3$ of simple natures in Rule 12.[39] In Rule 6, Descartes includes "whatever is viewed as being independent, a cause, simple, universal, single, equal, similar, straight, and other qualities of this sort" in his enumeration$_3$ of "absolutes," which are always first in the series of things to be known, and which he defines as "whatever has within it the pure and simple nature in question [*quidquid in se continet naturam puram et simplicem*]" (AT 10:381–2, CSM 1:21). It is not surprising that differences between Rule 6 and Rule 12 have motivated the debate about "realism" and "idealism" in *Rules*: the simple natures in Rule 6 seem to be *notions*, not things, whereas the material simple natures in Rule 12 seem to be *things*, not notions. I have not discussed the differences between these two enumeration$_3$ yet, but they are significant, and it turns out that they reveal important facts about the development of Descartes's theory of simple natures between *Rules*$_{\text{CM}}$ and *Rules*$_{\text{AT}}$.

The difference between these two enumerations$_3$ is undeniable, and they are in some respects incompatible, but *not* because Rule 6 enumerates only *notions* while Rule 12 enumerates *things*. Descartes describes the simple natures in Rule 12 as "*notions* of simple things [*simplicium rerum notiones*]" (AT 10:417, CSM 1:43; my emphasis), so it cannot be the case that the contrast between the simple natures enumerated$_3$ in Rule 6 and those enumerated$_3$ in Rule 12 hinges on the difference between "notions" and "things," especially not when "things" means "ontological elements." The real problem lies elsewhere. No member of Descartes's enumeration$_3$ of simple natures in Rule 6 is reproduced in Rule 12. Indeed, some of them even seem *incompatible* with the criteria that determine Descartes's enumeration$_3$ of the simple natures in Rule 12. For example, Descartes includes "universal" in his enumeration$_3$ of absolutes in Rule 6. Universals are abstracted from particulars. The simple natures in Rule 12, however, are not universals. They are not abstracted from particulars. There is nothing from which the simple natures in Rule 12 could be abstracted, since they are epistemically basic (see Section 7.2 above).[40] Descartes also includes "straight" in his enumeration$_3$ of simple natures in Rule 6. Unlike the simple natures enumerated$_3$ in Rule 12, however, "straight" can be further analyzed into the material simple natures of extension and shape. Indeed, as Descartes himself points out in Rule 14, "straight" can only be abstracted from the solid, for the "line and the surface are not conceived as being really distinct from the solid or from one another," so that if the line and the surface are "thought without respect to anything else," they are mere "abstractions of the intellect" (AT 10:448–9, CSM 1:63) (see Section 7.7 above). Thus, many of *the simple natures enumerated$_3$ in Rule 6 do not satisfy the criterion of cognitive indivisibility*, which, as we have seen, the simple natures in Rule 12 do (must) satisfy (see Section 7.2).

Furthermore, many of the absolutes enumerated$_3$ in Rule 6 are relatively absolute in a stronger sense than relative absolutes such as motion or shape in Rule 12. The absolutes in Rule 6 are only absolute *relative to the series in which they are a member*, and Descartes is very clear that what is absolute in one series may be relative in another, for "some things are more absolute than others from one point of view, and yet more relative from a different point of view" (AT 10:382, CSM 1 22). For example, what is

regarded as a cause in one problem may be regarded as an effect in another. This is not true of the simple natures in Rule 12. *Relative absolutes in Rule 12 like motion or shape always occupy the same place in the epistemic hierarchy.* They do not become more or less absolute. The class of absolutes in Rule 6 do become more or less absolute depending on the series in which they are a member. The class of absolutes in Rule 6 is wider than the class of simple natures in Rule 12 precisely because they are *series-relative.*

We may well wonder what accounts for these differences between Rules 6 and 12. *Rules*$_{\text{CM}}$ provides precious details here. Rule 12$_{\text{CM}}$ does not provide a complete solution to the problem of the limits of knowledge. The second part of the problem—the objects of knowledge—is defined, but not solved or even addressed in the body of Rule 12$_{\text{CM}}$. Descartes does not mention the simple natures, let alone develop a theory of simple natures in Rule 12$_{\text{CM}}$.[41] This means that *Descartes had not yet identified the criterion of cognitive indivisibility in* Rules$_{\text{CM}}$. This criterion would later enable him to enumerate$_3$ the intellectual, material, and common simple natures in subsequent drafts of Rule 12. To solve the problem of the limits of knowledge, a problem that Descartes is not concerned with in Rule 6, Descartes would have to enumerate$_3$ all and only those simple natures that can be intuited by the intellect (either alone or aided by the imagination) and *to which all other objects of knowledge must be reduced*. He would need the criterion of cognitive indivisibility. This criterion alone furnishes simple natures whose simplicity is such that they cannot be further divided by the mind into more distinctly known notions, and they compose the rest of human knowledge. This sort of theory is nowhere to be found in Rule 6, because it is not needed there. All that Descartes really needs in Rule 6 is a method for comparing things in terms of relations of epistemic priority and posteriority in *local* series relative to *local* problems. He needs much more in Rules 8 and 12. In order to solve the problem of the limits of knowledge, he needs a theory of all “the things it is possible to know” (AT 10:398, CSM 1:32). Once Descartes identified the criterion of cognitive indivisibility in order to complete his solution to the problem of the limits of knowledge, the simple natures in Rule 6 did not become defunct, but they could no longer be regarded as on a par with the simple natures enumerated$_3$ in Rule 12.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0008

1 Even though Descartes does not discuss them in any detail in *Rules*, the intellectual simple natures constitute the object of Descartes's metaphysics. This becomes fully explicit in *Principles* and in his correspondence with Princess Elizabeth. See Gouhier 1962, 329–34, Marion 1992, and the discussion in Chapter 11.

2 See Aristotle 1984, 1:6. On unity in Aristotle's theory of substance, see Lewis 1995 and Gill 1989.

3 See *Physics* III. 1 201^{a}10 in Aristotle 1984, 1:343.

4 Cf. *The World*, AT 11: 39–40, CSM 1: 94.

5 Cf. *Principles* I. 10, AT 8A: 8, CSM 1: 195–6.

6 It might seem that since the simple natures are notions and not propositions, they must be terms (as "man" is). The relevant terms in this case, however, would be universals, and universals both have definitions and, more importantly, are abstracted from individuals. See Aristotle 1984, 1:27: "I call universal that which is by its nature predicated of a number of things" (*De Interpretatione* 7 17^{a}39–41). See also *Principles* I. 59 (AT 8A:27–8, CSM 1:212–13), where Descartes argues that terms are universals that arise by abstracting from differences between things that resemble one another. This is not how the mind intuits the simple natures. In the case of material things, for example, no material thing I intuit does not already contain extension, shape, and motion (real or possible). The simple natures, as Descartes also indicates in *Principles* I. 48 are "simple notions which are the basic components of our thoughts" (AT 8A:23, CSM 1:208–9). Because they are notionally basic, these simple natures compose the intuition of any individual material thing, and so cannot be abstracted from any material thing. Since terms are so abstracted, simple notions or natures are not terms. On the contrary, in *Rules* as elsewhere in Descartes's corpus, both "terms" and "propositions" are purely *logical entities* whose true sense is entirely determined by and restricted to the scope of the content provided in the intuition of the simple natures they *express*.

7 Keeling 1937, 76–7 argues that the simplicity of simple natures consists in the fact that "chacune est *per se nota*" or self-evident, as do Joachim 1957, 71 and Beck 1952, 75. Beck (ibid.) also confuses the simple natures with propositions. For more discussion, see Chapter 3, Section 3.2.

8 O'Neil's (1974, 14) contention that "Descartes never defined what he meant by a simple nature" is hasty. O'Neil does not discuss Descartes's criteria of cognitive indivisibility or univocity.

9 "If, for example, Socrates says that he doubts everything, it necessarily follows that he understands at least that he is doubting […]" (Rule 12, AT 10:421, CSM 1:46).

10 O'Neil 1974, 14 complains that Descartes's enumeration$_3$ of the simple natures "unhappily […] ends with a tantalizing 'etc.,' 'and so forth,' or 'and the like.'" There is, however, no need for the enumeration$_3$ to be complete, but only sufficient. I discuss the role played by the intellectual simple natures in Descartes's ontology of mind in *Rules* in more detail in Section 7.7 and Chapter 8.

11 I discuss the difference between abstraction and exclusion in Section 7.7 below. See also Murdoch 1993.

12 There is an important difference between *Rules* and *Meditations* here. In *Rules*, Descartes recommends intuiting the material simple natures by means of the intellect aided by the imagination. In *Meditations* II, by contrast, he recommends intuiting extension by means of the pure intellect alone. One should not, however, make too much of this difference. In *Meditations* II, Descartes

recommends intuiting by the intellect alone, not extension per se, but rather the fact that it is "flexible and changeable" in an "immeasurable number" of ways, because the imagination can never run through each one individually. The point is that the immeasurable number of shapes any part of extension can assume exceed the limits of the imagination, not that extension, shape, and motion as such should always be intuited by the pure intellect alone. Even when the pure intellect inspects the idea of extension and determines that any bit of extension can take on an "innumerable number" of shapes, *it can only do so via the imagination via enumeration*$_3$; the mind could not even conceive extension without the imagination (see Meditations VI, AT 7:73, CSM 2:51). After *Meditations*, Descartes continues to insist on the necessity of employing the imagination in mathematics (see, e.g., letter to Elizabeth, June 28, 1643, AT 3:692, CSMK 3:227).

13 I discuss the material simple natures in more detail in Section 7.7 below and illustrate their use in mathematics and natural philosophy in more detail in Chapters 9 and 10. Like Descartes, Beeckman embraced what he termed "picturability" (*aanschouwelijkheid*) as his preferred criterion of intelligibility. For Beeckman, picturability is not defined by what can be seen by the eye, but rather by what can be intelligibly imagined *as if it were observable*. In an October 1629 letter to Mersenne, Beeckman writes: "In philosophy I allow nothing that is not represented to the imagination as if it were observable [*Nihil enim in philosophia admitto quam quod imaginationi velut sensile representatur*]" (cited in van Berkel 2013, 81). Similarly, in Rule 14, Descartes writes that "we generally do not recognize philosophical entities of the sort that are not genuinely imaginable" (AT 10:442, CSM 1:59). The difference is that Descartes extends intuition beyond the confines the imagination to the pure intellect in *Rules*. For more discussion, see Chapter 3, Section 3.2 and Chapter 10, Section 10.4.

14 As a common simple nature, unity should be distinguished from the mathematical concept of unity (the unit). The latter pertains exclusively to the material simple natures; it renders magnitudes measurable. The intellectual simple natures are not magnitudes and are not measurable in this sense.

15 See Descartes's letter to Arnauld (July 29, 1648): "I understand the successive duration of things in motion, and of the motion itself, no differently from that of things that are not in motion; for earlier and later in any duration are known to me by the earlier and later of the successive duration which I detect in my own thought, with which the other things co-exist" (AT 5:223, CSMK 3:358).

16 See Rabouin 2009, 319. Rabouin argues that despite their Euclidean origins Descartes's examples are examples of common notions because they illustrate the type of order required in *all* deduction. Descartes frequently illustrates order via mathematical examples. In Rule 6, for instance, he illustrates deduction via a series of continued proportionals: 3, 6, 12, 24, 48, etc. The order is an order of numbers, but it illustrates the type of order required in all deduction (mathematical or not). It is less the Euclidean common notions themselves than the type of order they exhibit that deserves the name "common notions."

17 See Aristotle 1984, 1:19 (*Categories* 12^{a}26–39) and Aristotle 1984, 2:1615 (*Metaphysics* V.22 1022^{b}23–30).

18 See Aristotle 1984, 1:388 (*Physics* V.6 229^{b}26–7). For late scholastic treatments, see Des Chene 1996.

19 See Costabel and Marion in Descartes 1977, 242 n.27. Descartes's decision to include rest as a simple nature enables him to state his first law of nature: a body in motion will continue in motion "unless checked by something else," and a body at rest will remain at rest "unless pushed into motion by some cause," "for rest is the opposite of motion, and nothing can by its own nature tend towards its opposite, or towards its own destruction" (*Principles* II. 37, AT 8A:62–3, CSM 1:241). For more discussion, see Chapter 10, Section 10.4.

[20] See also *Second Replies*: “It is impossible that nothing, a non-existing thing, should be the cause of the existence of anything, or of any actual perfection in anything” (AT 7:165, CSM 2:116). This common notion entails Descartes’s causal axiom in *Meditations* III that “it is manifest by the natural light that there must be at least as much [reality] in the efficient and total cause as in the effect of that cause” (AT 7:40, CSM 1:28). Descartes does not discuss these causal axioms in *Rules*, but he would have been familiar with them, since he learned them at La Flèche from the textbooks written by the Conimbricenses. On the history of these causal axioms, see Carraud 2002 and Schmaltz 2008.

[21] I postpone discussing the relation between “duration” and “instant” until Chapter 10, Section 10.4, where I discuss the role they play in Descartes’s laws of nature in *The World* and *Principles* in the context of his deduction of the shape of the anaclastic lens and the law of refraction in Rule 8.

[22] On Descartes’s confusion about the differences between the modal and rational distinctions in *First Replies*, see Wells 1965, 1966 and Ariew 2012. Descartes acknowledges his confusion about these distinctions in *Principles* I. 62, AT 8A: 30, CSM 1: 215.

[23] Beyssade 1997 argues that extension is only a mode of body in Rule 14, and that the rational distinction is not present in *Rules*. See also Beyssade and Kambouchner in Descartes 2016, 724, n. 337. The end of the footnote suggests that the seeds of the rational distinction can, in fact, be found in Rule 14.

[24] Ariew 2012 has argued that Descartes did not begin seriously reflecting on the theory of distinctions until *First Replies*. By contrast, Beyssade 1997 has convincingly shown that Descartes’s theory of distinctions originates in Rule 14, and Zepeda 2016 (following indications in Murdoch 1993) has convincingly shown that Descartes’s concept of real distinction originates in *Rules*. I think all three distinctions can already be found in *Rules*. In *Principles*, the ambiguity in Descartes’s definition and examples of necessary conjunction between “one-way inseparability” and “two-way inseparability” is cleared up such that the former corresponds to the modal distinction while the latter corresponds to the rational distinction. See AT 8A:28–30, CSM 1:213–15. For further discussion of the relation between Descartes’s theory of simple natures in *Rules* and his ontology of substance, attribute, and mode, see Chapter 11, Sections 11.5–11.6.

[25] See Euclid 1908, 1:316–22 (*Elements* I, prop. 32).

[26] See McRae 1972 and Wilson 1982, 150–65. While McRae conflates implicit and potential knowledge, Wilson correctly emphasizes that implicit knowledge is knowledge I already have, but am not attending to, whereas potential knowledge is knowledge I can have, but do not yet have.

[27] Unlike *Meditations*, in *Rules* the intellect, not the will, judges. The distinction between judgment and intuition as distinct acts nevertheless remains.

[28] “Our experience consists of whatever we perceive by means of the senses, whatever we learn from others, and in general whatever reaches our intellect either from external sources or from its own reflexive self-contemplation” (AT 10:422–3, CSM 1:46–7).

[29] Intuition via the senses is possible so long as the sensory qualities are excluded and only the material simple natures remain: “The intellect can never be deceived by any experience, provided that when the object is presented to it, it intuits it in a fashion exactly corresponding to the way in which it possesses the object, either within itself or in a phantasm [*phantasmate*],” where “phantasm” is a mental representation of an object caused by the phantasy, which is the organ in which the figures received from the common sense cause the intellect to have sensory perceptions (see Chapter 6, Section 6.3). So long as the intellect does not “judge that the imagination faithfully represents the objects of the senses,” there is no risk of error here (AT 10:423, CSM 1:47).

[30] The will, not the intellect, is the faculty whereby one has faith in anything that remains irreducibly obscure to the intellect. This is odd, since Descartes also maintains that the intellect is the

faculty of judgment in *Rules*, and it would seem that faith is an act of judgment. Descartes's claim is perhaps that the intellect can only judge about those things that can be intuited, while the will can only judge about those things that cannot be intuited. It is, therefore, not the case that only the intellect can judge in *Rules*. The will can judge too in those cases where the intellect cannot be a guide. Descartes's motivation here may be to ensure that since faith in divine revelation is a supernaturally caused act of will that cannot be rationally evaluated on the basis of intuition, it does not fall within the scope of the method, and so cannot be doubted according to the criteria of the method. The truth of faith consists in the fact that it is caused by a superior power, not in the fact that its contents are intuited. For more discussion, see Davenport 2006, 198. On the superior certainty, yet obscurity, of supernaturally revealed truths, and on the fact that they are believed by means of the will rather than the intellect, see Aquinas 1882–, 8:54 (*ST* II-II, q. 5, art. 1); trans. 1945, 2:1108 and 1110: "…the knowledge of faith is dark and obscure [*cognitio fidei est aenigmatica et obscura*]."; Aquinas 1882–, 8:27 (*ST* II-II, q. 2, art. 1); trans. Aquinas 1945, 2:1075–6: "The intellect of the believer is determined to one object, not by the reason but by the will [*intellectus credentis determinatur ad unum non per rationem, sed per voluntatem*]."; Aquinas 1882, 8:52 (*ST* II-II, q. 4, art. 8); trans. Aquinas 1945, 2:1106: "In this way, faith is more certain that these three virtues [wisdom, science, and understanding], because it is founded on the divine truth, whereas the aforesaid three virtues are based on human reason [*Et hoc modo fides est certior tribus praedictis, quia fides innititur veritati divinae, tria autem praedicta innituntur rationi humanae*]." See Beyssade and Kambouchner in Descartes 2016, 672–3, n. 60. See also Suárez 1856–1878, 12:67–79 (*De fide*, disp. 3, sec. 7–8), cited in Laporte 1945, 314. The thesis that "what is revealed by God is more certain than any knowledge" is one Descartes defends well after *Rules*. See, e.g., *Principles* I. 76: "But above all else we must impress on our memory the overriding rule that whatever God has revealed to us must be accepted as more certain than anything else" (AT 8A:39, CSM 1:221).

31 Following a suggestion in Brunschwig's translation of *Rules* in Descartes [1963] 1997: 146, n. 1, Araujo 2003, 75–8 argues that when impulsive beliefs caused by our own freedom are true (and they are mostly true, according to Descartes), Descartes is in fact referring to judgments the "plain man" makes when he is "always guided only by reason" (77), and not by the "authority of the teachers" (ibid.). This cannot be, since Descartes explicitly asserts that the impulsive beliefs are held "not because any reasons convince us of it" (AT 10:424, CSM 1:104). Marion 1975, 144 regards freedom in Rule 12 as "*la plus part trompeuse*," but this flatly contradicts the letter of the text.

32 See Montaigne 2003, 35.

33 Baillet reports that Descartes intended to write a short treatise on Socrates, *De deo Socratis.* See Baillet 1691, 2:408, cited in AT 4:532. See also Alquié in Descartes 1963, 679, n.1. For a similar interpretation, see Laporte 1945, 39–40 and Beyssade and Kambouchner in Descartes 2016, 712, n. 267. See also Deprun 1983. Further evidence can be found in Descartes's letter Elizabeth (October or November 1646, AT 4:529–30, CSMK 3:296–7).

34 As Gaukroger describes it, in the seventeenth century "melancholia" refers to a "quite severe disease involving persistent hallucinations, epileptic fits [e.g., sensory disturbances and convulsions due to abnormal brain activity], and lycanthropy [i.e., the delusion that one can transform into or is a non-human animal or perhaps even an inanimate thing]" (Gaukroger 1995, 19). For a history of melancholia from Hippocrates to the present, see Jackson 1986. On the history of the concept of madness, see Foucault [1961] 2006.

35 In *Sixth Replies*, Descartes distinguishes between three grades of sensory response. The first grade is limited to the "immediate stimulation of the bodily organs by external objects," and corresponds to the modification of the shape of the sensory organ described earlier in Rule 12 (see Chapter 6, Section 6.2). The second grade "comprises all the immediate effects produce in the mind as a result of its being united with a bodily organ which is affected in this way," and includes "perceptions of pain, pleasure, thirst, hunger, colors, sound, taste, smell, heat, cold and the like." The

second grade of sensory response corresponds to what is given in the imagination when *vis cognoscens* is affected by the phantasy. The third grade "includes all the judgments about things outside us which we have been accustomed to make from our earliest years – judgments which are occasioned by the movements of these bodily organs." Descartes restricts the possibility of error to the third grade (judgment) alone, exactly as he does in Rule 12. See AT 7:437, CSM 2:294–5. I will not discuss material falsity here, since Descartes does not discuss material falsity in *Rules*, and in any case it is not a problem that arises in the case of the simple natures.

36 On Aristotelian dialectic, see Irwin 1988. On scholastic *disputatio*, see Lawn 1993 and Novikoff 2014.

37 See also *Discourse* II, AT 6:12–13, CSM 1:117 and *Meditations* I (AT 7:17, CSM 2:12).

38 It was Keeling 1937 who initiated and framed the debate in terms of the distinction between "realism" and "idealism." Keeling maintains that simple natures are "real existences, ontological elements, the ingredients of the universe" (79), but does not address Descartes's claim in Rule 12 that he will "consider things in the order that corresponds to our knowledge of them," which Descartes explicitly distinguishes from "how they exist in reality" (AT 10:418, CSM 1:44). Beck 1952, 66–74 follows Keeling, but adds that "if we do not wish to go as far as this eminent commentator, we may suppose that the minimum interpretation of Descartes's words leads to the conclusion that these natures, as simple and singular entities, if they do not exist in the full ontological sense of this term, at least subsist." Beck does not clarify his distinction between "exist" and "subsist." In direct opposition to Keeling, Le Blond 1937 insists that "il ne s'agit pas de 'choses' mais de 'notions' : Descartes a soin de préciser, en commençant cette étude des objets de la connaissance, qu'il ne s'occupe pas des êtres '*prout revera existunt*,' mais seulement '*in ordine ad cognitionem nostram*" (165). Against Le Blond 1937, O'Neil's "central contention is that at least three simple natures are constituent of the physical world: figure, extension and motion," and that, consequently, the "simple natures – at least some of them – are not just concepts, notions or ideas," but "have a species of ontological independence and are constituent of the world" (O'Neil 1972, 167, 170). Hartland-Swan 1947 zeroes in on the common simple natures and, more specifically, the common notions, which he interprets as eternal truths (according to *Principles* I. 48–9), and which he argues move us "out of the sphere of ontological essences and back to mere concepts or logical principles," so that "entification on this basis becomes extraordinarily precarious," concluding that Keeling mistakenly "designate[d] all simple natures indiscriminately" as "ontal element[s]" (143). Mahnke 1967 criticizes Keeling's ontological interpretation of the simple natures on the neo-Kantian grounds that "[sie] sind nicht dinge […] sondern Beziehungsformen," i.e., "die Structuren, in denen der Verstand notwendig denkt," thereby rendering them Kantian categories (65–6). Marion 1975, 141–2 approves. Schuster 2013, 253, n. 69–70 mentions, but does not discuss the debate.

39 See, e.g., Keeling 1937, 76; Beck 1952, 71; Mahnke 1967, 62; O'Neil 1972, 162–7.

40 See also Laporte 1945, 102–13.

41 See CM fo. 17r.

8

The Origins of Cartesian Dualism in Rule 12

8.1 New Evidence, Old Problem

As we have seen, in Rule 12, Descartes describes *vis cognoscens* as a "purely spiritual" force "distinct from the whole body" (AT 10:415, CSM 1:42). *Vis cognoscens* can only intervene in and regulate the cognitive mechanism—i.e., determine when to employ the pure intellect alone or the intellect aided by the imagination—as *volition*, and since volition is an intellectual simple nature, *vis cognoscens* cannot be a part of the body. Nevertheless, it is not self-evident that Descartes embraces mind–body dualism in *Rules*.[1] Descartes makes no claims in *Rules* about mind and body as two distinct substances, and he certainly makes no claims about the immortality of the soul. At one point in Rule 12, he even describes the mind in an Aristotelian manner as the form of the body (AT 10:411, CSM 1:40).[2] If "dualism" accurately describes Descartes's theses about mind and body in *Rules*, a more refined sense of what "dualism" can and cannot mean in *Rules* is clearly needed. Things are further complicated by the fact that in the recently discovered Cambridge manuscript, Descartes does not describe *vis cognoscens* as "distinct from the whole body," but only as "distinct from the phantasy" (*Rules*$_{\text{CM}}$ fo. 16$^{\text{v}}$). This leaves open the possibility that *vis cognoscens* is not distinct from the whole body, but only from one part of it, perhaps as one body part is distinct from another.[3] If, as the evidence suggests,[4] the Cambridge manuscript is a copy of an earlier draft of *Rules*, then the possibility of Descartes's endorsing materialism about the mind is striking. Past discussions of Descartes's theory of mind in *Rules* have

turned *inter alia* on whether he embraces a dualist or a hylomorphic theory of mind there.[5] The Cambridge manuscript introduces a third possibility: viz., that Descartes may have embraced a materialist theory of mind in *Rules*. If Descartes did endorse materialism about the mind in the Cambridge manuscript, then it would mean that he initially regarded the mind as an object of natural philosophy alone, and only later came to regard it as an object of metaphysics.

In this chapter, I examine whether Descartes did, in fact, endorse materialism or hylomorphism about the mind in the Cambridge manuscript and *Rules*$_{\mathrm{AT}}$. My verdict is negative: he endorsed neither. I begin by providing a detailed interpretation of Descartes's mind–body dualism in *Rules*$_{\mathrm{AT}}$ (Section 8.2). I then argue that while the differences between *Rules*$_{\mathrm{AT}}$ and the *Rules*$_{\mathrm{CM}}$ are important, Rule 12$_{\mathrm{CM}}$ precludes interpretation along both materialist and hylomorphic lines (Section 8.3). I also offer an account of the development of Descartes's mind–body dualism between *Rules*$_{\mathrm{CM}}$ and *Rules*$_{\mathrm{AT}}$. What Descartes discovered in *Rules*$_{\mathrm{CM}}$ for the first time in his career, I argue, is that the operations and objects of the intellect in no way depend on the phantasy (a corporeal faculty located in the brain; see Chapter 6, Section 6.3). This discovery constituted a radical departure from mainstream, Aristotelian theories of intellection, according to which intellectual cognition depends on the presence of a phantasm in the imagination—a theory Descartes himself adhered to in the early 1620s (AT 10:217–19, CSM 1:45), but could now definitively abandon. Descartes's discovery of the autonomy of the intellect would lead him to enumerate$_3$ the intellectual simple natures and the material simple natures as two wholly distinct classes of object in *Rules*$_{\mathrm{AT.}}$ It would also enable him to demonstrate that there is no necessary conjunction between these two classes of simple natures, such that that mind and body are really distinct.[6] Descartes had not yet developed the theory of simple natures in Rule 12$_{\mathrm{CM}}$. Nevertheless, his mechanization of Aristotelian faculty psychology (see Chapter 7), his description of *vis cognoscens*, and his discovery of the autonomy of the pure intellect lead him to endorse a recognizable, albeit minimal form of mind–body dualism in *Rules*$_{\mathrm{CM}}$. This dualism is deducible from the following four properties of *vis cognoscens*: (1) its ontological simplicity or indivisibility; (2) its ability to be both active and passive; (3)

its *per se* existence; and (4) its independence, qua "pure intellect," from all other (corporeal) cognitive functions. What *Rules*$_{\mathrm{CM}}$ reveals is not Descartes "before dualism," but rather Cartesian dualism in its barest form, before even the theory of simple natures, let alone Descartes's ontology of substance, attribute, and mode.

8.2 Descartes's Dualism in *Rules*$_{\mathrm{AT}}$

Because Cartesian dualism developed over time, when comparing *Rules* to later texts, it is difficult to identify a definition of dualism that embraces them all. In this chapter, I focus on how many of the elements, if any, of Descartes's dualism in *Meditations* and *Principles* can be found in *Rules*$_{\mathrm{CM}}$ and *Rules*$_{\mathrm{AT}}$.[7] By "Descartes's dualism," I understand the thesis that mind and body are two really distinct things or substances, which can be understood apart from one another, and which enjoy per se existence (see *Meditations* VI, AT 7:78, CSM 2:54; *Principles* I. 60, AT 8A:28–9, CSM 1:213). There are three passages in *Rules*$_{\mathrm{AT}}$ where Descartes seems very clearly to endorse mind–body dualism. In Rule 7, he claims that the proposition "The rational soul [*animam rationalem*] is not corporeal" can be demonstrated by means of "sufficient enumeration" (enumeration$_3$), which here requires grouping "all bodies together into several classes so as to demonstrate that the rational soul cannot be assigned to any of these" (AT 10:390, CSM 1:26–27).[8] Later, in Rule 12, he writes: "The force through which we know things in the strict sense is purely spiritual, and is no less distinct from the whole body than blood is distinct from bone, or the hand from the eye [*vim illam, per quam res proprie cognoscimus, esse pure spiritualem, atque a toto corpore non minus distinctam, quam sit sanguis ab osse, vel manus ab oculo*]" (AT 10:415, CSM 1:42; translation modified). Finally, toward the end of Rule 12, Descartes writes that the proposition "I understand, therefore I have a mind distinct from body [*intelligo, ergo mentem habeo a corpore distinctam*]" is an example of a necessary conjunction between simple natures (AT 10:422, CSM 1:46; translation modified). Neither in these passages nor anywhere else does Descartes describe the mind as a "substance" in *Rules*. Thus, whatever dualism he

may be endorsing in *Rules* cannot be described as "substance dualism," at least not without serious qualifications.

Despite the fact that Descartes does not describe the mind as a "substance" in *Rules*, it is clear that he does regard *vis cognoscens* as "purely spiritual" and "distinct from the whole body." This strongly suggests that *vis cognoscens* is really distinct from body as one substance is distinct from another substance. Indeed, Descartes writes that *vis cognoscens* is no less distinct from body than one body (or body part) is distinct from another (blood from bone or hand from eye). Even when two body parts are parts of one and the same body, they can be separated from one another. This too suggests real distinction.[9] But note a residual ambiguity. Once a part of the human body is separated from the rest of the body, it ceases to function. Is the same true of *vis cognoscens*? Descartes does not indicate as much, but if *vis cognoscens* does functionally depend on the body in the way that a hand or any other body part does, then it is not really distinct from the body. In the case of mind, real distinction requires functionality *after* separation. If *vis cognoscens*, like severed or dead body parts, cannot perform any of its functions after separation from the body, then perhaps it is the "form" of the body in some sense after all.

Note also that Descartes refers to the mind–body union as a "composite" (*composito*) in Rule 12 (AT 10:411, CSM 1:40). This too is ontologically ambiguous, and does not definitively decide in favor of mind–body dualism. On the one hand, "composite" may mean that the human being is composed of two really distinct substances. Descartes often refers to the human being as an entity composed of two really distinct substances (see, e.g., *Comments on a Certain Broadsheet*, AT 8B:351, CSM 1:299 and letter to Hyperaspistes in August 1641, AT 3:422, CSMK 3:189). On the other hand, it may instead mean that the human being is a single substance composed of form (soul) and matter. The latter interpretation would bring Descartes closer to an Aristotelian theory of the soul.[10]

Similarly, read in isolation, the proposition in Rule 7, "The rational soul [*animam rationalem*] is not corporeal," could easily be interpreted along Aristotelian lines. Many scholastic Aristotelians employed the expression "*anima rationalis*" as shorthand to denote the rational *part* of the soul: the intellect.[11] No Aristotelian scholastic would have disagreed with Descartes about the incorporeity of the intellect. Nor would they necessarily have disagreed with the proposition Descartes introduces later in Rule 12, "I

understand, therefore I have a mind distinct from body." "Mind" and "intellect," *mens* and *intellectus*, are interchangeable terms in scholastic Aristotelianism.[12] If all Descartes means to assert is that the intellect is not corporeal, this is not a terribly controversial proposition, and it is certainly not one that clearly commits him to mind–body dualism in *Rules*.

One can already see from these considerations that the passages in which Descartes seems to endorse mind–body dualism in *Rules* are more ambiguous than they first appear to be. By themselves, these passages can be interpreted along either hylomorphic or dualist lines. However, when placed in their proper context and read as an ensemble, they add up to more than the sum of their parts. By paying closer attention to Descartes's theory of the faculties in Rule 12, it can be shown that these passages express a form of mind–body dualism that is distinctively Cartesian, and which coincides neither with materialism about the soul nor with an Aristotelian, hylomorphic theory of soul. On the contrary, these passages constitute enduring contributions to Descartes's mind–body dualism.

Above, I mentioned that no Aristotelian scholastic would have disagreed with Descartes about the incorporeity of the intellect. However, they would have strongly disagreed with anyone who believes that the intellect and the soul are *identical* to one another (or, equivalently, that the soul is *reducible* to the intellect), since they generally recognized the existence of other parts of the soul (the vegetative, appetitive, sensitive, and locomotive), which inhere in the human soul–body composite as their subject.[13] As we have seen in Chapter 6, in *Rules* all faculties other than the intellect and the will are reduced to parts of the body (sensory organs, nerves, and parts of the brain), and their respective functions are explained by their role in the reception, synthesis and transmission, retention, and production of figure. They are not parts of *vis cognoscens*, but rather parts of the body that communicate figures to *vis cognoscens*, where they produce the relevant sensory representations. Descartes's mechanization of Aristotelian faculty psychology in Rule 12 places the passage in Rule 7, "The rational soul is not corporeal," in a new light. This passage can only be interpreted along Aristotelian lines if the rational soul is an incorporeal part of the soul alongside other, corporeal (or corporeally inhering) parts. But as we have seen, there are no corporeal parts of the soul in Rule 12; the functions usually attributed to the soul by Aristotelians have been mechanized as functions of the body alone. The same holds for the passage later in Rule

12, “I understand, therefore I have a mind distinct from body.” “Mind” cannot be interpreted here as a part of the soul existing alongside other, corporeal parts. Once more, there simply are no such parts.

What about Descartes’s description of *vis cognoscens* in Rule 12? Due to its importance, I will reproduce it in its entirety:

> Fifthly, and lastly, the force through which we know things in the strict sense should be conceived as [*concipiendum est*] purely spiritual, and is no less distinct from the whole body than blood is distinct from bone, or hand from the eye [*vim illam, per quam res proprie cognoscimus, esse pure spiritualem, atque a toto corpore non minus distinctam, quam sit sanguis ab osse, vel manus aboculo*]. It is one single force [*unicamque esse*], whether it receives figures from the common sense at the same time as does the phantasy, or applies itself to those which are preserved in the memory, or forms new ones which so preoccupy it that it is often in no position to receive ideas from the common sense at the same time, or to transmit them to the power responsible for motion in accordance with a purely corporeal mode of operation. In all these functions the cognitive force [*vis cognoscens*] is sometimes passive, sometimes active [*interdum patitur, interdum agit*]; sometimes resembling the seal, sometimes the wax. But this should be understood merely as an analogy, for nothing quite like this power is to be found in corporeal things [*quod tamen per analogiam tantum hic est sumendum, neque enim in rebus corporeis aliquid omnino huic simile invenitur*]. It is one and the same force [*una et eadem est vis*], when applying itself along with imagination to the common sense, it is said to see, touch, etc.; when applying itself to the imagination alone, insofar as the latter is invested with various figures, it is said to remember; when applying itself to the imagination in order to form new figures, it is said to imagine or conceive; and lastly, when it acts on its own, it is said to understand [*intelligere*]. How understanding comes about I shall explain at greater length in the appropriate place. According to its different functions [*functiones*], then, the same force is called either pure intellect, or imagination, or memory, or sense-perception (AT 10:415–16, CSM 1:42–3; translation modified).

Here, Descartes enumerates$_3$ four properties of *vis cognoscens*, each of which distinguishes it from body: (1) *vis cognoscens*’s simplicity or indivisibility; (2) its ability to be both active and passive; (3) its *per se* existence; and (4) its independence, qua “pure intellect,” from all other (corporeal) cognitive functions. Regarding its simplicity or indivisibility, *vis cognoscens* remains “the same,” whether it is called “pure intellect, or imagination, or memory, or sense-perception.”[14] Descartes describes these faculties as “functions” of one and the same force. The names “sense-perception,” “common sense,” “memory,” “imagination,” and “pure intellect” denote functions of *vis cognoscens*, not parts of which it is composed,[15] and it performs these functions in collaboration with parts of the body (principally, the brain), except when it acts on its own as “pure intellect.” *Vis cognoscens* is essentially simple, and all of the other “modes

of knowing" (*cognoscendi modis*, AT 10:396, CSM 1:30) are only modes of *vis cognoscens's* activity. Modes are not parts.

Regarding its ability to be both active and passive, *vis cognoscens* can be affected by the phantasy and the common sense, and it can spontaneously intervene in and regulate how the other, corporeal faculties are employed.[16] This indicates that "nothing quite like this force is to be found in corporeal things." The property of *vis cognoscens* that most impresses Descartes here is the fact that it is "sometimes passive, sometimes active." This property is unique to *vis cognoscens*: one body can act on another or be acted upon, but no body can spontaneously decide if and when to be active or passive. Furthermore, in the case of the corporeal faculties, Descartes meant the seal/wax analogy literally.[17] But when it comes to *vis cognoscens*, the analogy "should be understood merely as an analogy." Descartes clearly regards *vis cognoscens*'s ability to be both active and passive as evidence of its being "purely spiritual," and he will soon include "volition" in his enumeration$_3$ of the intellectual simple natures.

Regarding the *per se* existence of *vis cognoscens*, this is strongly suggested by the fact that "nothing quite like this force is to be found in corporeal things." This means both that the existence of *vis cognoscens* could not have been *caused* by corporeal things, and that it does not *inhere* in corporeal things as in a subject, since it is nowhere to be found "in" them. The fact that *vis cognoscens* is neither caused by nor inheres in corporeal things is perhaps one of the strongest pieces of evidence in favor of its being regarded as a substance by Descartes, even though he does not explicitly assert that it is a substance in Rule 12 (I explain why he avoids the term "substance" below). *Per se* existence certainly constituted the preferred criterion of substancehood in standard scholastic Aristotelian definitions of substance.[18]

Finally, regarding independence from all other (corporeal) cognitive functions, all acts of sense-perception and imagination presuppose acts of *vis cognoscens*, but its acts do not presuppose acts of sense-perception or imagination whenever it acts alone.[19] In *Meditations* VI, Descartes similarly argues that "there is an intellectual act included in their [sense-perception and imagination] essential definition," but not vice versa (AT 7:78, CSM 2:54), such that imagination and sense-perception are modes of

the intellect when the latter is united to the body. This parallel between Rule 12 and *Meditations* VI strongly suggests that *vis cognoscens* is really distinct from body.

Properties (1)–(4) would eventually lead Descartes down the path of defining *vis cognoscens* as a simple, indivisible substance really distinct from body. Many of the principal components of Descartes's dualism in *Meditations* can already be found in *Rules*.

This concludes my interpretation of the ontology of *vis cognoscens* as described in the above-cited passage from Rule 12. I will now discuss the role played by Descartes's theory of simple natures and the theory of conjunction in Descartes's dualism in *Rules*. The theory of simple natures and the theory of conjunction play indispensable roles here. Later in Rule 12, Descartes argues that the proposition "I understand, therefore I have a mind distinct from body [*intelligo, ergo mentem habeo a corpore distinctam*]" (AT 10:422, CSM 1:46; translation modified) is an example of a necessary conjunction (AT 10: 421, CSM 1: 45). Descartes's reference to "necessary conjunction" here can easily be overlooked, but it is very important. As we have seen in Chapter 7, Section 7.7, Descartes's theory of conjunction contains a theory of distinctions. The theory of conjunction provides Descartes with the resources he needs to assert that there is a real distinction (or the equivalent of a real distinction) between mind and body in *Rules*. We already know that whether a simple nature belongs to the class of intellectual simple natures or the class of material simple natures depends on whether intuiting the relevant simple nature requires the intellect alone or the intellect aided by the imagination. The intellectual simple natures are intuited "without the aid of any corporeal image" or "corporeal idea" in the imagination. Each of the intellectual simple natures (knowledge, doubt, ignorance, understanding, volition) names an act of the intellect or will. Descartes would later define these acts ontologically as "modes" of mind. The principal act—understanding—is, as previously mentioned, presupposed by all acts of sense-perception and imagination, whereas acts of understanding do not depend on either of the latter two faculties. This means that the intellect can be intuited entirely on its own. The intellectual simple natures constitute what Descartes would later refer to as the "complete idea" of mind: I can intuit the mind entirely on its own by means of the intellectual simple natures, excluding imagination and sense-perception (as well as the material simple natures).[20] Similarly, the material

simple natures include extension. Descartes would later define extension ontologically as the "principal attribute" of body, which enables one to intuit body entirely on its own. The other two material simple natures (shape and motion) are what he would later define as "modes" of extension. As I argued in Chapter 7, Section 7.7, the intuition of extension has priority over shape and motion: the latter two simple natures epistemically presuppose the first, but not vice versa. The material simple natures, then, constitute what Descartes would later refer to as the complete idea of body: I can intuit body entirely on its own, excluding the intellectual simple natures. The intellectual and material simple natures may be contingently conjoined (e.g., in a human being), but they are not necessarily conjoined, which means that I can "conceive […] them distinctly if I judge them to be separate from each other" (AT 10:421, CSM 1:45). I can do this because only my intellect is required to intuit the intellectual simple natures, whereas both my intellect and my imagination are required to intuit the material simple natures. The distinction between these two faculty configurations (intellect alone; intellect aided by the imagination) is what ultimately grounds the thesis that there is no necessary conjunction between the intellectual and material simple natures.

In both *Meditations* VI and *Principles* I. 60, Descartes similarly infers the real distinction between mind and body from the fact that the ideas of mind and body can be clearly and distinctly perceived or intuited entirely on their own, independently both of one another as well as the idea of any other thing. Thus, in *Meditations* VI, Descartes concludes his argument for mind–body dualism as follows:

> Hence the fact that I can clearly and distinctly understand one thing apart from another is enough to make me certain that the two things are distinct […]. I have a clear and distinct idea of myself, insofar as I am simply a thinking thing; and on the other hand I have a clear and distinct idea of body, insofar as it is simply an extended, non-thinking thing. And accordingly it is certain that I am really distinct from my body, and can exist without it (AT 7:78, CSM 2:54).

And in *Principles* I. 60, he writes:

> From the mere fact that each of us understands himself to be a thinking thing and is capable, in thought, of excluding from himself every other substance, whether thinking or extended, it is certain that each of us, regarded in this way, is really distinct from every other thinking substance and from every corporeal substance (AT 8A:29, CSM 1:213).[21]

The parallels between *Rules*, *Meditations*, and *Principles* are manifest. In all three texts, Descartes infers the real distinction between mind and body from the fact that he can clearly and distinctly perceive or understand each entirely on its own. Nevertheless, Descartes's dualism in *Rules* is not identical to his dualism in *Meditations* VI and *Principles* I. 60. Descartes does not express his dualism via an ontology of substance, attribute, and mode in *Rules*, whereas he does do so in *Meditations* VI and *Principles* I. 60. Descartes does not apply the category of substance to *vis cognoscens*, but rather describes it as a *power* or a *force*, which has diverse modes of activity, depending on whether and how it interacts with parts of the body.[22] The precise ontological sense of this "*vis*" remains undefined in *Rules*. In fact, there are two ontologically significant categories and concepts Descartes does not employ in his description of *vis cognoscens* in Rule 12: substance and soul. Why?

In Rule 6, Descartes emphasizes that the method "instructs us that all things can be arranged serially into various groups, not insofar as they can be referred to some ontological genus (such as the categories into which philosophers divide things), but insofar as some things can be known on the basis of others" (AT 10:381, CSM 1:21). Here, Descartes explicitly rejects Aristotle's categories—among which the most important is undoubtedly the category of substance—as relevant to the type of epistemic order he prescribes in *Rules*. Descartes rejects the "isolated natures of things" (ibid.) as defined by some combination of Aristotelian categories in favor of knowledge based on the combination of simple natures, which are not Aristotelian ontological categories. In *Rules*, Descartes regarded the category of substance as an *impediment* to his methodological project. In other words, he felt that the category of substance needed to be *rejected*, not *reconstructed*. This placed constraints on his ability to express, in an ontologically precise vocabulary, his mind–body dualism in *Rules*.

Descartes's reasons for not defining *vis cognoscens* as a soul in *Rules* are related to his reasons for not defining it as a substance, but different, since they depend on considerations about the relation between form and matter. *Vis cognoscens* is not the form of the body, at least not in any recognizably Aristotelian sense: *vis cognoscens* is not responsible for any of the body's physiological functions. As I mentioned earlier, Descartes passingly asserts that the mind "informs" the body in Rule 12, but he does not further determine what "informs" means there. In later texts, such as *Principles* IV.

89 (AT 8A:315, CSM 1:279), he means that the soul is not confined to one part of the body, but that, despite having its seat in the pineal gland, it is located everywhere in the body via the nerves, which connect the pineal gland to the body as a whole.[23] He certainly seems to have had a similar conception in Rule 12. If so, "informs" in Rule 12 does not mean what it would have meant to an Aristotelian. For most Aristotelians (including Aquinas, Suárez, and the Conimbricenses), the soul and the body are "partial" or "incomplete substances" that need one another (as form needs matter and vice versa) in order to compose a "complete" substance. The body needs the soul in order to perform its vital functions, and the soul needs the body in order to perform its own activities (including, crucially, intellection, since intellection requires a phantasm in the imagination, and phantasms are material).[24] The soul is not a complete substance, but rather a part of a substance: the form of a substance whose matter is the body.[25] The soul and the body together compose one substance, not two. The substance they compose is the "human being."[26] *Vis cognoscens* cannot be neatly placed into either slot of the form/matter distinction. Thus, it does not seem to me that Descartes embraced an Aristotelian definition of the mind as the "form" of the body in Rule 12.[27]

Vis cognoscens satisfies no definition of the soul Descartes would have been familiar with in *Rules*. Indeed, even had Descartes decided to describe *vis cognoscens* as a substance in *Rules*, he would have had some difficulty describing it as a soul, since, as we have seen, in scholastic Aristotelian theories of the soul, souls are *not* complete substances, but partial or incomplete substances (the form of the form-matter composite substance). Conversely, even had Descartes decided to describe *vis cognoscens* as a soul, he would have had some difficulty describing it as a substance, for the same reasons. To connect *vis cognoscens*, the concept of soul, and the category of substance to one another, Descartes would need to reconstruct the fundamental concepts of Aristotelian metaphysics, and he had not yet done so in *Rules*. His project in *Rules* is principally methodological, not metaphysical. Consequently, in *Rules* the soul is replaced by a *vis* whose properties Descartes happily, if somewhat briefly, enumerates$_3$, but whose ontological sense he does not fully clarify. Thus, if we adhere to the letter of *Rules* and do not read it in light of Descartes's later texts, it is clear that Descartes had definitively broken with Aristotelian theories of the soul on a

number of fundamental points, and it is also clear that he possessed the conceptual resources with which to articulate his position (the theory of simple natures). Nevertheless, the precise ontological sense of his position remained unclear, due to his rejection of the category of substance and its associated categories in *Rules*.

8.3 Dualism in *Rules*$_{\text{CM}}$

As I mentioned in Section 8.1, in Rule 12$_{\text{CM}}$ Descartes does not describe *vis cognoscens* as a "purely spiritual" force "distinct from the whole body." Rather, he writes: "The force by which we know things should be conceived as something in us no less distinct from the phantasy than is the eye, or the hand" [*Concipiendum est vim illam, per quam res cognoscimus esse aliquid in nobis [a] phantasia non minus distinct[a]m quam sit oculus, vel manus*] (*Rules*$_{\text{CM}}$ fo. 16$^{\text{v}}$). Here, Descartes describes *vis cognoscens* as distinct from the phantasy, but not from the body as a whole. This leaves open the possibility that *vis cognoscens* is a part of the body; the phrase "no less distinct from the phantasy than is the eye, or the hand" leaves open the possibility that *vis cognoscens* is distinct from the phantasy as one body part is distinct from another. Without the expressions "purely spiritual" and "distinct from the whole body," there is no obvious reason to read the distinction between *vis cognoscens* and the phantasy dualistically. Nor, however, is there any special reason to read the distinction between *vis cognoscens* and the phantasy materialistically. *Vis cognoscens* can be distinct from the phantasy in one of two ways: either because it is part of the body, or because it is not. To be sure, Descartes does not endorse the latter possibility. But he does leave it open. He also does not endorse the former possibility.

One must, therefore, look elsewhere in order to determine which of these two possibilities Descartes endorsed. In the same paragraph in Rule 12$_{\text{CM}}$, Descartes writes (as he will in Rule 12$_{\text{AT}}$) that the "cognitive force is sometimes active, sometimes passive; sometimes resembling the seal, sometimes the wax. But this should be understood merely as an analogy. Nothing quite like this force is to be found in corporeal things [*vis cognoscens interdum patitur, interdum agit, et modo sigillum, modo ceram*

imitator, quod tamen par analogiam tantum hic est sumendum. Neque enim in rebus corporeis aliquid omnino huic simile invenitur…]” (*Rules*$_{CM}$ fo. 16^{v}; cf. AT 10:415–416, CSM 1:42). Descartes does not explicitly affirm the “pure spirituality” of *vis cognoscens* here, but it seems clear that a force that cannot be found in corporeal things, and whose mode of action and passion cannot be literally analogized to any corporeal process, is incorporeal. As I argued in Section 8.2, in this passage Descartes is effectively claiming that *vis cognoscens* is neither caused by nor inheres in any corporeal entity. In other words, *vis cognoscens* enjoys per se existence. In fact, every other relevant feature of the passage in Rule 12$_{AT}$ can be found in Rule 12$_{CM}$:

> It is one single force [*unicamque esse*], whether it receives figures from the common sense at the same time as does the phantasy, or applies itself to those which are preserved in the memory, or forms new ones which so preoccupy it that it is often in no position to receive ideas from the common sense at the same time, or to transmit them to the power responsible for motion in accordance with a purely corporeal mode of operation. In all these functions the cognitive force [*vis cognoscens*] is sometimes passive, sometimes active [*interdum patitur, interdum agit*]; sometimes resembling the seal, sometimes the wax. But this should be understood merely as an analogy. Nothing quite like this force is to be found in corporeal things. It is one and the same force [*una et eadem est vis*], when applying itself along with imagination to the common sense, it is said to see, touch, etc.; when applying itself to the imagination alone, insofar as the latter is invested with various figures, it is said to remember; when applying itself to the imagination in order to form new figures, it is said to imagine or conceive; and lastly, when it acts on its own, it is said to understand [*intelligere*]. How understanding comes about I shall explain at greater length in another place. According to its different functions [*functiones*], then, the same force is called either pure intellect, or imagination, or memory, or sense-perception. (*Rules*$_{CM}$ fo. 16^{V}; cf. AT 10:415–16, CSM 1:42–3).[28]

In both *Rules*$_{AT}$ and *Rules*$_{CM}$, Descartes enumerates$_{3}$ four properties of *vis cognoscens*, each of which distinguishes it from body: (1) *vis cognoscens*’s simplicity or indivisibility; (2) its ability to be both active and passive; (3) its per se existence; and (4) its independence, qua “pure intellect,” from all other (corporeal) cognitive functions (see Section 8.2). Thus, it seems that Descartes did indeed embrace mind–body dualism in *Rules*$_{CM.}$ Nevertheless, a major difference between *Rules*$_{CM}$ and *Rules*$_{AT}$ remains. Rule 12$_{CM}$ contains no theory of simple natures, and so no theory of conjunction. Descartes’s solution to the problem of the limits of knowledge, while clearly stated in Rules 8$_{CM}$ and 12$_{CM}$, comes to an abrupt halt

immediately after he defines *ingenium* in Rule 12. The distinction between the intellectual and material simple natures is the foundation of Descartes's dualism in *Rules*$_{AT}$, but Descartes's dualism in Rule 12$_{CM}$ rests on no such foundation. Without the theory of simple natures and the theory of conjunction, many of the parallels between Rule 12 and Descartes's dualism in *Meditations* VI and *Principles* I. 60 are lost. The absence of the theory of simple natures and the theory of conjunction in *Rules*$_{CM}$ renders the type of distinction between mind and body Descartes endorses in Rule 12$_{CM}$ much more difficult to define. Since the theory of simple natures and the theory of conjunction are absent in *Rules*$_{CM}$, these theories cannot play a role in interpretations of Descartes's dualism in *Rules*$_{CM}$.

Is it, then, really appropriate to claim that Descartes's description of *vis cognoscens* in Rule 12$_{CM}$ constitutes an endorsement of mind–body dualism? Yes. Despite the fact that Descartes does not describe *vis cognoscens* as a "purely spiritual" force "distinct from the whole body" in Rule12$_{CM}$, the remainder of his description of *vis cognoscens* in Rule 12$_{CM}$ is *identical* to the parallel description of *vis cognoscens* in Rule 12$_{AT}$ analyzed in Section 8.2. Descartes's description of *vis cognoscens* in Rule 12$_{CM}$ suffices to demonstrate that Descartes embraced mind–body dualism in *Rules*$_{CM}$, even if he had not yet found a way to express it more clearly via the theory of simple natures and the theory of conjunction. Thus, Descartes's dualism, while certainly less developed in *Rules*$_{CM}$, is nevertheless clearly asserted. The discovery of *Rules*$_{CM}$ does not testify to the existence of a Descartes "before dualism," but rather to Descartes's earliest known endorsement of dualism, before even the theory of simple natures, let alone Descartes's ontology of substance, attribute, and mode. When Descartes writes in Rule 12$_{CM}$ that "the force through which we know things should be conceived as something in us no less distinct from the phantasy than is the eye, or the hand," he means to assert that *the intellect is distinct from the only corporeal faculty with which it would have made any sense to associate the operations of the intellect* in early seventeenth-century Aristotelianism: the phantasy or imagination.

The only remaining question, then, is what factors led Descartes to endorse mind–body dualism in *Rules*$_{CM}$? Prior to *Rules*$_{CM}$, Descartes was

firmly committed to the thesis that the imagination must aid the intellect *even when the latter understands non-corporeal or spiritual things*. Indeed, prior to *Rules*$_{CM}$, Descartes did not clearly distinguish between the intellect and the imagination.[29] The very idea that the intellect can act (i.e., understand) on its own, without a phantasm in the imagination, was deemed naturally impossible in early seventeenth-century Aristotelianism, and was definitively ruled out by Aristotle in *De anima* III, 429^{a}1. "It is impossible," Aquinas would later argue on Aristotelian grounds, "for our intellect to understand anything actually, except by turning to the phantasms" located in the imagination.[30] For Aquinas (and many other scholastics), the intellect is an incorporeal part of the soul, but it cannot understand incorporeal objects on its own without a phantasm because "the proper object of the human intellect, which is united to a body, is the quiddity or nature existing in corporeal matter."[31] A quiddity or nature exists only in matter and "cannot be apart from corporeal matter." This is why the proper act of the intellect—understanding—requires a phantasm: "For the intellect to understand actually its proper object, it must of necessity turn to the phantasms in order to perceive the universal nature existing in the individual."[32] The intellect cannot understand any substance—material or immaterial—without turning to phantasms. Regarding knowledge of immaterial substances, Aquinas writes: "[In] Aristotle's opinion, which experience corroborates, our intellect in its present state of life has a natural relation to the natures of material things; and therefore it can understand only by turning to the phantasms, as we have said above. Thus it clearly appears that immaterial substances, which do not fall under sense and imagination, cannot be known by us first and essentially, according to the mode of knowledge of which we have experience."[33] These are clearly not the conditions in which the intellect operates in *Rules*. In both *Rules*$_{AT}$ and *Rules*$_{CM}$, cognition is not a process whereby intelligible species are abstracted from material things via the intermediary of phantasms in the imagination, but rather a process that consists in the transmission of figures from one part of the body to another. Understanding has no need for phantasms: the elimination of intelligible species means that phantasms have no function to perform as material carriers of such species. Instead, motions in the brain cause sensory representations in *vis cognoscens*. Descartes's elimination of Aristotelian species in *Rules* explains why he

was no longer beholden to the thesis that the intellect understands nothing without a phantasm in the imagination.

The autonomy of the intellect, explicitly asserted for the first time in Descartes's career in Rule 12_{CM}, therefore constituted a radical discovery: the intellect need not be separated from the body in order to act wholly on its own; the intellect could and, indeed, should understand incorporeal things without the help of the imagination even in this life. The prospect of understanding incorporeal objects incorporeally, without phantasms in the imagination, was recognized by seventeenth-century Aristotelians as an incomparably superior, but naturally impossible mode of cognizing incorporeal things, available to human beings only after the separation of the soul and the body by God after death. Descartes discovered that there is no need to wait for the afterlife or to reserve such acts for God and the angels. Trained by the method, human intellects can understand incorporeal things without the imagination *hinc et nunc*.

This is precisely what Descartes would later assert far more forcefully in *Rules*$_{AT}$, when he finally returned to complete his solution to the problem of the limits of knowledge. As we have seen, Descartes left this problem unsolved in Rule 12_{CM}, which ends immediately after his definition of *ingenium*, and moves immediately into the mathematical illustration of the method in Rules 13–16_{CM}, where the manuscript ends. After *Rules*$_{CM}$, Descartes describes the problem of the limits of knowledge as the "first problem of all that should be examined by means of the Rules described above" in *Rules*$_{AT}$ (AT 10:398, CSM 1:31). He could no longer avoid offering a fully general theory of "the actual things it is possible to know," i.e., a theory of simple natures. The only simple natures Descartes actually discussed in *Rules*$_{CM}$—the material simple natures, discussed in Rule 14_{CM}—must be intuited by the intellect aided by the imagination, and are only employed in mathematics and natural philosophy. The problem of the limits of knowledge, however, is the most general problem Descartes addresses in *Rules*, and its generality is such that it requires an enumeration$_3$ of *all* possible objects of intuition. Reflection on the operations of the intellect yielded the intellectual simple natures, each of which denominates an act of the pure intellect. Consequently, the distinction between the intellectual and material simple natures was officially baptized in Rule 12_{AT}. The faculty configurations required for intuiting each class (intellect; intellect aided by

the imagination) also became clear. Finally, now that the theory of simple natures was on the table as an explicit area of investigation, Descartes explored the connections between the simple natures and articulated these connections via a theory of conjunction. This enabled him to introduce the proposition "I understand, therefore I have a mind distinct from body" as an example of a necessary conjunction. All of these developments show up in *Rules*$_{AT}$. Descartes considered his task complete: he had enumerated$_3$ all of the objects of knowledge (as he had not in *Rules*$_{CM}$) and the problem of the limits of knowledge was solved in a way that was consistent with the rest of the treatise. He modified his comparatively weaker assertion of dualism in Rule 12$_{CM}$ by including the expressions "purely spiritual" and "distinct from the whole body" in his description of Rule 12$_{AT}$, for in the interim he had enumerated$_3$ the intellectual simple natures and laid out a theory of conjunction in which the mutual exclusion of the intellectual and material simple natures could be fully expressed.

The development of *Rules* between *Rules*$_{CM}$ and *Rules*$_{AT}$ reveals that the conquest of the theory of simple natures and the theory of conjunction discussed in Chapter 7 constituted an essential step toward a more robust, but early conceptual expression of Cartesian dualism. For Descartes's dualism to achieve a more canonical form, Descartes would have to (1) redefine the category of substance, not as a form/matter composite, but rather as the subject of a rationally distinct principal attribute; (2) demonstrate that the mind or the soul is such a substance. This may have occurred as early as late 1629, in the "little treatise on Metaphysics," which Descartes began while in Friesland, and in which he "set out principally to prove *the existence of God and of our souls* when they are separate from the body, from which their immortality follows" (letter to Mersenne, November 25, 1630, AT 1:182, CSMK 3:29; see also letter to Gibieuf, July 18, 1629, AT 1:17, CSMK 3:5). The first public presentation of Descartes's dualism would have to wait until *Discourse on Method* (1637), and even then, Descartes's dualism would require a more detailed elaboration in *Meditations* (1641) and *Principles* (1644). While the term "*vis*" does not disappear from Descartes's metaphysical lexicon after *Rules*, it is largely replaced by the term "*substantia,*" which more clearly expresses its *per se* existence (and, therefore, its immortality). Descartes's decision to include the intellectual simple natures in *Rules*$_{AT}$ indicates that he had begun

crossing over into metaphysics for the first time in his career in *Rules*. But it was only a beginning. Even after they make their appearance in *Rules*$_{AT}$, the intellectual simple natures, while enumerated, are not systematically explored, at least not in *Rules*. Where they would fit into Descartes's developing system remained an open question. That they would remain enduring parts of his system nobody can deny. The simple natures reappear in revised forms either implicitly or explicitly in *Meditations*, *Principles*, and in Descartes's correspondence with Elizabeth in the 1640s (see AT 8A:22–4, CSM 1:208–10; letter to Elizabeth, May 21, 1643, AT 3:665–8, CSMK 3: 217–20).[34]

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0009

[1] Beck 1952, 283–4; and Schuster 2013, 315; and Rozemond 2014, 237 assume that Descartes embraced mind–body dualism in *Rules*, but it is not obvious that he did and others have denied it. See Alquié 1950, 72–3 and, later, Hoffman 1986, 351 and Clarke 2003, 200. Marion 1975, 126 remains agnostic.

[2] See AT 10:411, CSM 1:40. Descartes continued to describe the mind as a "substantial form" even after *Meditations*, in his letter to Regius in January 1642 and in *Principles* IV. 189 (AT 3:505; CSMK 3:208): "…the soul, which is the true substantial form of man." AT 8A:315, CSM 1:279: "…the human soul…informing the entire body."). For a hylomorphic interpretation of these passages, see Hoffman 1986, 349–69. For a dualist interpretation of these passages, see Rozemond 1998, 139–72. The mere use of the term "form" does not by itself make Descartes an Aristotelian about the relation between the soul and the body, neither in *Rules* nor anywhere else. On the contrary, it raises the question of whether there is a distinctively "Cartesian" sense of the term "form" when Descartes employs it in these contexts.

[3] Richard Serjeanston (2013) first raised the question of whether Descartes embraced mind–body dualism in *Rules*$_{CM}$ in an unpublished paper delivered in 2013 at the Centre for Research in the Arts, Social Sciences, and Humanities (CRASSH) at Cambridge University, "Descartes Before Dualism: New Evidence." Garber 2015, 6 claims that the absence of "apparent reference to Descartes's dualism" in *Rules*$_{CM}$"suggests that the Cambridge manuscript may have been written before Descartes came to be a dualist."

[4] See Appendix.

[5] For hylomorphic interpretations, see Alquié 1950, 72–3; Hoffman 1986, 351.

[6] On Descartes's theory of conjunction in *Rules* and its relation to his theory of distinctions, see Chapter 7, Section 7.7.

[7] For definitions of Descartes's dualism, see Rozemond 1998, 1–38 and Baker and Morris 1996, 11–23, 59–69.

[8] On sufficient enumeration, see Chapter 4, Section 4.4 and Chapter 5, Section 5.2.

[9] In *Fourth Replies*, Descartes argues that the hand, when considered on its own and not referred to the body, is a complete substance: "Thus a hand is an incomplete substance when it is referred to the whole body of which it is a part; but it is a complete substance when it is considered on its own" (AT 7:222, CSM 2:157).

[10] See *De anima* in Aristotle 1984, 1:656 (412^{a}16–17), where Aristotle argues that "every natural body which has life in it is a substance in the sense of a composite [*συνθέτη*]" of matter and form. See also Aquinas 1882–, 5:200–1 (*ST* I, q. 75, art. 4); trans. Aquinas 1945, 1:688. See also Pasnau in Aquinas 2002, xvii and Pasnau 2011, 99–102.

[11] See, e.g., Aquinas 1945, 1:734 (*ST* I, q. 78, art. 1): "There are five genera among powers of the soul," which Aquinas previously enumerated as the vegetative, the sensitive, the appetitive, the locomotive, and the intellectual. "Of these," he continues, "three are called souls," including the intellectual. See also Aquinas 1882–, 5:202 (*ST* I, q. 75, art. 5); trans. Aquinas 1945, 1:690: "The intellectual soul itself is an absolute form, and not something composed of matter and form [*Anima igitur intellectiva est forma absoluta, non autem aliquid compositum ex materia et forma*]." Here, "intellectual soul" refers to the intellectual *part* of the soul. For Ockham, the intellectual soul is not a part of the soul, but rather a soul of its own alongside other really distinct souls (the corporeal and sensitive souls). See Ockham 1967–1988, 9:62–6 (*Quodlibeta* VII, quod. 1, q. 10) and the discussion in Adams 1987, 633–71. For other references, see Rozemond 1998, 38–64.

[12] See, e.g., Aquinas 1882–, 5:196 (*ST* I, q. 75, art. 2); trans. Aquinas 1945, 1:685: "The intellectual principle, which we call the mind or intellect, has essentially an operation in which the body does not share [*Ipsum igitur intellectuale principium, quod dicitur mens vel intellectus, habet operationem per se, cui non communicat corpus*]." The synonymy of "mind" and "intellect" is very common amongst scholastic Aristotelians. For further references, see Gilson 1979, 93, 179–80.

[13] See, e.g., Conimbricenses [1598] 1604, 145 (*In De anima*, lib. 2, cap. 3, q. 2, art. 1): "*Priori parti controversiae respondemus quinque esse primas, et generales potentias rerum viventium, nimirum vim vegetandi, sentiendi, loco movuendi, appetendi, intelligendi*." This division is very common (see n. 11 above). The Conimbricenses rarely use the expression "*vis cognoscens*," but when they do, it functions as a generic term for any cognitive faculty (sensitive or intellectual). See, e.g., Conimbricenses [1598] 1604, 533 (*In tres libros De anima Aristotelis Stagiritae*, lib. 3, cap. 13, q. 1, art. 1); ibid., 547 (lib. 3, cap. 13, q. 3, art. 1).

[14] See also *Meditations* VI, where Descartes argues that "the faculties of willing, understanding, of sense-perception and so on, these cannot be called parts of the mind, since it is one and the same mind that wills, and understands and has sensory perceptions" (AT 7:86, CSM 2:59). He takes the indivisibility of the mind to confirm his thesis that it is really distinct from body (see AT 7:85–6, CSM 2:59).

[15] See also Rozemond 2014, 237, who interprets the relevant passages in Rule 12 along similar lines.

[16] For more detailed discussion of *vis cognoscens*'s regulation of the cognitive mechanism, see Chapter 6, Section 6.4.

[17] See AT 10:412, CSM 1:40.

[18] See Aquinas: "*Est autem substantia* ens per se existens […]," cited in Gilson 1979, 275. Suárez 1856–1878, 26:313 (*DM*, disp. 32, sec. 1.5). "…*ens autem per se constituit substantiam.*" Eustachius 1609, 96: "*Substantia in genere definiri potest: Ens subsistens seu per se existens*," cited in Gilson 1979, 277. For an overview of scholastic definitions of substance, see Pasnau 2011, 99–115.

[19] Clarke 2003, 200 denies that the pure intellect can act independently of "brain activity" in *Rules*. However, Descartes explicitly argues that what distinguishes the pure intellect from all other cognitive functions is the fact that it can act on its own (*sola agat*, AT 10:416, CSM 1:42). Not only the *content* of the thought (as Clarke alleges), but also the *act* does not depend on brain activity. This is consistent with Descartes's position in *Fifth Replies*, where he explicitly argues that the "mind can operate independently of the brain; for the brain cannot in any way be employed in pure understanding, but only in imagining or perceiving by the senses" (AT 7:358, CSM 2:248). See also Des Chene 2006, 315–40 and Cottingham 2017, 46.

[20] In *First Replies*, Descartes argues that a real distinction only obtains when the relevant ideas are ideas of "complete things," such that no other ideas are needed to clearly and distinctly perceive them. See AT 7:121, CSM 2:86. See also *Fourth Replies*, AT 7:222, CSM 2:156 and the letter to Gibieuf of January 19, 1642, AT 3:475, CSMK 3:202.

[21] "Exclusion" is a technical term for Descartes, and he distinguishes between exclusion and mere abstraction. In an important letter to Gibieuf, he writes: "Intellectual abstraction consists in turning my thought away from one part of the contents of [a] richer idea the better to apply it to the other part with greater attention. Thus, when I consider a shape without thinking of the substance or the extension whose shape it is, I make a mental abstraction. I can easily recognize this abstraction afterwards when I look to see whether I have derived this idea of the shape on its own from some other, richer idea which I also have within myself, to which it is joined in such a way that although one can think of the one without thinking of the other, it is impossible to deny one of the other when one thinks of both together" (January 19, 1642, AT 3:474–5; CSMK 3:202). Exclusion or denial involves judgment (i.e., the assertion of a proposition of the form "Fa &~Ga"). Abstraction, by contrast, does not involve judgment, but merely a consideration of one part of a richer idea (e.g., "Fa") on its own. A merely abstract consideration of mind apart from body does not entail that they are really distinct substances; I must conceive each "entirely on its own, and deny of it everything else of which I have an idea," and I must do this when considering both simultaneously in one judgment. The real distinction between mind and body in *Meditations* VI and *Principles* I. 60 depends on exclusion, not abstraction. Similarly, in *Rules*, contingent conjunction requires that I "judge them [the relevant simple natures] to be separate from each other." See Chapter 7, Section 7.7 above. See also Murdoch 1993, 39–40 and Rozemond 1998, 17–18.

[22] One should not make too much of the fact that Descartes describes *vis cognoscens* as a "force" in *Rules*, but refers to mind as a "substance" in *Meditations* VI and *Principles* I. 60. In the case of mind, Descartes did not regard the difference between "force" and "substance" as radical. In a letter to More (February 5, 1649), Descartes writes that "no incorporeal substances are in any strict sense extended. I conceive them as sorts of powers or forces [*virtutes aut vires quasdam*], which although they can act upon [*se applicent*] extended things, are not themselves extended" (AT 5:270, CSMK 3:361). Thinking substance is a force, and since thinking is the principal attribute of mind, force constitutes its essence as a substance.

[23] For more discussion, see Chapter 11, Section 11.5.

[24] The thesis that the soul is not a complete substance can be found in Aquinas 1882, 5:196–9 (*ST* I, q. 75, art. 2), trans. Aquinas 1945, 1:685; Suárez 1856–1878, 25:499 (*DM*, disp. 25, sec. 1.6), trans. Suárez 2000, 20; Conimbricenses [1598] 1604:75–6 (*In De anima*, lib. 2, cap. 1, q. 6, art. 2). As Suárez puts it: "Hence, this soul is a substantial form because […] the term 'substantial form' signifies nothing other than a certain partial substance which can be united to matter in such a way that it composes with it a substance that is whole and essentially one, such as a human being" (Suárez 1856–1878, 25:499 (DM, disp. 25, sec. 1.6), trans. Suárez 2000, 20). The thesis that intellection (or understanding) requires phantasms in the imagination can be found in Aquinas 1882–, 5:325–8, trans. Aquinas 1945, 1:808–10 (*ST* I, q. 84, art. 7) and Conimbricenses [1598] 1604:423–4 (*In De

anima, lib. 3, cap. 5, q. 2, art. 1). Even in the case of Suárez, for whom phantasms do not causally interact with the intellect, the soul still requires phantasms to serve as a "model" or "exemplar" by which to produce, via its own spiritual power, intelligible species in the passive intellect. See South 2012, 133. Note that while the intellect needs the body in order to operate in this life, it can operate without the body after death. For more discussion, see Chapter 3, Section 3.2.1.

25 The thesis that the soul is the form of the body was widespread and can be found in Aquinas 1882–, 5:194–235 (*ST* I, q. 75–6), trans. Aquinas 1945, 1:682–719; Suárez 1856–1878, 3:467–71 (*De anima*, lib. 1.1) and Suárez 1856–1878, 25:499 (DM, disp. 25, sec. 1.6), trans. Suárez 2000, 20; Conimbricenses [1598] 1604:100–3 (*In De anima*, lib. 2, cap. 1, q. 6, art. 2), among many others. All of these texts draw on Aristotle's *De anima*: Aristotle 1984, 1:656 ($412^{a}20$). For an historical reconstruction of late Aristotelian theories of soul, see Des Chene 2000.

26 See Aquinas 1882–, 5:200–1 (*ST* I, q. 75, art. 4), trans. Aquinas 1945, 1:687–9; Suárez 1856–1878, 25:499 (DM, disp.25, sec.1.6), trans. Suárez 2000, 20; Conimbricenses [1598] 1604:76 (*In De anima*, lib. 2, cap. 1, q. 6, art. 2).

27 *Pace* Hoffman 1986, 351. For a similar criticism, see Rozemond 1998, 139–72, esp. 152.

28 This is the only passage in *Rules*$_{\mathrm{CM}}$ relevant to Descartes's dualism, and it suffices to establish that Descartes embraced mind–body dualism there. The other passages discussed in Sections 8.1–8.2 are absent.

29 As Baillet notes in his description of the contents of *Studium bonae mentis* (c. 1619–23): "*Il [Descartes] semblait douter que la mémoire fût distinguée de l'entendement et de l'imagination*" (Descartes 2013, 134). If memory cannot be distinguished from the intellect or the imagination, then it is not clear that the imagination and the intellect can be distinguished from one another. Carraud and Olivio rightly infer that "*Descartes n'était pas davantage en mesure de distinguer l'entendement de l'imagination*" in *Studium bonae mentis* (ibid., 150, n. 45). Similarly, in *Olympian Matters* (c. 1619–1620), Descartes anchors the intellect in the imagination: "Just as the imagination employs figures in order to conceive of bodies, so, *in order to frame ideas of spiritual things, the intellect makes use of certain bodies which are perceived through the senses*, such as wind and light." Elsewhere in the same notebook, he writes: "Man has knowledge of natural things only through their resemblance to the things which come under the senses" (AT 10:217–19, CSM 1:4–5; my emphasis). Thus, the broadly Aristotelian thesis that the intellect cannot understand anything without a phantasm in the imagination is a thesis Descartes remained very much committed to in the early 1620s. For a thorough discussion of Descartes's development during this period, see Sepper 1996, 2013.

30 Aquinas 1882–, 5:325–8 (*ST* I, q. 84, art. 7); trans. Aquinas 1945, 1:808. For Suárez and the Conimbricenses, see n. 24 above. For some exceptions in the case of the intuitive cognition of one's own mental acts without the intermediary of phantasms, see my discussion in Chapter 3, Section 3.2.1.

31 Ibid.

32 Ibid.

33 Aquinas 1882–, 5:364–6 (*ST* I, q. 88, art. 1); trans. Aquinas 1945, 1:844.

34 On the role played by the simple natures in *Meditations*, see, e.g., Marion 1999 and Kambouchner 2005. See also Chapter 11, Section 11.5.

PART IV

APPLICATIONS

Perfectly and Imperfectly Understood Problems

9

Perfectly Understood Problems

Method and Mathematics in Rules 13–21

9.1 The Cartesian Scientific *Habitus* in Mathematics

The Cartesian scientific *habitus* has acquired three degrees of perfection since its production in the human *ingenium* via practice in the so-called "feminine arts" and recreational mathematics (see Chapter 3, Section 3.6). From these arts and sciences, the human *ingenium* acquires the first degree of perspicacity and sagacity, the intellectual virtues or *habitus* that perfect intuition and deduction, respectively. Practice in these arts and sciences produces the ability to abstract problems from subject-matters; enumerate$_1$ all and only those conditions relevant to the solution of a problem (reduce problems to their simplest component parts); and deduce the solution. In *mathesis universalis*, these intellectual virtues or *habitus* acquire their second degree of perfection (see Chapter 4). The human *ingenium* perfects its ability to deduce the unknown from the known via practice in finding continuous and mean proportionals (e.g., 3:6 = x:12). Furthermore, by "sagaciously reflecting" on solutions to problems in *mathesis universalis*, the human *ingenium* also produces an enumeration$_3$ or classification of problems as well as the procedures whereby they may be solved. Finally, in the solution to the problem of the limits of knowledge—the first problem of all that should be solved by means of the method—the human *ingenium* measures the limits of its ability to solve problems (see Chapters 5–8). The solution to the problem of the limits of knowledge yields a sufficient enumeration$_{2–3}$ of the faculties and the objects of science (the simple

natures) as well as what I have termed an habitual theory of the faculties, which prescribes how the faculties must be employed in order to intuit the simple natures. Any problem about any object that is reducible to simple natures can be solved by means of the Cartesian scientific *habitus* (see Chapters 5–8). In short, the human *ingenium* has finally acquired the scientific *habitus* needed to solve particular problems in "the somewhat more advanced sciences" (Rule 4, AT 10:379, CSM 1:20), beginning with perfectly understood problems (discussed in Chapter 3, Section 3.5), which are less complex than imperfectly understood problems, and which typically arise in mathematics.

In this chapter, I show how Descartes's requirement that the material simple natures be intuited by the intellect aided by the imagination determines the activity of the Cartesian scientific *habitus* in mathematics. Descartes introduces several problem-solving techniques in Rules 13–21. Sections 9.2–9.3 discuss the reduction of problems about subject-matters to problems about relations between pure magnitudes (Rules 13–14). Section 9.4 discusses the representation of these magnitudes by means of the material simple nature of figure alone via the unit (Rule 15). Section 9.5 discusses Descartes's algebraic notation, which symbolically represents relations between pure magnitudes (Rule 16). Sections 9.6–9.7 discuss Descartes's geometrical interpretation of addition, subtraction, multiplication, division, and root extraction (the "geometrical calculus" in Rule 18). These techniques are all based on the intellect aided by the imagination, and are meant to preserve the possibility of intuition at every step of the solution to any mathematical problem. In mathematics, intuition requires *the geometrical construction of a figure that embodies the relevant mathematical operations and satisfies the solution conditions expressed in the problem.* As we will see in more detail in Sections 9.7–9.8, it is precisely here that Descartes's method in *Rules* encounters a number of problems that lead him to end the treatise prematurely at Rule 21. The main problem lies in his geometrical interpretation of root extraction in the case of incommensurate continuous magnitudes. Extracting roots in this case requires the use of the circle, which Descartes *excludes* from his geometrical calculus in *Rules* because it has no use in the case of the extraction of the roots of commensurate magnitudes (both discrete and continuous), and so is not universal. In the end, Descartes's geometrical calculus of lines and rectangles in *Rules* proves too limited to embrace the

operations required to solve even the simplest mathematical problems that require root extraction. In the *Geometry*, Descartes has no difficulty introducing the circle, since his project in the *Geometry* is purely geometrical. Root extraction in the *Geometry* pertains exclusively to *continuous magnitudes*. In *Rules*, however, one and the same procedure in the imagination must serve as the basis of root extraction *irrespective of the species of magnitude.*[1]

9.2 The Unity of Discrete and Continuous Magnitude in the Imagination

In *Rules*, the object of mathematics is simply the material simple nature of extension as intuited by the intellect aided by the imagination (see Rule 14, AT 10:447, CSM 1:62). Even arithmetic, which deals with discrete magnitude (number), really only deals with extension. As we have seen in Chapter 7, Section 7.4, Descartes's habitual theory of the faculties and theory of conjunction/distinctions in *Rules* defines an implicit "ontology of mathematics" in which there can be no real distinction between number and things numbered in the imagination (see AT 10:446, CSM 1:61). Number is not the object of a science distinct from geometry, but rather *a symbol that represents continuous magnitudes insofar as they are measurable by a unit*; number is simply *commensurate* continuous magnitude. Consequently, in contrast to the Aristotelian view of many scholastics, the distinction between discrete and continuous magnitude in *Rules* does not entail a distinction between two heterogeneous mathematical sciences (one mathematical science per species of magnitude). On the contrary, as Descartes insists in Rule 14, "nothing can be ascribed to magnitude in general that cannot also be ascribed to any species of magnitude" (AT 10:440, CSM 1:58). Further evidence that Descartes seeks to transcend the distinction between discrete and continuous magnitude can be found in his reflections on *mathesis universalis* in Rule 4, which clearly indicate that what interests him is not the different subject-matters of the branches of mathematics, but rather the reason why arithmetic, geometry, and the other mathematical sciences "are called branches of mathematics." His answer is "order and measure." Order and measure are not objects, but rather meta-

objective "respects" (*respectus*) in which both discrete and continuous magnitudes can be considered by the mind. Both arithmetic and geometry deal with perfectly understood problems about order and measure, and the fact that the former deals with discrete magnitude while the latter deals with continuous magnitude—a crucial difference for any theory of mathematics according to which discrete and continuous magnitude are irreducibly distinct "species" of "quantity"—has zero significance, at least in principle. Thus, following his own prescription in Rule 1 not to "[distinguish] the sciences by the differences in their objects" as understood in Aristotelian theories of science (AT 10:360, CSM 1:9), Descartes unifies arithmetic and geometry by undermining the distinction between discrete and continuous magnitude. The unity of mathematics in *Rules* is defined by the material simple natures (extension and figure); the *respectus* in terms of which the mind considers extension and figure (order and measure); the types of problems mathematics deals with (perfectly understood problems); and the types of procedures whereby these problems may be solved (directly or indirectly).[2] For Descartes, arithmetic, geometry, and the remaining mathematical sciences that depend on them do not differ in any of these regards (see Table 9.1).

Table 9.1 The unity of mathematics in *Rules*

Sciences: Arithmetic and geometry
Object: Material simple natures (extension and figure)
Faculties: Intellect aided by imagination
Respectus: Order and measure
Problem: Perfectly understood
Solution: Direct/indirect

9.3 Abstracting Problems from Particular Subject-Matters

The title of Rule 13 condenses some of the lessons learned from recreational mathematics and *mathesis universalis*, discussed in Chapters 3–4: "If we perfectly understand a problem we must abstract it from every superfluous conception, reduce it to its simplest terms and, by means of an

enumeration[1], divide it up into the smallest possible parts" (Rule 13, AT 10:430, CSM 1:51). Abstracting a problem "from every superfluous conception" requires disregarding the particular subject-matter in which the problem is expressed so that the problem may be perspicuously defined in terms of relations between magnitudes alone. In Chapter 3, Section 3.4.1.1 and Section 3.6 and Chapter 4, Section 4.6, I discussed an example in recreational mathematics (the Tantalus cup) in which the solution depends on abstracting the problem from the subject-matter and defining it as follows: "How must the bowl be constructed if it lets out all the water as soon as, but not before, it reaches a certain height?" (AT 10:436, CSM 1:55). Descartes briefly discusses two other examples in Rule 13: the "nature of the magnet" and the "nature of sound." He does not aim to solve either problem, but rather to show how the problem should be abstracted from the particular subject-matter in which it is expressed. Abstracting "from every superfluous conception" here means (1) disregarding the fact that in the former case we are dealing with a magnet and in the latter case that we are dealing with sound, and (2) reducing the problems "to such a form that we are no longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparisons [i.e., proportional relations] between them" (AT 10:431, CSM 1:52). For Descartes, the difference between these subject-matters is irrelevant: solutions to both problems can be deduced from extension, shape, and motion: the material simple natures. Thus, abstracting these problems from their respective subject-matters is the first operation one must perform in order to solve either problem, whereas retaining their subject-matters constitutes a failure to distinguish between conditions that are relevant to the solution of the problem and conditions that are not relevant, such that the problem is not clear and distinct.[3]

The abstraction of problems from subject-matters permits one to reduce all perfectly understood problems to problems in which the solution consists in finding one or more magnitudes on the basis of its relation other magnitudes:

> Let us then take it as firmly settled that perfectly determinate problems [*quaestiones perfecte determinatas*] present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures. Accordingly, we shall dismiss everything else from our thoughts and deal exclusively with these until we reach Rule 25 (AT 10:441, CSM 1:58).

In perfectly understood problems, the relation between the relevant magnitudes can be exactly stated as a proportion or equation. As I showed in Chapter 4, Section 4.6, proportions and equations are convertible (e.g., $(3:x = x:12) = (x^2 = 12 \cdot 3)$, etc.). In all such problems, the relation between the knowns and the unknowns is exact because the unknown (the mean or continuous proportional) is related to the known by a definite measure. All perfectly understood problems can be solved as problems in *mathesis universalis* are solved. No matter how obscure their subject-matter, all problems can be abstracted from their subject-matter and rendered perspicuous to intuition as relations between pure magnitudes. Vague differences in degree between phenomena that are difficult to discern by means of the senses can always be represented as proportional relations between magnitudes and examined by the intellect aided by the imagination: "One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly [*exacte definire*] whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape" (AT 10:441, CSM 1:58). To abstract problems from subject-matters, a "faculty shift" from the senses to the intellect aided by the imagination renders the problem open to an exact solution by means of intuition and deduction.

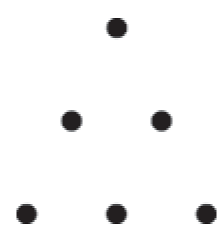
9.4 The Unit

In all perfectly understood problems, one must compare magnitudes to one another in order to determine unknown magnitudes on the basis of their relation to known magnitudes. Comparison depends on the *unit*; I cannot compare one magnitude to another unless I have a unit by which to measure each: "Unity is the common [simple] nature which, we said above, all of the things which we are comparing must participate in equally" (AT 10:449, CSM 1:63). In Rule 14, Descartes writes that "there are but two kinds of things which are to be compared with each other: sets and magnitudes" (AT 10:450, CSM 1:64), and he distinguishes between two distinct spatial representations of the unit: one for continuous magnitudes, and another for discrete magnitudes (sets or multiplicities). Continuous magnitudes are

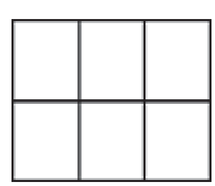
represented by means of *linear or rectilinear units*, and multiplicities are represented by means of *points*:

> If no determinate unit is specified in the problem, we may adopt as unit either one of the magnitudes already given or any other magnitude, and this will be the common measure of all the others. We shall regard it as having as many dimensions as the extreme terms which are to be compared. We shall conceive of it either simply as something extended, abstracting it from everything else – in which case it will be the same as a geometrical point […], or as some sort of line, or as a square (AT 10:450, CSM 1:63–64).
>
> For example, we shall depict the unit in three ways, viz., by means of a square, □, if we think of it only as having length and breadth; by a line, —, if we regard it as having just length; or, lastly, by a point, . , if we view it as the element which goes to make up a set (AT 10:453, CSM 1:65).

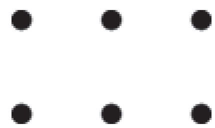
The choice of unit depends on the magnitudes to be compared. To compare two or more numbers to one another, I must display them on the screen of the imagination as points. "For example, the points



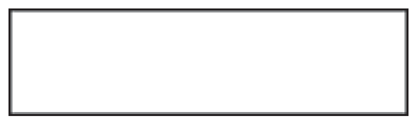
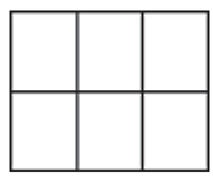
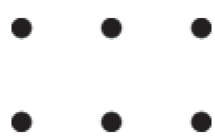
[…] represent a triangular number," i.e., a number whose units can be arranged in the form of a triangle (AT 10: 450, CSM 1: 64). "Figures such as these represent sets; while those which are continuous and unbroken, such as Δ, □, etc., illustrate magnitudes" (ibid.). To compare two continuous magnitudes to one another, I must represent them as two sides of a rectangle. For example, the rectangle,



represents two magnitudes (2 and 3, respectively) measurable by a unit square. In all cases in which the two continuous magnitudes are commensurate (i.e., measurable by a common unit), the rectangle must be divided into unit squares. The same magnitudes can be equivalently represented by unit points:



In cases in which the two sides of the rectangle are incommensurate, the rectangle is not composed of unit squares or points. As Descartes puts it in Rule 15:

> If we have to attend simultaneously to two different magnitudes belonging to the terms, we shall display them visually as a rectangle, two sides of which will be the two magnitudes in question. If they are incommensurable with the unit, we shall represent them thus:
>
> 
>
> If commensurable, thus:
>
> 
>
> or thus:
>
> 
>
> (AT 10:453, CSM 1:66).

The divided rectangle and the points below it can represent either continuous or discrete magnitudes, provided that the sides of the relevant magnitudes are commensurate or measurable by a common unit. The points can represent a continuous magnitude, and the divided rectangle can represent a discrete magnitude. Thus, in the case of discrete and continuous commensurate magnitudes, it is a matter of indifference which of these representations is chosen. The only circumstance in which Descartes requires an undivided rectangle is when the two magnitudes are *incommensurable with the unit*. This reveals that the distinction between the two schematisms (lines and rectilinear surfaces, on the one hand, and points, on the other) is not absolute; they depart in only one case:

incommensurate continuous magnitudes. The division in *Rules* is not between continuous and discrete magnitudes per se, but rather between continuous and discrete *commensurate* magnitudes, on the one hand, and continuous *incommensurate* magnitudes, on the other. Descartes insists that "with the aid of the unit we have adopted, it is *sometimes* possible completely to reduce continuous magnitudes to a set and […] this can always be done *partially* at least" (AT 10:452, CSM 1:65; my emphasis). That this can always be done, but only "partially," when one reduces incommensurate magnitudes to commensurate magnitudes, clearly indicates that any such reduction can only be approximate, not exact.[4]

Beyond the fact that these two distinct spatial representations of the unit satisfy Descartes's requirement in Rule 12 that all magnitudes be represented by the intellect aided by the imagination, it is not immediately clear what role these representations play in actual solutions to problems in mathematics (e.g., in computation), and in Rules 14–15, Descartes does not assign them any such role. Descartes's spatial representation of magnitude in Rules 14–15 is purely foundational, not operative; it represents magnitude so that it may be intuited by the intellect aided by the imagination, but how it can be employed in mathematics remains unclear.[5]

It is especially unclear what role the representation by points plays in solutions to problems in mathematics, since Descartes seems to drop the representation by points in Rule 18, where he introduces his geometrical calculus (i.e., his geometrical interpretation of the operations of arithmetic and algebra) (see Sections 9.6–9.7). Descartes's geometrical calculus in Rule 18 seems to be based on the manipulation of lines and rectangles alone.[6] Furthermore, oddly, in Rules 14–15, Descartes seems to *prioritize* the representation of magnitudes by means of points *over* the representation of magnitudes by means of lines and rectilinear surfaces. Why? Because he insists on the *reducibility* of continuous magnitudes to discrete magnitudes, and the latter can always be represented by means of points. Once the reduction to points has been executed, the "set of units [points] can then be arranged in such an order that the difficulty involved in discerning a measure becomes simply one of scrutinizing the order. *The greatest advantage of our method lies in this progressive ordering*" (AT 10:452, CSM 1:65; my emphasis). The prioritization of points over continuous magnitudes in Rules 14–15 is correlated to the prioritization of *order* over *measure* in Rule 14. For example, the series of points (Figure 9.1):

Figure 9.1 Points in continued proportion

are in continuous proportion, and by scrutinizing the order I can see the measure and easily continue the series. This is much less clear in the case of the series of lines (Figure 9.2):

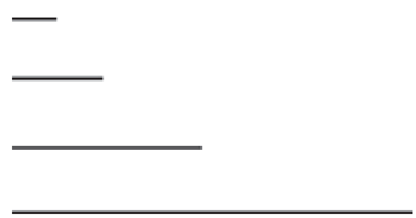


Figure 9.2 Lines in continued proportion

which are equivalent in length to the points and also in continuous proportion. In this case, I cannot as easily see the measure (the ratio) simply by scrutinizing the order because the unit and the relation between the unit and the remaining magnitudes is not obvious. Descartes's representation of magnitudes by means of points is not an idle representation intended only to satisfy his requirement in Rule 12 that all magnitudes be represented by means of the intellect aided by the imagination. On the contrary, it reflects his desire to reduce problems about continuous magnitudes to problems about discrete magnitudes, i.e., problems in which "the difficulty involved in discerning a measure becomes simply one of scrutinizing the order." In Rule 14, Descartes expressly indicates that problems in mathematics can be solved *either* by order *or* measure: "[All] the relations which may possibly obtain between entities of the same kind should be placed *under one or other of two categories: viz., order or measure*" (AT 10:451, CSM 1:64; my emphasis). Thus, it is not surprising that he would prioritize the former over the latter in certain cases.

A clear example of problems that can be solved simply by “scrutinizing the order” of points can be found in *De solidorum elementis* (AT 10:257–77), most probably written during the early 1620s.[7] *De solidorum elementis* contains parallel passages about figurate numbers that shed light on the problems Descartes has in mind when he recommends representing discrete magnitudes by means of points in Rule 14. In the Pythagorean tradition, polyhedral and polygonal numbers are referred to as “figurate numbers,” since their units can be represented as geometrical figures. As we have already seen above, the numbers 3 and 6 are triangular numbers because their units can be represented as triangles. Points are necessary when it comes to representing figurate numbers and the relations between them. The general problem Descartes addresses in *De solidorum elementis* II is how to obtain the “weight” (*pondera*, or number of points) of any polyhedral (solid) number, but he begins by discussing the simpler problem of how to obtain the weight of any given polygonal (plane) number. We can order a series of figurate numbers by means of what Euclid termed the “gnomon” in *Elements* II. def. 2, which Heron of Alexandria would later define as “any figure, which, when added to any figure whatever, makes the whole figure similar to that to which it is added.”[8] Repeated addition of the gnomon yields a series of figurate numbers in continued proportion. For example, in the case of triangular numbers, one begins with one point. The gnomon required to produce the next triangle in the series will be two points, and the gnomon required to produce the subsequent triangle in the series will be three points, etc. (Figure 9.3):

Figure 9.3 Triangular numbers in continued proportion

The general equation for the gnomon is as follows:

$$(a-2)n-(a-3),$$

where “*a*” is the number of sides, and “*n*” is the number of points that compose the radix (base). Descartes obtains this equation in *De solidorum elementis* by producing the sequence of polygonal numbers. For example,

triangles have three sides, and the first triangular number has two points on its radix. By inserting these values into the general equation for the gnomon, one obtains the following: (3 – 2)2–(3 – 3) = 2. Therefore, the gnomon on the first triangular number in the series is two points. Using this equation, I can define the gnomon for any figure, given its number of sides and the number of points on its radix. By adding this gnomon to the figurate number immediately preceding it in the series, I obtain the next member of the series, etc.

To obtain the "weight" of any given figurate number, Descartes employs the equivalent of the following general equation:

$$p(a, n) = (a - 2)n^2/2 - (a - 4)n/2,$$

where "*p*" is the "weight" of the relevant figurate number, "*a*" the number of sides, and "*n*" the number of points on its radix.[9] For example, to determine the "weight" of a pentagon that has three points on its radix (see Figure 9.4), the general equation yields:

$$p(a, n) = (5 - 2)3^2/2 - (5 - 4)3/2 = 12.$$

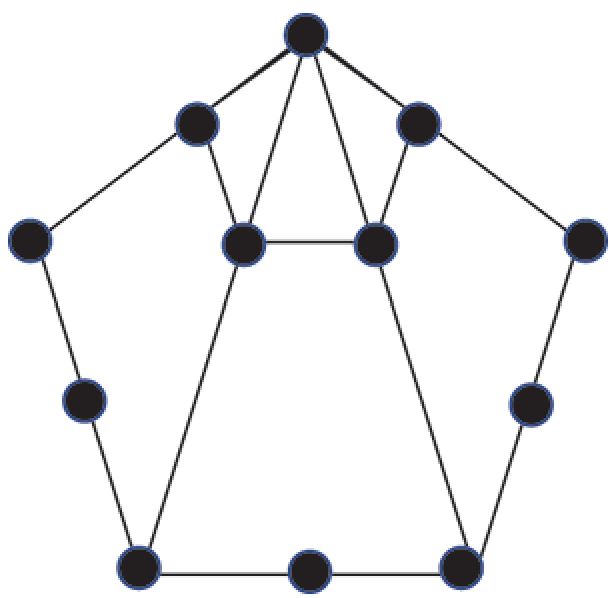


Figure 9.4 Figurate number (pentagon)

Thus, all figurate numbers can be produced and serially ordered according to these general equations, and their weights can be measured by means of points. These equations algebraically express the relations exhibited in the ordered series of figurate numbers geometrically represented by means of points. This explains why Descartes correlates points to the representation of triangular numbers in Rule 14. These points perspicuously represent a series of figurate numbers via the gnomon. It is clear that such a display enables the intellect to define the measure simply by scrutinizing the ordered sequence of figurate numbers, exactly as Descartes recommends in Rule 14. Each triangular number being larger than, but similar to the triangular number immediately preceding it in the series, the order is perfectly transparent. One literally counts the number of points by which each successive triangle exceeds the triangle immediately preceding it in the series. One determines the measure by scrutinizing the order.

In another striking parallel with Rule 14, in *De solidorum elementis* Descartes writes that figurate numbers can be imagined according to either *order* or *measure*. When they are imagined as measurable, then, as in Rule 14, they must be measured by a unit that has "as many dimensions as the extreme terms which are to be compared" (Rule 14, AT 10:450, CSM 1:63–4).

> If one imagines these figures as measurable, then all the units are understood *as being of the same kind as the figures themselves*: that is for triangles a triangular unit; pentagons are measured by a pentagonal unit, etc. Then the proportion between a plane and its radix is the same as the square to its radix; and a solid as a cube: so if the radix is 3, the plane will be 9, the solid 27, etc., for example. […] From which it is observed that our mathematical progression, n, n^2, n^3, etc., is not attached to linear, square, cubic, figures, but is designated generally by the diverse species of measure (AT 10:271; trans. Descartes 1982, 95; translation slightly modified; my emphasis).

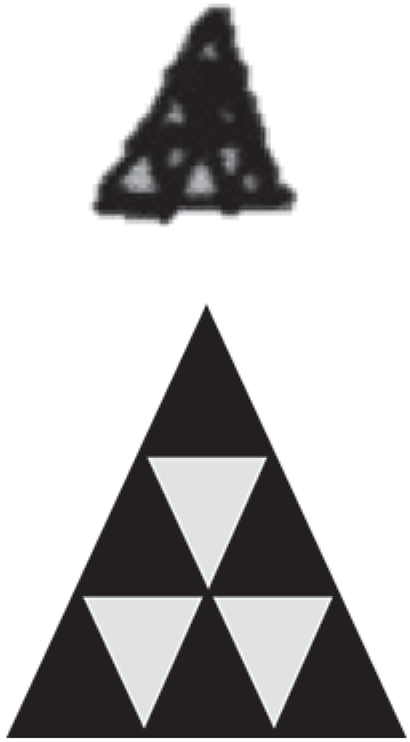


Figure 9.5 Triangle composed of triangular units (from Leibniz's Notes) (Niedersächsische Landesbibliothek, Hanover, LH IV, I, 4b, fol. 1 v.)

In *De solidorum elementis*, the unit measure of a triangle *must itself be triangular* (see Figure 9.5). The advantage to representing triangles in this way is that it perspicuously represents relations between the root, the square, the cube, etc., *without having to employ these figures themselves and, therefore, without having to worry about how to spatially represent exponents higher than three.* Instead, *these relations are represented as relations between the radix and the area of the figure*. A triangle that has an area of 9 triangular units has a radix of 3. The *radix* is the *root*, and the *area of the triangle* is the *square*. This is very different both from the more economized and sophisticated representation of roots, squares, cubes, etc., by means of lines and rectangles in Rule 18 and also from the even more economized and sophisticated representation of roots, squares, cubes, etc., by means of lines alone in the *Geometry*. Nevertheless, a common concern underlies these different representational schemas: they are all intended to overcome the classical Euclidean dimensional interpretation of the mathematical powers, so that they may be *constructed in intuition*.

Returning to Descartes's geometrical representations of the unit in Rules 14–15, the representation based on lines and rectilinear surfaces (including triangular and other such units) seems to correspond to measure (e.g., the measure of areas), while the representation based on points seems to correspond to order (e.g., an ordered series of figurate numbers). The difference between these representations suggests that the *unity of mathematics does not play a major role in Descartes's conception of mathematics in Rule 14* (I explain why below). On the contrary, *Descartes acknowledges that there remains an irreducible barrier between discrete and continuous magnitude*. It is clear that at this point in the treatise Descartes cannot wholly reduce the distinction between continuous and discrete magnitudes, and that the difference between them and their corresponding spatial representations must, therefore, be maintained. Descartes's envisioned reduction encounters definite limits in the case of incommensurable continuous magnitudes, which can only be reduced to representation by means of points "partially." By definition, these reductions cannot be exact (clear and distinct), and so their status is not properly "geometrical," at least not by Descartes's standards in the *Geometry*, where he distinguishes between the properly "geometrical," which is "precise and exact," and the merely "mechanical," which is neither precise nor exact.[10]

This interpretation of Rules 14–15 is further reinforced by evidence from the Cambridge manuscript. Apart from some minor differences, the Cambridge manuscript contains Rule 14 in its entirety, but no mention whatever of *mathesis universalis* in Rule 4. This means that Descartes does not explicitly reflect on the unity of mathematics in Rule 4 or anywhere else in the Cambridge manuscript. Furthermore, the Cambridge manuscript ends after Rule 16. Rules 14–15 contain the two spatial representations of the unit discussed above, and Rule 16 contains Descartes's algebraic notation (discussed in Section 9.5 below). Rule 18 is not in the Cambridge manuscript. Consequently, the geometrical calculus based on lines and rectangles had simply not yet appeared on the scene. Without the geometrical calculus, the dual representations of the unit in Rules 14–15 and Descartes's reluctant acknowledgement that it is impossible to completely reduce continuous magnitudes to discrete magnitudes suggest that in the Cambridge manuscript he either did not yet regard the mathematical sciences as a unity or that he did not yet know how to

establish the unity of the mathematical sciences. As we will see in Section 9.7, Descartes does not entirely evade the problem even after the Cambridge manuscript, but in Rule 18 he makes considerable progress toward a more unified spatial representation of the operations and objects of arithmetic and geometry based on the manipulation of lines and rectangles alone, with points playing a subsidiary and even negligible role.

9.5 Symbolic Intuition? Algebraic Notation in *Rules*

The abstraction of problems from subject-matters does not stop at magnitude. Too many magnitudes burden the imagination and memory (and, therefore, intuition), and can themselves be more easily represented by means of algebraic notation, which clearly and distinctly represents these magnitudes by means of *symbols*. There is, then, a *double abstraction* required in solutions to perfectly understood problems: (1) the abstraction of problems from subject-matters, so that the problem is reduced to relations between pure magnitudes; (2) the representation of these magnitudes and their relations themselves by means of an algebraic notation that relies exclusively on *the manipulation of symbols* (letters, numbers, and other symbols).

Algebraic notation plays a number of roles in *Rules*:

(1) *Gnoseological*. Algebraic notation permits the mind to provisionally disregard those magnitudes that “do not require the immediate attention of the mind” *without relying on memory* (AT 10:454, CSM 1:66). Once all relevant relations between the magnitudes have been enumerated$_1$ and assigned a unique symbol, the mind can focus on the one or two dimensions that demand its immediate attention. By “dimension” Descartes has in mind, not spatial dimensions, but rather “simply a mode or aspect in respect of which some subject is considered as measurable,” such as “length, breadth, and depth,” “weight,” “speed,” and in general any respect in which a subject can be measured, both purely mathematical and physical (whether a dimension is real or not “is something for the physicists to consider”) (AT 10:448, CSM 1:62–3). Attending to more than two dimensions at once is strictly prohibited: the mind “should not contemplate, in one and the same

visual or mental gaze, more than two of the innumerable different dimensions which it is possible to depict in the imagination" (AT 10:454, CSM 1:66–7). All other relations can be materially archived on paper, so that the mind can return to them whenever the need arises. By reducing the number of relations that need to be retained in the present, algebraic notation reduces the possibility of distraction, which divides attention and, therefore, undermines intuition (see Chapter 3, Section 3.2). Algebraic notation isolates "whatever is to be viewed as one thing from the point of view of the problem" (AT 10:455, CSM 1:67), such that every condition relevant to the solution of the problem can be perspicuously represented "in accordance with Rule 9" and enumerated$_1$ "in accordance with Rule 11" (AT 10:455, CSM 1:67). Algebraic notation enables the mind to "run through" each condition "with the swiftest sweep of thought and intuit as many as possible at the same time" (AT 10:455, CSM 1:67). With algebraic notation, the initially inordinate number of magnitudes and relations yields a well-ordered enumeration$_1$ of every condition relevant to the solution of the problem. Each condition can then be surveyed by the mind individually.

(2) *Epistemological*. Algebraic notation yields definite epistemological results, since by disburdening the mind it yields certain and evident cognition, or *science*: every condition of the problem is cleanly isolated by a unique symbol, so that in principle perfect clarity and distinctness reigns throughout.

(3) *Mathematical*. Rule 16 contains the seeds of a transformation in the history of mathematical notation. Already in Rule 4, Descartes praises ancient analytic geometry and modern algebra but complains about "the multiplicity of numbers and incomprehensible figures which overwhelm it," as opposed to "that abundance of clarity and simplicity which I believe the true mathematics ought to have" (AT 10:377, CSM 1:19). Prior to *Rules*, Descartes employed cossic notation (see Figure 9.6):[11]

Figure 9.6 Cossic notation in Rudolff's *Coss* (1525)

In cossic notation, $\mathfrak{r}$ symbolizes the unknown "thing" or quantity; $\mathfrak{z}$ symbolizes the square (census, in German); $\mathfrak{ce}$ symbolizes the cube; $\mathfrak{zz}$ symbolizes the "square-square," etc.[12] What is written in post-Cartesian notation as $(x^2+x+2)^2$ is written by mathematicians such as Stiefel as[13]

$$(1\mathfrak{z} + 1\mathfrak{r} + 2) \cdot (1\mathfrak{z} + 1\mathfrak{r} + 2)$$

and developed as:

$$1\mathfrak{z}\mathfrak{z} + 2\mathfrak{ce} + 5\mathfrak{z} + 4\mathfrak{r} + 4.$$

For a Cartesian, cossic notation suffers from a number of limitations. (1) The square only attaches to terms contained in the parentheses, not to the expressions delimited by them. Consequently, the square of the sum contained in the parentheses cannot be symbolically represented, but only effectuated: the expression must be multiplied by itself. The square "is unrepresentable in the cossic system" and "is not capable of being individuated, that is, it cannot be objectified" in the signs themselves.[14] (2) To carry out the multiplication, supplementary rules that determine how to multiply heterogenous quantities are required, and they must be memorized by the learner accordingly. For example, in the present case,

 is equal to  (“*Thing*” *multiplied by* “*thing*” makes “*square*”)

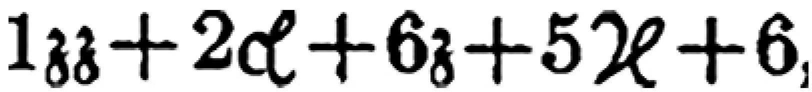
and

 is equal to  (“*Thing*” multiplied by “*square*” makes “*cube*”).[15]

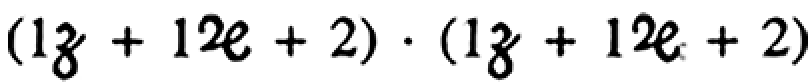
As can be seen in Figure 9.6, higher-order powers have different symbols, and must be calculated according to different rules. One could not simply multiply by adding the exponents in an exponential notation ($a^b \cdot a^c = a^{b+c}$). (3) Cossic notation does not perspicuously represent that one and the same unknown quantity is the root of all of the squares and the cubes, since different symbols designate the root, the square, the cube, etc. These symbols do not resemble one another.[16] (4) Cossic mathematics regards the root, the square, and the solid as irreducibly different species of magnitude, which cannot be easily compared to one another as x, x^2, x^3, etc. can in a Cartesian exponential notation by numerical superscript. Consequently, “nothing can be deduced from the visual examination of the signs themselves.”[17] Finally, (5) one cannot arbitrarily change the unknown in cossic notation. From a Cartesian perspective in and after *Rules*, the principal insufficiency of cossic notation consists in how it “confuses” the unknown and the mathematical power of the unknown in one and the same symbol. The unknown is not perspicuously represented as such. Cossic mathematics regards the mathematical powers as *primitive, not as composed of a relation between an unknown quantity and the number of times that unknown quantity is multiplied by itself*.[18]

In Rule 17, Descartes lays down as a principle that “whatever is to be viewed as one thing from the point of view of the problem we shall represent by a unique symbol,” which may be arbitrarily chosen, so long as it perspicuously represents or isolates every condition relevant to the solution of a problem. He employs the minuscule “letters *a*, *b*, *c*, etc. to express magnitudes already known, and *A*, *B*, *C*, etc. for ones that are unknown” (AT 10:455, CSM 1:67).[19] These letters are preceded by numerals (e.g., 1a or 1A) “to indicate how many of them there are” (ibid.), and “we shall also append these as suffixes to indicate the number of relations which they are to be understood to contain” (ibid.). For example, “if I write ‘$2a^3$,’ that will mean ‘twice the magnitude symbolized by the letter *a*, which contains three relations’” (ibid.). The symbol “$2a^3$”

perspicuously distinguishes between the known magnitude (*a*), the number of times it occurs (2), and the number of proportional “relations” it contains (i.e., $1:a = a:a^2 = a^2:a^3$). In short, *the numerical superscript clearly indicates where in a series of proportional relations a given exponent or power belongs*.[20] Descartes’s notation perspicuously represents the distinction between the indeterminate, but known magnitudes (*a*, *b*, *c*, etc.) and the unknown magnitudes (*A*, *B*, *C*, etc.). Perhaps most importantly, the exponent is distinctly represented by means of a numerical superscript. Where Stiefel writes:



Descartes writes: $A^4 + 2A^3 + 6A^2 + 5A + 6$ (as he later will $x^4 + 2x^3 + 6x^2 + 5x + 6$), and where Stiefel writes:



Descartes writes: $(x^2+x+2)^2$. The square (or any other exponent) attaches to both terms and expressions. Both the unknown quantity and the exponent are uniformly represented throughout. Finally, Descartes’s algebraic notation abstracts “just as much from numbers as […] from geometrical figures […] – or from any matter whatever” (AT 10:455–6, CSM 1:67). This both “avoid[s] the tedium of long and unnecessary calculation and, most importantly, [ensures] that the parts of the subject relevant to the nature of the problem are kept separate at all times and are not bogged down with pointless numerical expressions” (AT 10:456, CSM 1:67). To express the hypotenuse of a triangle whose sides are 9 and 12, arithmeticians typically write $\sqrt{225}$ or 15. “We on the other hand will substitute *a* and *b* for 9 and 12, and will find the hypotenuse to be $\sqrt{a^2 + b^2}$, which keeps distinct the two parts a^2 and b^2 which the numerical expression conflates” (AT 10:456, CSM 1:67–8). The numerical expression constitutes an obstacle to perspicuous representation of “whatever is to be viewed as one thing from the point of view of the problem” (AT 10:455, CSM 1:67), while the algebraic expression clearly distinguishes between the dimensions of the triangle relevant to determining the hypotenuse. Descartes’s algebraic notation is not only perspicuous, it is also completely

general: $c = \sqrt{a^2 + b^2}$ is the general equation of the hypotenuse, no matter what the numerical value *a* and *b* may be in any one case.

Algebraic notation in hand, Descartes writes that in order to solve any perfectly understood problem, one must (1) enumerate$_1$ the "the terms of the problem as they were stated in the first place" (AT 10:458, CSM 1:69). One must then (2) "note down the way in which they may be abstracted, and the symbols […] use[d] to represent them" (ibid.). "The purpose of this is that" (3) "once we have found the solution in terms of these symbols, we shall be able to apply it easily to the particular subject we are dealing with, without having recourse to memory" (ibid.). The role algebraic notation (and the equations expressed by its means) plays in solutions to perfectly understood problems consists in symbolization in which the solution to the problem is provided *in symbolic form*, which must then be translated into the initial terms of the problem and *geometrically constructed* so that the solution may be *intuited*. *The solution to the problem does not consist in an algebraic equation*, *but rather in the geometrical construction based on it*. Algebraic notation introduces the possibility of a purely "symbolic intuition" into Cartesian science. It is "symbolic" because the object of intuition in mathematics is magnitude, and algebraic notation only *represents* magnitudes; it does not *present* them. It is "intuition" because the notation can be employed in geometrical constructions in which the solution to the problem is intuited by the intellect "face to face" in the imagination (see Chapter 3, Section 3.2).

9.6 The Geometrical Calculus

In Rule 18, Descartes provides a geometrical interpretation of the operations of arithmetic (addition, subtraction, multiplication, and division) and algebra (root extraction). Addition and subtraction are interpreted as operations performed on line segments. Two line segments *a* and *b* being given, I add them to obtain their sum *c* as follows (Figure 9.7):

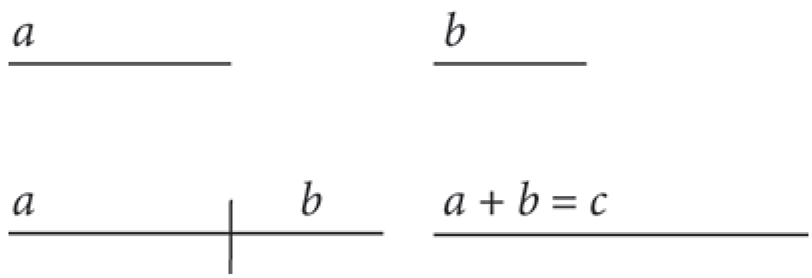


Figure 9.7 Addition in Descartes's geometrical calculus (see AT 10:464)

I simply combine the two lines in one act and display their combination on the screen of the imagination, where they can be intuited "face to face" by the intellect. In subtraction, I place line segment *a* above line segment *b* in order to obtain the difference *c* as follows (Figure 9.8):

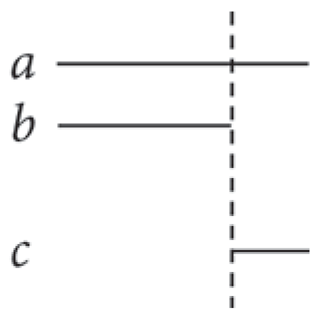


Figure 9.8 Subtraction in Descartes's geometrical calculus (see AT 10:464)

I simply remove from *a* whatever parts extend beyond *b* in order to obtain the difference *c*. These are exceedingly simple examples, but they clearly illustrate how the operations of arithmetic can be intuited by the intellect on the screen of the imagination. Multiplication and division are slightly more complex operations, but they must intuited by the intellect on the screen of the imagination in exactly the same sense as the operations of addition and subtraction are. Everything must be "seen" by intuition "face to face."

Descartes interprets the product of two line segments *a* and *b* as forming a rectangle with sides *a* and *b* (Figure 9.9):

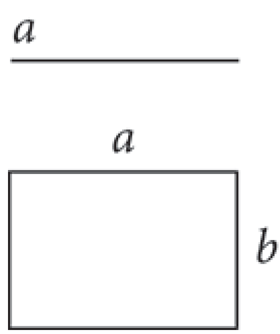


Figure 9.9 Multiplication in Descartes's geometrical calculus (see AT 10:465)

In the case of the multiplication of n>3 lines, Descartes insists that relations between magnitudes should never be represented in more than two dimensions because otherwise the product of n>3 lines could not be geometrically constructed. For Euclid, the product of two lines must be constructed as a plane area in two dimensions, and the product of three lines must be constructed as a solid in three dimensions.[21] The product of n>3 lines, however, could not be geometrically constructed, since the solid exhausts all three dimensions of space (see Figure 9.10).

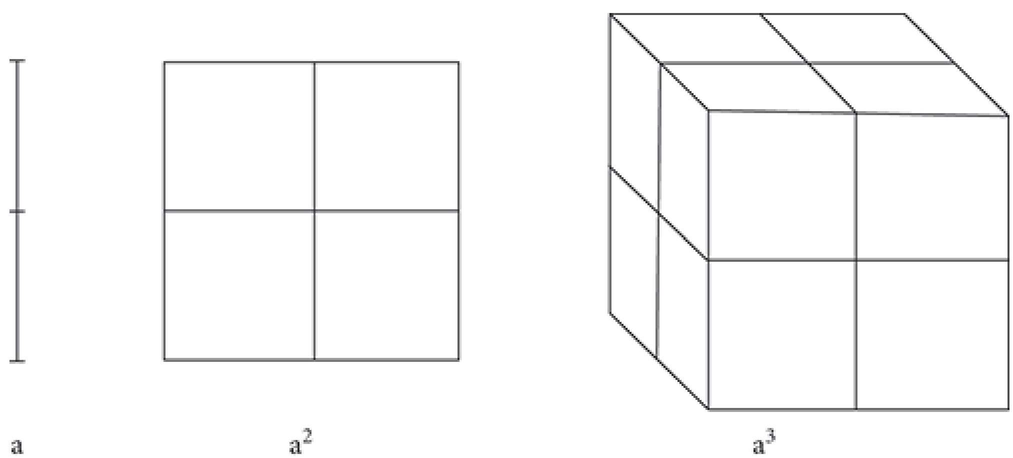


Figure 9.10 Multiplication according to line, square, and cube (the problem of dimensionality)

There are no more dimensions in which to represent the product of n>3 lines.[22] Suppose the problem is to raise a line to the fourth power (in Cartesian notation, $x = a^4$). For Descartes's predecessors, this made

absolutely no geometrical sense. The product of $n>3$ numbers, by contrast, presented no analogous difficulty; the product of $n>3$ numbers is always another number. These differences between discrete and continuous magnitude strongly reinforced the Aristotelian thesis that arithmetic and geometry constitute two irreducibly heterogeneous sciences permanently separated by Aristotle's ban on genus-crossing in the sciences (see Chapter 1, Section 1.2). The "problem of dimensionality," as it has since come to be known, constituted a serious obstacle to the use of algebra in geometry. For Descartes in *Rules*, by contrast, the product of any number of lines always yields only another line segment.

To multiply the rectangle *ab* in Figure 9.9 by the line segment *c*:

c

We should transform the rectangle *ab* in Figure 9.9 into a line:

ab

In order to obtain the rectangle *abc* (see Figure 9.11).

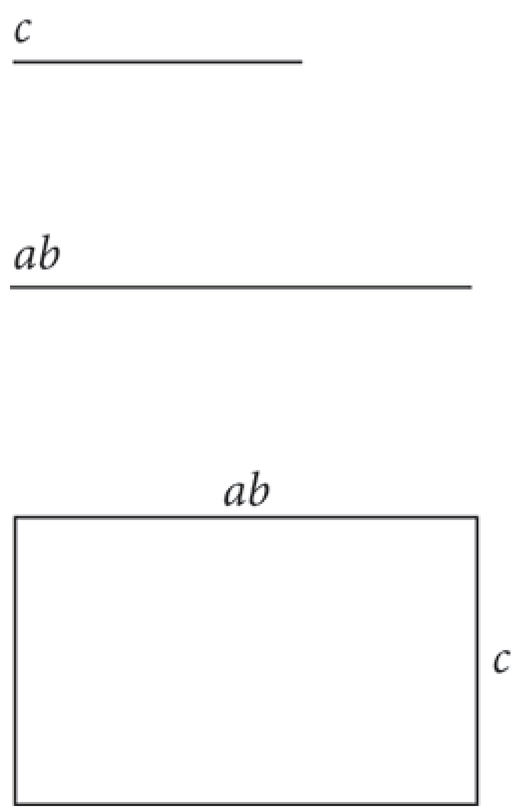


Figure 9.11 Multiplication of rectangle *ab* in Figure 9.8 by line *c* (see AT 10: 466)

This raises two questions: (1) What does it mean to transform the rectangle *ab* into the line segment *ab*? Rectangles have areas, lines do not. How can they be compared to one another and regarded as equal? (2) How—by what procedure—do I know that they are, in fact, equal?[23]

Regarding (1), Descartes defines an equivalence relation between rectangles and line segments in the following way in *Rules*. Any line segment *a* is equivalent to a rectangle one of whose sides is *a*, and whose other side is the unit, in this case the arbitrarily chosen unit *u* (Figure 9.12):

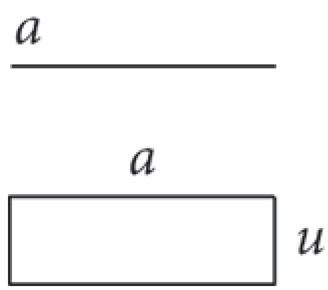


Figure 9.12 Equivalence relation between rectangles and line segments (Bos 2009, 12)

According to the equivalence relation, to transform the rectangle *ab* into the line segment *ab* is to conceive the rectangle *ab* as equivalent to *another rectangle du*, one of whose sides *d* is equal to *ab*, while the other is the unit *u* (Figure 9.13):

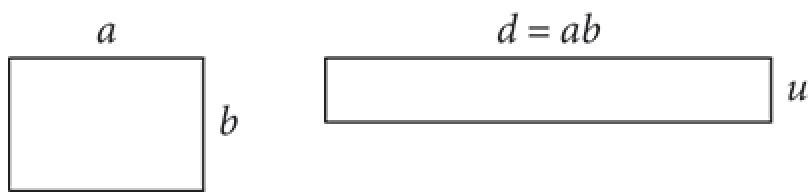


Figure 9.13 Transformation of rectangle *ab* into line segment d = *ab* (see Bos 2009, 13)

Thus, the multiplication of any number of line segments never exceeds two dimensions, and only one of these dimensions is the relevant product.

Regarding (2), as we have seen, according to the equivalence relation, to transform the rectangle *ab* into the line segment *ab* is to transform the rectangle *ab* into *another rectangle*, one of whose sides *d* is equal to *ab*, while the other is the unit. How can the equivalence between the rectangle *ab* and the rectangle *du* be *intuited*? The areal equivalence between these two rectangles is not immediately evident. But it can be rendered evident by means of a definite procedure. Descartes obliquely describes this procedure toward the end of Rule 18:

> It is therefore important to explain here how every rectangle can be transformed into a line, and conversely how a line or even a rectangle can be transformed into another rectangle, one side of which is specified. Geometers can do this very easily, provided they recognize that in comparing lines with some rectangle (as we are now doing), we always conceive the lines as

> rectangles, one side of which is the length which we adopted as our unit. In this way, *the entire business is reduced to the following problem: given a rectangle, to construct upon a given side another rectangle equal to it* (AT 10:468, CSM 1:76; my emphasis).

The procedure Descartes describes here—the "application of areas"—requires transforming a given rectangle into another rectangle equal in area. It can be found in Euclid's *Elements* I.44 and I.45. The problem is stated in *Elements* I.44: "To a given straight line to apply, in a given rectilinear angle, a parallelogram equal to a given triangle."[24] In the case of rectangles, one constructs a rectangle equal in area to a given rectangle by employing the gnomon. Euclid provides an illustration of the relevant gnomon property in *Elements* I.43. The diagonal line segment divides the rectangle into two equal triangles (Figure 9.14):[25]

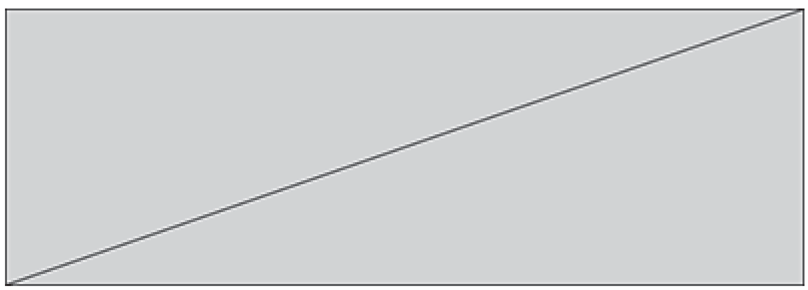


Figure 9.14 Application of areas (Bos 2009, 15)

The two shaded triangles in the inscribed rectangle are also equal (Figure 9.15).

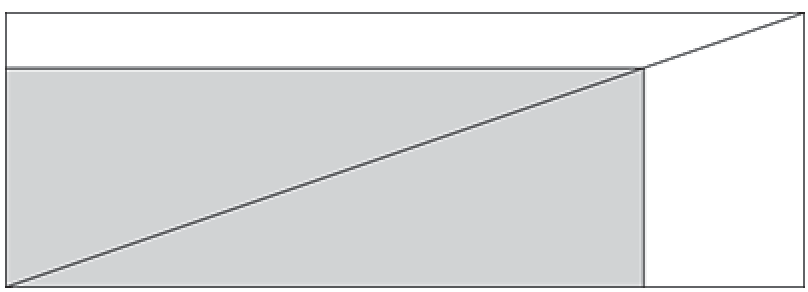


Figure 9.15 Application of areas (Bos 2009, 15)

The removal of the two inscribed triangles leaves two equal shaded areas in virtue of the common notion (simple nature) that equals taken from equals are also equal (see AT 10:419, CSM 1:45 discussed in Chapter 7, Section 7.5) (Figure 9.16):

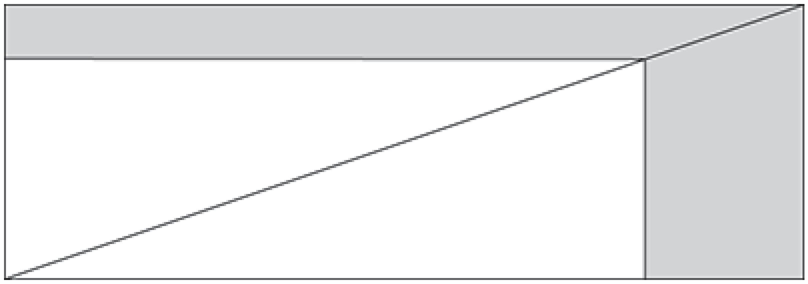


Figure 9.16 Application of areas (Bos 2009, 16)

For the same reason, the two shaded triangles in the upper right-hand corner are also equal (Figure 9.17):

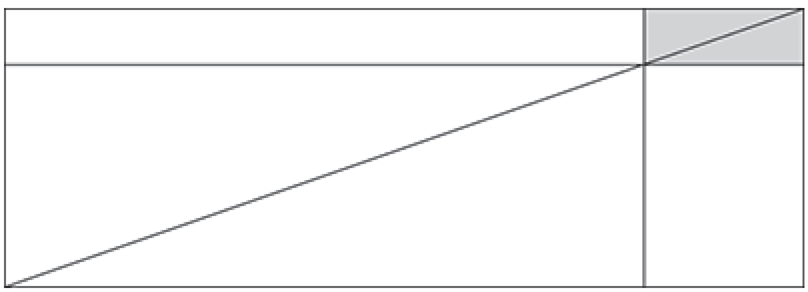


Figure 9.17 Application of areas (Bos 2009, 16)

Again for the same reason, these two shaded rectangles are equal (Figure 9.18):

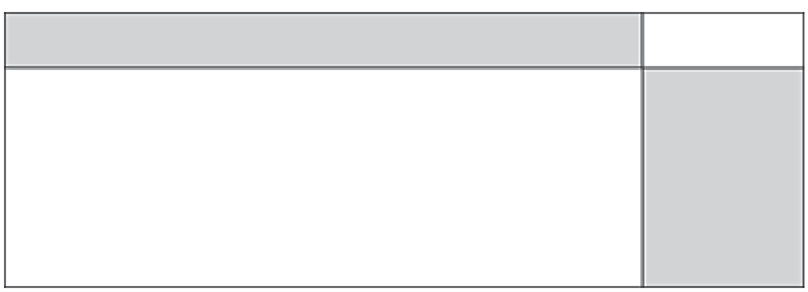


Figure 9.18 Application of areas (Bos 2009, 16)

The gnomon property intuitively reveals a series of areal equalities that are not initially evident. Descartes exploits the gnomon property in order to transform the rectangle *ab* in Figure 9.9 into the line segment $d = ab$:

I am searching for a line one of whose sides is *d*, and since $d = du$, I drop a line segment *u* from the rectangle (Figure 9.19):

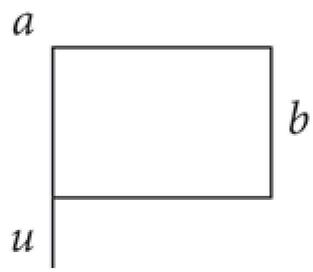


Figure 9.19 Line segment *u* added to rectangle *ab*

I then construct the gnomon so that the figure in the lower right-hand quadrant is equal to *ab* (Figure 9.20):

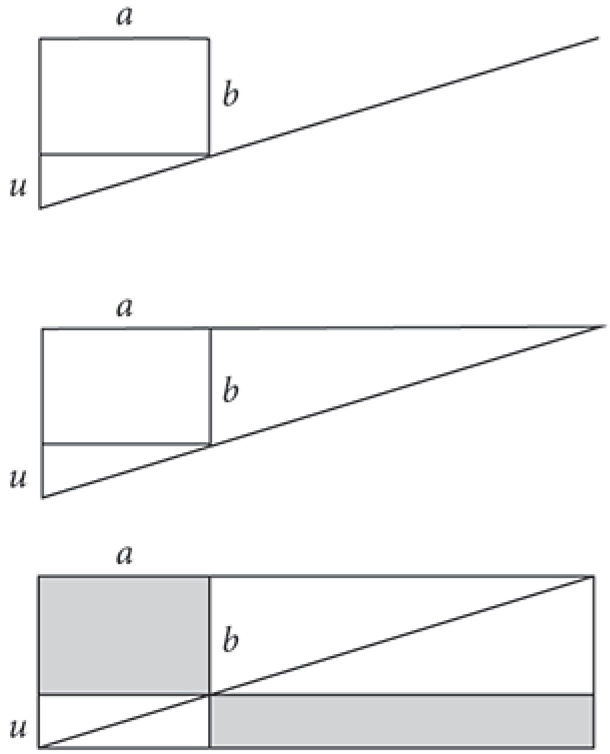


Figure 9.20 Construction of the gnomon

Flipped over 180º, the rectangle in Figure 9.20 looks like Figure 9.21:

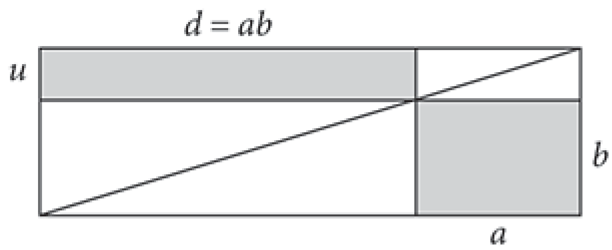


Figure 9.21 Equivalence between line *d* and rectangle *ab*

The lower right-hand shaded quadrant *ab* is equal in area to the upper left-hand shaded quadrant, which is equal to *ab*. Therefore, the upper left-hand quadrant is $du = ab$, and the side *d* is the required line.

Descartes also exploits the gnomon property in order to divide the line *a* by *b*. In this case, I must calculate $a/b = c$. Since $(a/b = c) = (a = bc) = (au = bc)$, I must find a square one of whose sides is *c*. To do this, I place *a* and *b* next to one another (Figure 9.22):

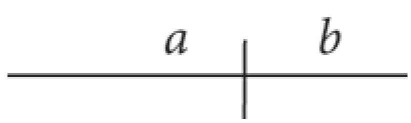


Figure 9.22 Lines *a* and *b* (see Bos 2009, 18)

I then construct a rectangle on *a* by multiplying it by *u* (Figure 9.23):

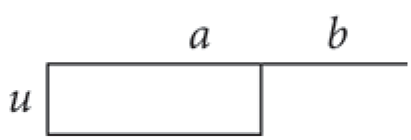


Figure 9.23 Construction of a rectangle on line *a* (see Bos 2009, 18)

I can already see how to exploit the gnomon property in order to find the quotient. It will be the quadrant in the lower right-hand corner of the figure (Figure 9.24):

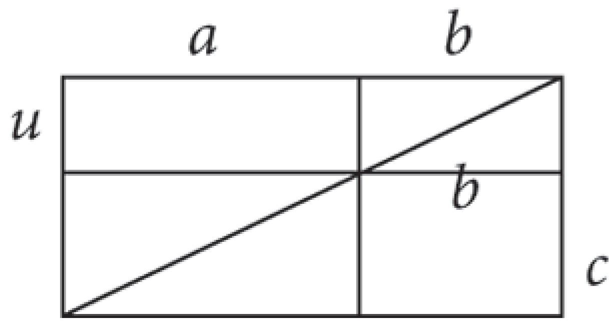


Figure 9.24 Construction of the gnomon (see Bos 2009, 18)

The required quotient is *c*.

I will discuss root extraction in more detail in Section 9.7, since it raises problems of its own.

Descartes's geometrical interpretation of the operations of arithmetic effectively ensures that these operations *can always be intuited by the intellect aided by the imagination in two spatial dimensions.* To reach a non-dimensional interpretation of the elementary operations of arithmetic, Descartes relates magnitudes that are in continued proportion to one another in a way that effortlessly falls within the power of the imagination. All magnitudes, whatever their degree, can be represented as straight lines and rectangles, and the degree of each line will be determined by how proportionally longer or shorter it is relative to an arbitrarily chosen "unit" length (see AT 10:456–7, CSM 1:68). The manipulation of lines and rectangles (either by themselves or via the application of areas) in the imagination renders relations between magnitudes in addition, subtraction, multiplication, and division completely transparent. Descartes's geometrical interpretation of the operations of arithmetic forcefully illustrates the principally methodological orientation of his theory of the faculties in Rule 12. In mathematics, the material simple natures are directly displayed on the screen of the imagination, where they are intuited by the intellect as it manipulates lines and rectangles in two dimensions in "real time." The intellect and the imagination run parallel to one another: the operations of the intellect are directly reflected in the objects manipulated in the imagination. All the intellect has to do is "see" the solution to problems in the manipulations of the magnitudes themselves. Cartesian mathematics is not *formal*, but rather *intuitive*.

9.7 The Problem of Root Extraction

Root extraction raises special problems in *Rules*. It is not clear how the application of areas via the gnomon property can be employed in order to extract roots. For example, in order to extract the square root of the line segment *a*, I must multiply *a* by the unit line segment *u* in order to form a rectangle and construct upon that rectangle another rectangle $d = \sqrt{a}$. This equation is equivalent to the equation $d^2 = a$, which is equivalent to the equation, $d \bullet d = a \bullet u$. Since $d \bullet d$ is a square, I must construct a square upon the given rectangle *au*.

To construct $a \bullet u$, I simply multiply *a* by the unit line segment *u* and obtain (Figure 9.25):

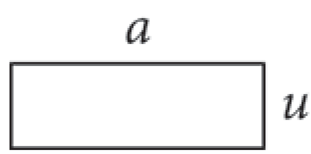


Figure 9.25 Multiplication of line *a* by the line (unit) *u* (see Bos 2009, 19)

In the cases of multiplication and division discussed in Section 9.6, I constructed the gnomon by means of two given magnitudes *a* and *b* and the unit line segment *u*. In this case, I only have one given magnitude *a* and the unit line segment *u*. It is not clear how I am supposed to construct the gnomon here. How long should I drop a line from *u* or extend *a*? Suppose I drop the following line from *u* (Figure 9.26):

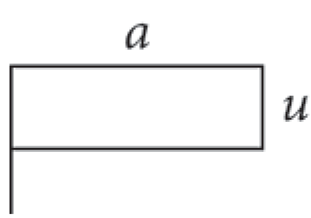


Figure 9.26 Line segment dropped from line segment *u* (see Bos 2009, 19)

I extend the diagonal from this line up to the point where it intersects the line extended from *a*. This yields (Figure 9.27):

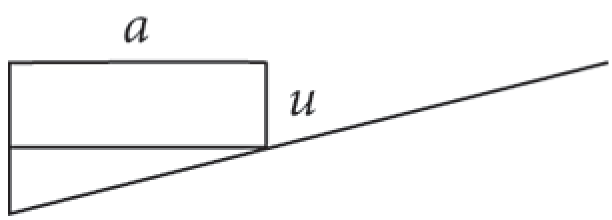


Figure 9.27 Construction of the gnomon (see Bos 2009, 19)

Will the parallelogram in the lower-right-hand corner be a square? I complete the gnomon (Figure 9.28):

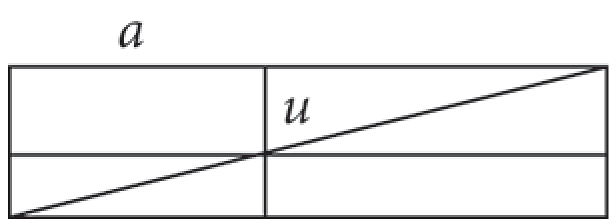


Figure 9.28 Construction of the gnomon (see Bos 2009, 19)

Clearly, the parallelogram in the lower-right-hand corner is not a square. There are no well-defined parameters by which to construct the gnomon, and every construction based on only one given magnitude and the unit line segment will be arbitrary.[26]

In Euclid, one extracts the roots of a given magnitude by means of the *semicircle*, not just lines and rectangles. To extract the root of *a*, I must combine it with the unit line segment *u* and construct upon the line *au* a semicircle, extending a line from the point where *a* and *u* meet up toward where it intersects with the semicircle (Figure 9.29). This line will be *d*.

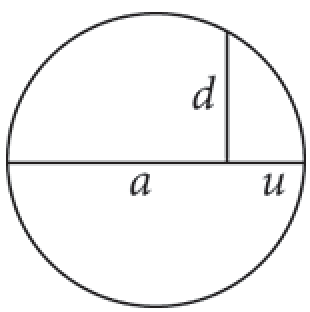


Figure 9.29 Root extraction in Euclid (see Bos 2009, 20)

Clearly, this is not nearly as intuitive as the constructions via the gnomon discussed in Section 9.6 are. The construction does not perspicuously display *why* d is the square root of a. By adding two more lines, however, the construction does become more intuitive (Figure 9.30):

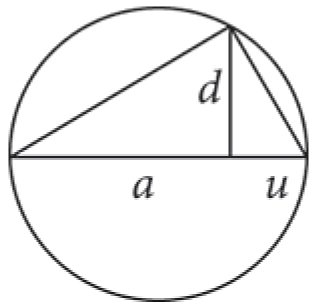


Figure 9.30 Root extraction in Euclid (with similar triangles) (see Bos 2009, 21)

This construction exploits the similarity of the triangles inscribed within the circle. All sides of similar triangles are proportional to one another. Root extraction is a species of division, and division is equivalent to the problem of finding one or more mean proportionals. In this case, $a{:}d = d{:}u$. Therefore, $d^2 = au$, and the line d is the required root.[27]

Some commentators have argued that the construction in Figure 9.30 is not intuitive by Descartes's standards.[28] I disagree. The construction by semi-circle exploits the theory of proportions, and those who have practiced solving problems in *mathesis universalis* should have no difficulty seeing how the relevant proportional relations exhibited in the construction yield the required solution. The problem is that Descartes does not himself explicitly provide this construction in *Rules*. He does not provide *any* geometrical interpretation of root extraction in *Rules*, but instead explicitly postpones it:

> As for those divisions in which the divisor is not given but only indicated by some relation, as when we are required to extract the square root or the cube root, etc., in these cases we must note that the term to be divided [i.e., the square or the cube], and all the other terms, *are all to be conceived as lines which form a series of continued proportionals, the first member of which is the unit, and the last the magnitude to be divided.* We shall explain in due course how to find any number of mean proportionals between the latter two magnitudes. For the moment we must be content to point out that we are assuming that we have not yet quite done with these operations, *since in order to be performed they require an indirect and reverse movement of the imagination*, and at present we are dealing only with problems which are to be treated in the direct manner (AT 10:467, CSM 1:75; my emphasis).

Descartes reserves the application of areas via the gnomon property for the operations of arithmetic, but when it comes to root extraction, he seems to have another geometrical procedure in mind. He clearly indicates that the members of a proportional series must be “conceived as lines” *alone*, not as rectangles, and that these lines must “form a series of continued proportionals, the first member of which is the unit, and the last the magnitude to be divided.” The procedure described here is exhibited in the Euclidean construction by means of lines (similar triangles) and semicircles, a construction embodied in Descartes’s mesolabe or proportional compass (1619) and later incorporated into Descartes’s geometrical interpretation of root extraction in the *Geometry* (1637) (Figure 9.31):

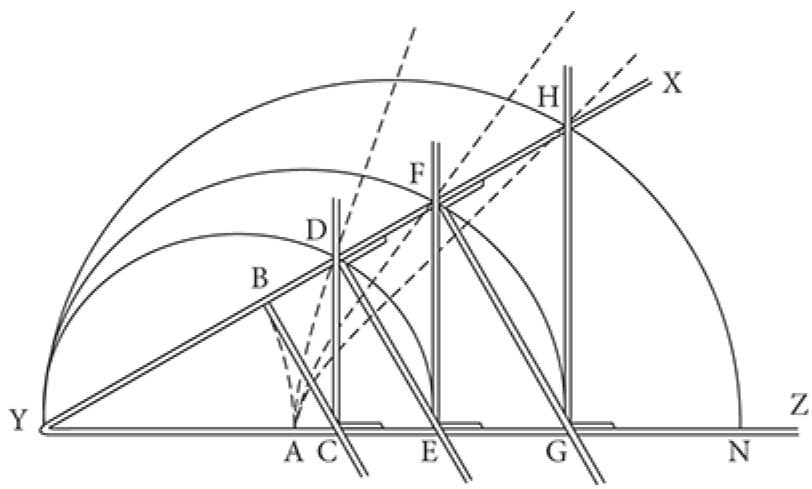


Figure 9.31 Descartes’s proportional compass (AT 6:391)

As the compass expands, it produces a series of semicircles, and in each semicircle there is inscribed a triangle similar to every other inscribed triangle in the compass. The sides of each similar triangle form a series of continued proportionals:

$$YB/YC = YC/YD = YD/YE = YE/YF$$

Let the unit be YB = YA = 1. This yields:

$$1 : x = x : x^2 = x^2 : x^3 = x^3 : x^4, \text{ etc.}$$

The proportional compass presents exactly what Descartes describes in Rule 18: a series of "lines which form a series of continued proportionals, the first member of which is the unit, and the last the magnitude to be divided." To find a mean proportional, I need only intuit the relation between the lines in the compass. For example, to extract the root of x^2 = YD, I immediately see that x = YC.[29]

If Descartes had the proportional compass in mind in Rule 18, it is curious that he does not specifically mention it. It is also curious that he does not mention the role played by the *circle* in root extraction. As we have seen, the circle is by no means coincidental in root extraction, and it is built into the very mechanism of the proportional compass. Readers ignorant of Descartes's mathematical research before *Rules* would be in no position to interpret Descartes's description of root extraction in Rule 18 as a reference to Euclidean construction by means of lines *and semicircles.* The reason why Descartes omits referring to the circle in the second part of *Rules* is that *the circle is only necessary in extracting the roots of continuous incommensurate magnitudes*, *not discrete magnitudes*. To extract the square root of discrete magnitudes (e.g., $\sqrt{9}$), I only need to produce a square composed of the relevant number of points or unit squares. Any side of the square clearly exhibits the root. There is no need for a circle, as there is in the case of continuous incommensurate magnitudes.

Descartes needs to completely overcome the distinction between continuous and discrete magnitude and show how his method can solve all perfectly understood problems by means of his geometrical schematism in Rules 14–15 and Rule 18. This schematism, however, lacks an essential ingredient: the circle,[30] required in geometrical constructions of root extraction, which, as we have seen, is equivalent to the problem of finding one mean proportional: $a:x = x:b$, or $ab = x^2$. Root extraction is an elementary algebraic operation, and without a corresponding geometrical construction, algebra almost entirely loses its applicability in geometry; without the semicircle, one cannot solve even the simplest algebraic equations in two degrees. Lines and rectangles do not suffice here, and neither do points. Clearly, Descartes knew, as anybody even minimally versed in Euclidean geometry would have known, that Euclid uses the semicircle in root extraction. However, as I indicated above, the problem is that the circle has no use in root extractions in the case of discrete

magnitudes. *The introduction of the circle would lead to a different geometrical interpretation of mean proportionals in the case of discrete and continuous magnitudes*. One would extract square roots by means of points or ruled lines in the case of discrete magnitudes, but by means of non-ruled lines and semicircles in the case of continuous magnitudes. The latter case would be irreducible to representation by means of points. For Descartes to have introduced the circle into his geometrical schematism in *Rules* would have undermined its universality and, therefore, the unity of mathematics based on *mathesis universalis* or the theory of proportions. In the *Geometry*, Descartes has no difficulty introducing the circle, since his project in the *Geometry* is purely geometrical. Root extraction in the *Geometry* pertains exclusively to *continuous magnitudes*. In *Rules*, however, one and the same procedure in the imagination must serve as the basis of root extraction *irrespective of the species of magnitude.*

9.8 The Collapse of Descartes's Methodological Enterprise in *Rules*

The dilemma created by Descartes's exclusion of the circle from his geometrical calculus in Rule 18 reflects deeper problems in *Rules*, and sends shockwaves throughout the treatise that ultimately undermine the basic design of the treatise as a whole. The success of Descartes's method in *Rules* depends on the possibility of reducing all imperfectly understood problems to perfectly understood problems. This presupposes that all perfectly understood problems can be solved by means of one procedure. Unfortunately, Descartes's geometrical calculus in Rules 13–21 *prohibits* him from solving all perfectly understood problems by means of one procedure. Problems whose solution requires root extraction cannot be solved by means of lines and rectangles alone (application of areas). This explains why the second part of *Rules* ends precisely on this problem. It also explains why the *third* part of *Rules* never saw the light of day: it makes no sense to reduce all imperfectly understood problems to perfectly understood problems unless perfectly understood problems can themselves be solved by means of one procedure.

Not only does the project of reducing imperfectly understood problems to perfectly understood problems fall apart, but so does another related project: the project of reducing all problems that can be solved indirectly to problems that can be solved directly. Root extraction requires “an indirect and reverse movement of the imagination,” since instead of continuing a series of continuous proportionals in which I only need to consider one relation (as I do in multiplication), I must find one or more mean proportionals between the unit and the magnitude to be divided by considering two or more relations. Geometrically, the indirect movement of the imagination in this case requires root extraction by means of lines and semicircles. Since Descartes cannot provide these means of construction in *Rules*, he does not show how all problems that can be solved indirectly can be reduced to problems that can be solved directly, as he had promised in Rule 6 (see Chapter 4, Section 4.4). Consequently, the system of reductions on which the method depends does not, in the end, succeed. *Even within the class of perfectly understood problems*, there are problems that remain irreducibly heterogeneous to one another. Descartes realized this, and it is precisely at this point that the treatise comes to an end.

Descartes’s Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0010

[1] *Pace* Schuster 2013, 307–49, I do not argue that in Rules 13–21 Descartes lays the foundations of *mathesis universalis*, which Schuster further identifies with Beeckmanian “physico-mathematics.” The techniques introduced in Rules 13–21 far exceed in complexity anything included under the heading of *mathesis universalis*. For more discussion, see Chapter 4, Section 4.7. I examine the appropriateness of “physico-mathematics” as an historiographical category in more detail in Chapter 10.

[2] For more discussion, see Chapter 3, Section 3.5; Chapter 4, Sections 4.2–4.4; Chapter 7, Section 7.4.

[3] Interestingly, both of the problems Descartes introduces in order to illustrate his point—the nature of the magnet and the nature of sound—are imperfectly understood problems. Anticipating the third part of *Rules* (Rules 25–36, never completed), Descartes introduces them as examples in Rule 13 in order to show how imperfectly understood problems can be reduced to perfectly understood problems. For more discussion, see Chapter 3, Section 3.5 and Chapter 10, Section 10.3.

[4] See Rabouin 2020.

[5] On the distinction between “foundational” and “operative” uses of Descartes’s geometrical schematism in *Rules*, see Rabouin 2010.

[6] This, however, is not the real problem. The representation by points is *not* completely absent in Descartes’s geometrical calculus in Rule 18, since in the case of commensurate magnitudes, the lines

are *ruled*, thereby indicating that they are *equivalent* to the representation of magnitudes by means of points. The representation by points creates serious problems in Descartes's geometrical calculus in Rule 18, especially in the case of division. For example, it is not clear how to divide three points by two points. I will not discuss these problems further, but they are discussed in Rabouin 2020.

7 See Costabel in Descartes 1987b, followed by Rabouin 2010. *De solidorum elementis* only exists via a copy made by Leibniz in 1676. It contains a number of exercises in solid geometry (polyhedra) (Part I) and "figurate numbers" (Part II), both subjects explored by a number of German mathematicians in the early 1620s, including Johannes Faulhaber, Johannes Remmelin, and Peter Roth. Descartes is known to have lived in Germany during this time, and his mathematical research in *De solidorum elementis* suggests that it was written in Germany or at least in relation to the mathematical research being conducted in Germany during time he spent there in 1620–21. Many scholars have explored the relation between the type of mathematics Descartes pursues in *De solidorum elementis* and German mathematics in the early 1620s. See Schneider 2008, 1993; Mehl 2001; Penchèvre 2004; and Manders 2006, 1995, cited in Rabouin 2010, 446. Leibniz's copy does not contain representations of figurate numbers by means of points, but as many have noted, the representation of figurate numbers by means of points can be inferred from the first definition in *De solidorum elementis* II: "Solids are best of all formed by superimposing gnomons," where the relevant gnomons are composed of points. See Costabel in Descartes 1987b and Frederico in Descartes 1982. For an excellent reconstruction of *De solidorum elementis*, see Sasaki 2003, 132–49. See also Rabouin 2010 and Warusfel's notes in Descartes 2016.

8 See Heath in Euclid 1908, 1:371.

9 See Frederico in Descartes 1982, 83–91.

10 See Rabouin 2020. On criteria of "exactness" in early modern geometry (including Descartes's), see Bos 2001.

11 See, e.g., AT 10: 236. German and English mathematicians in the sixteenth century referred to algebra as the "cossic art." See Rudolff 1525 and Cajori 1993, 107, 169. The word "cossic" derives from the Italian word "*cosa*," employed by Italian mathematicians in the sixteenth century to designate the unknown "thing" or quantity in an algebraic problem. "*Cosa*" translates the Latin "*res*," which in turn translates the Arabic "*shai* (شيء)," employed by al-Khowârizmî to designate the unknown "thing" or quantity in an algebraic problem. See Cajori 1993, 336. On the "symbolic revolution" in mathematical writing, see Serfati 2005. Descartes first encountered cossic notation as a student at La Flèche in the works of Clavius.

12 Rudolff 1525, reproduced in Cajori 1993, 134.

13 Cited in Serfati 1998, 250.

14 Serfati 2010, 114.

15 Cited in Cajori 1993, 139–49 and Serfati 2010, 113.

16 The example is discussed in Serfati 1998, 250–1.

17 Serfati 1998, 251.

18 Serfati 1998, 253 and Serfati 2010.

19 In the *Geometry*, Descartes expresses unknown magnitudes by means of the lower-case letters *x*, *y*, *z*, etc.

20 Viète, by contrast, represents the powers of known and unknown magnitudes by means of majuscule and minuscule letters alone. Majuscule letters (*A*, *B*, *C*, etc.) designate the relevant magnitude (consonants for known magnitudes, vowels for unknown magnitudes), and minuscule letters representing the relevant power (*q* for *quadratus* or square, *c* for *cubus* or cube). Dq represents

the square of a known magnitude, and Aqqc represents the unknown square of the square of the cube. See Cajori 1993, 181–7 and Serfati 1998, 261, n. 68. For example, Viète writes what in post-Cartesian notation is written as $a^8+3bx = 2z^3$ purely verbally and without abbreviation as "A cubus + B plano 3 A, aequari Z solido 2" (cited in Cajori 1993, 184). Viète's geometrical representation of the powers, moreover, is Euclidean and, therefore, dimensional: Dq can only be geometrically represented as a square, and Dc as a cube, etc. A magnitude such as Aqqc exceeds the three dimensions of space and cannot be geometrically represented (the suffix "qqc" designates what Viète terms an "abstract dimension"). As we will see in Section 9.6, in *Rules* Descartes can geometrically represent any power, since he represents all magnitudes, whatever their power, by means of lines and rectangles in one or two dimensions maximum. In the *Geometry*, he represents all magnitudes by means of lines in one dimension alone (see AT 6:369–70; Descartes 2001, 177–9).

21 See Euclid 1908, 2:287–91 (*Elements* VII, def. 15–19).

22 See Mancosu 2008, 112.

23 Descartes only employs lines and rectangles divided into units in Rule 18. By counting the units in the lines and the rectangles, one can easily see the equality between square *ab* and line *ab* (they have the same number of units). I have deliberately omitted representation by means of units here because Descartes's geometrical calculus in Rule 18 applies to both commensurable and incommensurable magnitudes.

24 Euclid 1908, 1:341.

25 These diagrams can be found in Bos 2009, to which the argument in this section is indebted.

26 This example is discussed in Bos 2009.

27 See Bos 2009.

28 Ibid., 371.

29 See also Mancosu 2008.

30 See Bos 2009; Rabouin 2009, 333; Rabouin 2010. See also Schuster 1977 and 2013, 343.

10

Imperfectly Understood Problems

Descartes's Deduction of the Law of Refraction and the Shape of the Anaclastic Lens in Rule 8

10.1 Neither "Mixed Mathematics" nor "Physico-Mathematics"

While Descartes focuses exclusively on mathematical problems in Rules 13–21, his principal ambition in Rules 13–21 is methodological, not mathematical:

> [These] Rules [13–21] are so useful in the pursuit of deeper wisdom [*altiorem sapientiam*] that I have no hesitation in saying that this part of our method was designed not just for the sake of mathematical problems; our intention was, rather, that the mathematical problems should be studied almost exclusively for the sake of the excellent practice which they give us in the method [*hanc partem nostrae methodi non propter mathematica prolemata fuisse inventam, sed potius haec fere tantum hujus excolendae gratia esse addiscenda*] (Rule 14, AT 10:442, CSM 1:59).

In Rule 12, Descartes stresses that "those who desire a perfect mastery of the latter part of my method (which deals with the other [imperfectly understood] sort of problem) should be advised that a long period of study and practice [in solving perfectly understood problems] is needed in order to acquire this technique [*in hac arte addiscenda diutius versari debere et exerceri illos, qui posteriorem hujus methodi partem, in qua de alijs omnibus tractamus, perfete cupiant possidere*]" (AT 10:430, CSM 1:51). Had Descartes completed *Rules* according to the plan laid out in Rule 12, his readers would be in a position to see exactly how "a long period of study and practice" in dealing with perfectly understood problems enables

us to acquire “perfect mastery” in dealing with imperfectly understood problems, in part because we would have learned “how imperfect problems can all be reduced to perfect ones,” a subject Descartes promised to address in the third projected part of *Rules* (Rules 25–36), which he never completed. Nevertheless, Descartes’s plan for *Rules* clearly places mathematics in an intermediary position between the “simple propositions” discussed in Rules 1–12 and the method for solving imperfectly understood problems in Rules 25–36. For Descartes, the utility of Rules 13–21 ultimately consists in the fact that they perfect the Cartesian scientific *habitus* (and, therefore, the human *ingenium*) so that it may finally solve the most difficult type of problem: imperfectly understood problems.

In this chapter, I reconstruct Descartes’s proposed deduction of the law of refraction and the shape of the anaclastic lens (the lens from which parallel rays of light are refracted toward a common focus) (see Figure 3.1), which is one of the most important imperfectly understood problems in *Rules*, and which illustrates how he would have solved the problem in the projected, but never-completed third part of this text.[1] In the remainder of this section, I introduce and frame the problem of the anaclastic in Rule 8, and in Section 10.2, I discuss the motivations behind and shortcomings in other reconstructions of Descartes’s discovery of the law of refraction and the shape of the anaclastic lens. In Sections 10.3–10.8, I execute the deduction of the law of refraction and the shape of the anaclastic lens according to the order of research Descartes prescribes in Rule 8. I integrate Descartes’s physics and optical research in 1618–1628 into the deduction, and I respect the chronology of the relevant documents (I do not import discoveries Descartes made after *Rules* into *Rules*).[2] Having shown that the law of refraction and the anaclastic lens can indeed be discovered by means of the method as described in Rule 8, in Section 10.9, I conclude that there is no reason to regard Descartes’s method as irrelevant to the practice of Cartesian science. Descartes’s method—i.e., the Cartesian scientific *habitus* —yields serious results in natural philosophy, and cannot be reduced to rhetoric alone, as some have argued.[3]

Descartes’s discovery of the law of refraction and the shape of the anaclastic lens has been the subject of controversy ever since the publication of Descartes’s *Dioptrics* in 1637. While Descartes does provide a demonstration of the law of refraction in *Dioptrics* II, he does not reveal how he discovered the law there. Instead, he deduces the law from premises

based on comparisons to the oblique motion of tennis balls in air and water. He does not clarify the relation between these comparisons and the principles of his physics. Descartes's most extensive discussion of the order of research one must follow in order to discover the law of refraction and the shape of the anaclastic lens is contained in Rule 8 (AT 10:393–5, CSM 1:28–9). Curiously, no reconstruction of Descartes's discovery of the law of refraction and the shape of the anaclastic lens takes Rule 8 as its primary basis.[4] On the contrary, most reconstructions either ignore or positively dismiss Rule 8 as the least likely indicator of Descartes's actual path to these discoveries. For over a century, there has been near-unanimous agreement that Descartes discovered the law of refraction by purely mathematical means, and only later provided the underlying physical rationale (via comparisons or analogies) in *Dioptrics* II.[5] The upshot of many of these reconstructions is that Descartes's method conceals, rather than reveals, how he discovered the law of refraction and the shape of the anaclastic lens; Descartes's method is a species of dissimulating rhetoric, not a cognitive technology that yields serious scientific results. In Rule 8, Descartes expressly denies that the law of refraction can be discovered by purely mathematical means, and he requires that the law of refraction be deduced from physical principles about natural power or force, the nature of the action of light, and the behavior of light rays in a variety of transparent media. One of my purposes in this chapter is to argue that Rule 8 reflects Descartes's actual path to the discovery of the law of refraction and the shape of the anaclastic lens.

When he introduces the problem of the anaclastic in Rule 8,[6] Descartes also discusses the problem of the limits of knowledge (see Chapters 5–8). The problem of the anaclastic, he argues, is not a problem that lies beyond the limits of human knowledge. It is a problem that can be solved, but in which one can nevertheless commit a "sin of omission" or "fail to take account of some condition necessary for defining a problem, a factor which is either explicitly stated in it or is in some way implied by it" (see Rule 13, AT 10:436, CSM 1:55 and my discussion in Chapter 3, Section 3.4.1.1). In the case of the anaclastic, Descartes insists that knowledge of both mathematics and physics is required; mathematics alone does not suffice. Exclusion of physics constitutes a "sin of omission." Take "someone whose studies are confined to mathematics [and who] tries to find the line called the 'anaclastic' in optics. […] [H]e will easily see, by following Rules Five

and Six, that the determination of this line depends on the ratio of the angles of refraction to the angles of incidence" (AT 10:394, CSM 1:29). The ratio between the angle of incidence and the angle of refraction is the law of refraction. However, the pure mathematician "will not be able to find out what this ratio is, since it has to do with physics rather than mathematics" (ibid.). The problem of the anaclastic cannot be solved by the pure mathematician, so "he will be compelled to stop right at the outset" (ibid.). By contrast, someone "whose studies are not confined to mathematics," or whose studies include both mathematics and physics, "will discover when he goes into it" both the reduction of the problem to its simplest component parts as well as how to solve each one:

> Now take someone whose studies are not confined to mathematics and who, following Rule 1, eagerly seeks the truth on any question that arises: if he is faced with the same [1] problem [of the anaclastic], he will discover when he goes into it that [2] the ratio between the angles of incidence and the angles of refraction depends upon [3] the changes in these angles brought about by differences in the media. He will see that [4] these changes depend on the manner in which a ray passes through the entire transparent body, and that [5] knowledge of this process presupposes also a knowledge of the nature of the action of light. Lastly, he will see that [6] to understand the latter process he must know what a natural power in general is – this last being the most absolute term in the series. Once he has clearly ascertained this through mental intuition, he will, in accordance with Rule 5, retrace his course through the same steps. If, at the second step, he is unable to discern at once what the nature of light's action is, in accordance with Rule 7 he will make an enumeration$_3$ of all the other natural powers, in the hope that a knowledge of some other natural power will help him understand this one, if only by way of comparison – but more of this later. Having done that, he will investigate the way in which the ray passes through the whole transparent body. Thus he will follow up the remaining points in due order, until he arrives at the anaclastic itself (AT 10:394–395, CSM 1:29).

In Chapter 3, Section 3.4.1, I represented Descartes's reduction of the problem of the anaclastic to its simplest component problems via enumeration$_1$ as follows (Table 10.1):

Table 10.1 The structure of Descartes's deduction of the anaclastic lens (see Garber 2001, 37)

Enumeration$_1$:
Q1 What is the shape of a line (lens) that focuses parallel rays of light to the same point?
Q2 What is the relation between the angle of incidence and the angle of refraction (i.e., the law of refraction)?
Q3 How is refraction caused by light passing from one medium to another?
Q4 How does a ray of light penetrate a transparent body?
Q5 What is the nature of the action of light?
Q6 What is a natural power?
Intuition and deduction:
Q1 A natural power is…
Q2 The nature of the action of light is…
Q3 A ray of light penetrates a transparent body by…
Q4 Refraction is caused by light passing from one medium to another when…
Q5 The relation between the angle of incidence and the angle of refraction is…
Q6 The shape of the line (lens) that focuses parallel rays of light to the same point is…
Q1 must be solved by means of intuition, and the solution to Q2–Q6 must be deduced from the solution to Q1 in order.

Since deducing the anaclastic requires knowledge of both physics and mathematics, it is clear that the problem of the anaclastic is not a problem in "mixed mathematics." Aristotle distinguishes between "pure" and "mixed" sciences, and he regards optics as a mixed science subalternate to geometry, which he regards as a pure science.[7] Mixed mathematics relies exclusively on mathematical principles. Physical principles are excluded. For example, optics, regarded as a species of mixed mathematics, can only provide a mathematical description of physical phenomena such as reflection ($\angle i = \angle r$). It is up to the physicist to determine the physical causes of reflection. For Aristotle and most Aristotelians, optics deals with mathematical magnitudes, not qua mathematical, but rather qua physical, and it inherits its principles from geometry alone, not natural philosophy. Consequently, the principles of natural philosophy can play no role in optics when the latter is regarded as a species of mixed mathematics. Aristotle's ban on

genus-crossing in the sciences aggressively polices the boundary between mathematics and natural philosophy (see Chapter 1, Section 1.2).

In Rule 8, Descartes intentionally defines the problem of the anaclastic in such a way as to deny that the problem can be solved by mathematics alone ("pure" or "mixed"). On the contrary, in Descartes's reduction of the problem, physics necessarily comes before and aids mathematics: the law of refraction must be deduced from physical principles about natural power or force, the nature of the action of light, and the behavior of light rays in a variety of transparent media. This is why Descartes immediately mentions Rule 1 when he reduces the problem of the anaclastic to its simplest component parts. In Rule 1, Descartes states that "all the sciences are so closely interconnected that it is much easier to learn them all together than to separate one from the other" (AT 10: 361, CSM 1:10). In the case of the anaclastic, mathematics and physics are the relevant "interconnected" sciences. This indicates that the problem of the anaclastic is not a problem in mixed mathematics, which only employs mathematical principles in demonstrations about physical magnitudes.

If the order of research prescribed in Rule 8 is interpreted as reflecting Descartes's path to the discovery of the law of refraction and the shape of the anaclastic lens, then the problem of the anaclastic is also not a problem in "physico-mathematics." As Schuster defines it, physico-mathematics brings mixed mathematics into natural philosophy so that solutions to problems in mixed mathematics are regarded as themselves containing clues about the underlying causes (in Descartes's case, corpuscular-mechanical causes) of the relevant physical phenomena.[8] In practical terms, in the case of optics the (Cartesian) physico-mathematician discovers a purely mathematical law of refraction represented in a well-grounded geometrical diagram and then carefully (and sometimes not so carefully) reads corpuscular-mechanical causes into it. Schuster's "physico-mathematical" reconstruction of Descartes's discovery of the law of refraction and the shape of the anaclastic lens requires dismissing the order of research Descartes prescribes in Rule 8.[9] Since I believe that Rule 8 must be taken more seriously, I argue that Descartes does not set out from a mathematical law toward physical premises, but rather sets out in the reverse direction from physical premises toward a mathematical law. Descartes is not, in my view, a "physico-mathematician" as Schuster defines the term, at least not in optics.

10.2 Problems in Previous Reconstructions

As I mentioned in Section 10.1, most scholars prioritize mathematics over physics in their reconstructions of Descartes's discovery of the law of refraction and the shape of the anaclastic lens. Indeed, ever since the publication of the *Essays* (1637), prominent critics have argued that Descartes's demonstrations of the laws of reflection and refraction in *Dioptrics* II are both incoherent and ad hoc. For example, Descartes's tennis-ball analogy in *Dioptrics* II is a dynamic analogy, based on the local motion of projectiles. Projectiles have speed (distance covered over time), whereas, according to Descartes, light is an instantaneously transmitted mechanical pressure, and so has neither local motion nor, therefore, speed. How, then, can light, which is not in motion, obey the laws of motion, as Descartes asserts in *Dioptrics* II? This has seemed incoherent to many critics, beginning with Fermat and Morin.[10] Furthermore, Descartes's thesis that the parallel component determination of the force of light is not affected by refraction because it does not come into contact with the refracting surface seems entirely designed to preserve the mathematical integrity of the sine law; were the parallel component determination to be affected by refraction, the required proportion between the sines of the angles of incidence and refraction would not obtain. This has seemed ad hoc to many critics, also beginning with Fermat.[11]

Reading critiques of Descartes by Fermat and Morin, one has some difficulty seeing how Descartes could have discovered the law of refraction on the basis of the analogies and comparisons employed in *Dioptrics* II. This naturally leads to the suspicion that Descartes discovered a purely mathematical law of refraction to which he later applied a physical rationale, biting the bullet no matter how ad hoc the explanation seemed to be. One can also understand why Rule 8, where Descartes clearly requires that both physics and mathematics be used in the discovery of the law of refraction and the shape of the anaclastic lens, may not have seemed terribly helpful, since it merely lays out a research program that Descartes does not execute. When one combines these facts with the fact that Descartes likely discovered the law of refraction in 1626/1627, before he had developed his systematic physics in *The World* (1629–1633), it seems reasonable to assume that Descartes must have discovered the law of refraction by purely

mathematical means, providing a physical interpretation of the law afterwards.

The prioritization of mathematics over physics in reconstructions of Descartes's discovery of the law of refraction can be traced back to the pioneering work of Gaston Milhaud.[12] Milhaud argues that Descartes's demonstration of the sine law in *Dioptrics* II is incoherent and ad hoc, much along the lines discussed above. This leads him to seek an alternative reconstruction of Descartes's path to the discovery of the law of refraction, based on mathematics alone. He argues that Descartes discovered the sine law by operating on geometrical representations of the anaclastic ellipse. Now, as Schuster correctly argues, however elegant this reconstruction is, there is no evidence that Descartes actually discovered the sine law in this manner, or that he knew that the ellipse is an anaclastic prior to his discovery of the law of refraction.[13] Nevertheless, one can disagree with Milhaud's thesis that Descartes discovered the law by operating on geometrical representations of the anaclastic ellipse and agree with him that Descartes must have discovered the law by purely mathematical means (as Schuster and many others do).

Much of the scholarly debate on Descartes's discovery of the law of refraction after Milhaud makes sense in this context. The main disagreements have been about *which* purely mathematical procedures Descartes employed in order to discover the law, not *whether* he employed purely mathematical procedures in order to discover the law.[14] Most reconstructions begin by showing why Descartes's demonstration of the law in *Dioptrics* II is problematic and, therefore, misleading as an indicator of how he actually discovered it. The road is then open to arguing that Descartes must have discovered the law by purely mathematical means, since he did not have a systematic physics when he discovered the law in 1626/1627. Once this argument has been made, the debate narrows to disagreements about which purely mathematical procedures Descartes employed. Did he rely on the properties of the ellipse?[15] Did he discover it by using image-location principle well-known in geometrical optics since Euclid?[16] Did he rely on the properties of the prism?[17] Or did he instead rely on the no less geometrical medieval perspectivist optics?[18] Once the relevant purely mathematical route has been reconstructed, the task is then to show how Descartes developed the underlying physical rationale in a

manner that would preserve the integrity of his mathematical law. Showing how Descartes developed the underlying physical rationale has the added advantage of explaining why Descartes's demonstration of the law in *Dioptrics* II seems so incoherent and ad hoc: the physical rationale is read into a mathematical law that Descartes discovered without recourse to physical considerations.

The attractiveness of the strategy described above is obvious, but it leads to a major problem: dismissing Rule 8, arguably the most important document detailing Descartes's reflections on how to discover the law of refraction and the shape of the anaclastic lens. In no other document does Descartes so explicitly prescribe the order one should follow in making these discoveries. This order leads *from physics to mathematics*, not vice versa. Since the reconstructions prioritizing mathematics over physics themselves face a number of serious difficulties, I will enumerate them here in order to motivate my reconstruction based on Rule 8 in Sections 10.3–10.8.

Schuster maintains that Descartes discovered a purely mathematical law of refraction with the French mathematician Claude Mydorge around 1626/1627, when Descartes lived in Paris and is known to have collaborated with Mydorge.[19] By employing the image-location principle (a purely mathematical principle well-known in mathematical optics since Euclid and Ptolemy), Schuster argues, Descartes discovered a cosecant law, and only later hit upon the sine law.[20] As evidence, Schuster cites a document that Mydorge sent to Mersenne in 1631.[21]

There are a number of problems with Schuster's reconstruction. First, as Heeffer has shown, the Mydorge document does not mention Descartes and is not a report on the research Descartes and Mydorge conducted in 1626/1627 (as Schuster claims), but rather part of a treatise Mydorge intended to publish (Book VIII of *Prodromi catoptricorum et dioptricorum*).[22] Thus, while it certainly seems to be the case that Descartes was familiar with Mydorge's research, the Mydorge document should be read with more caution, and should not be regarded as representative of Descartes's path to the discovery of the law of refraction.[23]

Second, as Heeffer also points out, Mydorge does not refer to the cosecants of the incident and refracted rays in Propositions I–II, as Schuster claims.[24] Mydorge only writes that "leurs angles d'inclinations et de

refractions doivent estre comparez ensemble suivant la raison de la droite GE à la droite EF" or (also) HE and GE (R1 and R2 in Figure 10.4 below), a ratio that can be expressed either as a ratio of sines or cosecants.[25] Mydorge consistently refers to the proportion that obtains between the sines of the incident and refracted rays, and he never refers to cosecants anywhere in the document. To be sure, Schuster argues that the existence of a "cosecant" version of the law is clearly indicated by the relevant diagram (see Figure 10.4 below), since it is based on two unequal semicircles (and, therefore, unequal radii) in which the sines of the angles (i.e., the parallel component determinations of the ray before and after refraction) are equal and the cosines (i.e., the normal component determinations of the ray before and after refraction) are unequal but in constant proportion to one another. Schuster is right that the difference between the diagrams is relevant, but even he does not deny that the difference between the sines and the cosecants in the first diagram are trigonometrically equivalent and, therefore, trivial.

Third, Schuster's claim that Descartes first discovered a cosecant law on the basis of the image-location principle requires dismissing what he (Schuster) himself regards as a "difficulty in this [his own] reconstruction": viz., that Descartes, having read Kepler, "was well aware of Kepler's new theory of vision, which case grave doubt on the use of the traditional [image-location] rule," and that Descartes's "work depended on an optical principle he could no longer accept"[26]

Fourth, as I will show in Section 10.6 in more detail, Schuster interprets Descartes's early optical research in the optical fragment of 1619 (AT 10:242–3) in ways that place some strain on the evidence contained within the document. Interpreting Descartes's optical fragment as a physico-mathematical interpretation of a diagram representing refraction from rarer to denser media in Kepler,[27] Schuster argues against Sabra that in the fragment Descartes asserts that there is a proportional relation between the normal components of light rays before and after refraction. This means that, however one interprets the relation between the parallel components, the optical fragment can only yield a law of tangents, and so Descartes's early optical fragment does not provide insight into Descartes's path to the discovery of the sine law. However, as Schuster would no doubt acknowledge, Descartes does not explicitly refer to either Kepler or to the normal components of light rays in the optical fragment. Descartes does not

seem to be reasoning with such components in the optical fragment at all. As I understand it (and I think this is the more natural interpretation), the document provides clear evidence that Descartes already possessed a key premise of the sine law: viz., that there is a direct proportion between the force of light and the density of the medium. This premise, I also argue, is based on purely physical, not mathematical considerations.

In sum, Schuster's claim that Descartes employed the image-location principle in order to discover a cosecant law by purely mathematical means, and only later hit upon the sine law and developed the underlying physical (corpuscular-mechanical) rationale as a "physico-mathematician," is not the only interpretation warranted by the available evidence, and it sometimes dismisses or, in my view, places undue strain on the available evidence, such as Rule 8 and the optical fragment of 1619.

Heeffer does not discuss Rule 8, but argues that Descartes discovered the law of refraction by examining two diagrams in Kepler's *Paralipomena*, one of which can be analyzed to yield the sine law, and the other of which is very similar to the equal-arm balance diagram that Descartes communicates to Beeckman in October 1628 in order to illustrate the sine law.[28] Like Schuster, Heeffer's interpretation provides a plausible reconstruction of Descartes's path to discovery of the sine law, but beyond the very suggestive similarity between the equal-arm balance diagrams in Kepler and Descartes, there is no direct evidence to support his reconstruction other than the fact that Descartes refers to Kepler as his "first master in optics."[29] In addition to not discussing Rule 8, Heeffer does not discuss Descartes's optical fragment of 1619 or the entry in Beeckman's journal of October 8, 1628.

Smith impressively argues that Descartes discovered a purely mathematical law of refraction (first a cosine law, followed by a sine law) by manipulating diagrams based on the principles of medieval perspectival optics.[30] The relevant principles are three in number.[31] The first principle asserts a direct proportionality between distance and intensity/speed of the light ray, such that of any two rays, the first of which covers twice the distance in the same direction over the same time as the second, the first must have twice the speed as the second.[32] The second principle asserts a direct proportionality between obliquity of incidence and intensity of the normal component of the ray, such that as the angle of incidence increases

relative to the surface, the intensity of the normal component also increases.[33] The third principle is the principle of vertical conservation, according to which the intensity of the vertical component of the light ray must be conserved after refraction.[34]

On the basis of these three principles, Smith argues that Descartes first hit upon a cosine law, which generated problems that forced him to adopt a sine law as a solution. The cosine law asserts a direct proportionality between the cosines of the angles of incidence and refraction (i.e., the intensity of the normal components of the light ray before and after refraction, which must, according to the third principle, always be equal to one another) (see Figure 10.1). The cosine law, however, leads to problems. First, it is "inconsistent with the rule of vertical conservation," since the lines SB and BR are not equal to one another "even though they should be."[35] Any ray transmitted along SB will be *shorter* after refraction along BR. Second, as the incident ray MB is rotated toward S, it will eventually reach a point X at which the normal "dropped from X to EO will be equal to BR," which would require it to refract along BR according to the principle of vertical conservation.[36] That such a ray should *not* refract along BR is obvious, since only a ray transmitted along SB should refract along BR. Third, as one further rotates the incident ray MB toward S, the normal dropped from MB "will actually *exceed* BR, so that even if the light were to refract along BR, the rule of vertical conservation would be violated."[37] To solve these problems, Smith argues, Descartes only had to rotate the entire diagram 90º clockwise (see Figure 10.2) so that the relevant proportion obtains between the sines rather than the cosines of the incident and refracted rays. The diagram now represents the length of the refracted ray as *longer* than the incident ray, which means that it must have *gained* intensity after refraction in a denser medium. This yields the first premise of Descartes's sine law, according to which the more density the medium has, the more intensity or force the light ray has (*F*i:*F*r). Second, in the rotated diagram, the principle of conservation pertains to the parallel components, not the normal components. This yields the second premise of Descartes's sine law, according to which the parallel component determination is conserved after refraction (*sin* i: *sin* r). Both premises show up in *Dioptrics* II, and the remaining problems mentioned above vanish.[38]

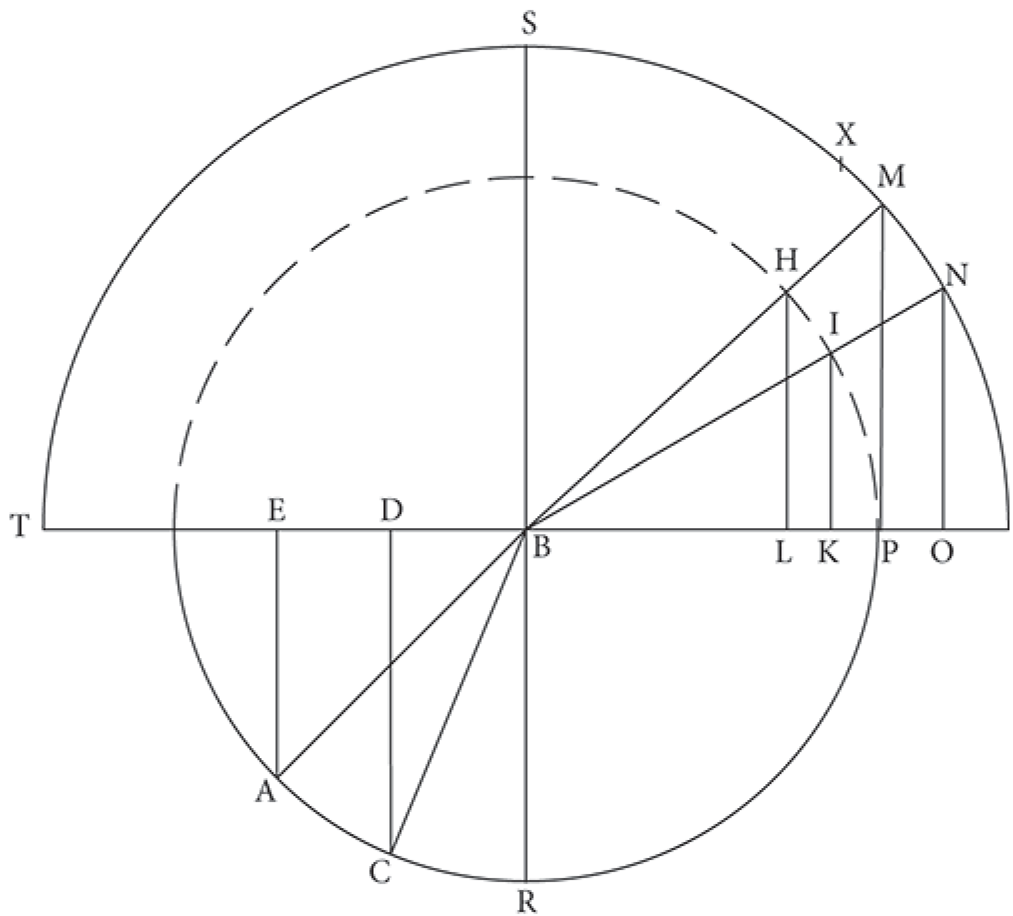


Figure 10.1 Cosine law of refraction derived from principles of perspectivist optics (Smith 1987, 61)

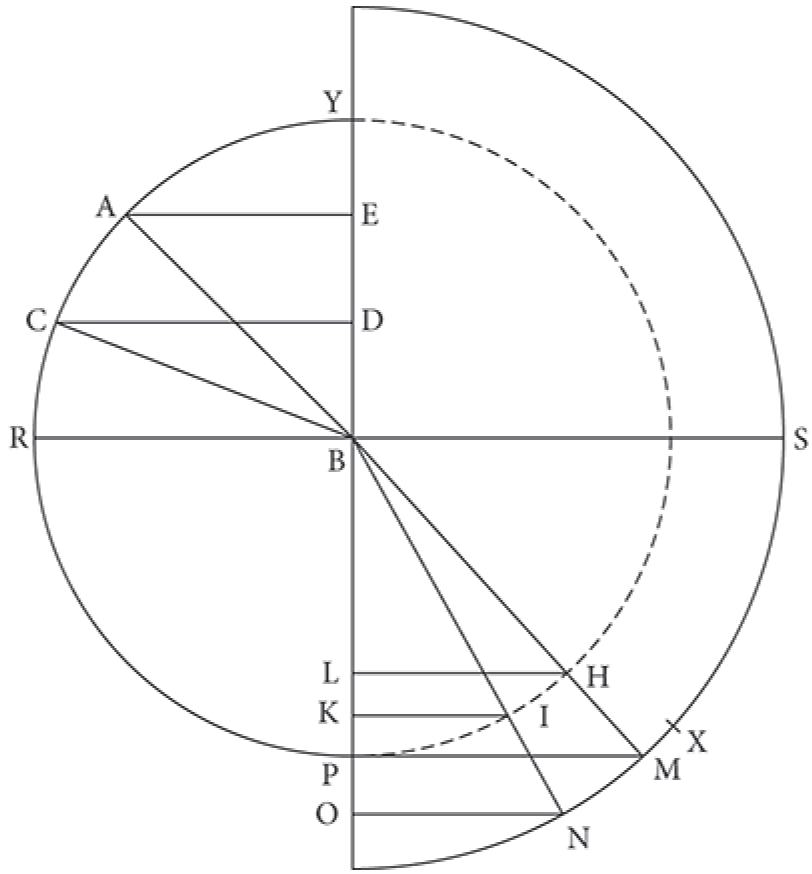


Figure 10.2 Sine law of refraction derived of principles of perspectivist optics (Smith 1987, 63)

One problem with Smith's interpretation is that Descartes asserted the proportional relation between force and density in the optical fragment of 1619 independently of any verifiable engagement with medieval perspectival optics and long before he discovered any mathematical law of refraction. The optical fragment makes no mention of any medieval perspectival optical principles. On the contrary, by asserting that the force of light *increases* in denser media, the optical fragment explicitly contradicts these principles. More importantly, since the optical fragment does not contain the law of refraction, it is clear that Descartes asserted the proportion between the force of light and the density of the medium *before he possessed any mathematical law of refraction*. Descartes did not, as Smith argues, discover the law of refraction and then infer, on the basis of a diagram representing the law, that the relation between density and force is directly proportional. On the contrary, he discovered the relevant

proportionality *before* he discovered the law. Consequently, the first premise of the sine law does not seem to have been discovered in the way that Smith suggests. Regarding the second premise, as Smith himself recognizes, the perspectivists did *not* maintain that light conserves its vertical intensity after refraction, but only that it does so *as much as possible*.[39] This means that *the vertical components of the incident and refracted rays cannot be equal to one another in any mathematical representation of the perspectival principles Smith discusses*. Without such an equality, however, there can be neither a cosine law nor a sine law as Smith understands the relation between the two in his reconstruction of Descartes's path to discovery. There is no principle of conservation of vertical intensity implicitly built into medieval perspectival theories that Descartes would merely have to turn on its head in order to arrive at his own formulation of the sine law via a cosine law. At best, there is a principle of *maximum conservation*, which is both physically and mathematically a far cry from *complete conservation*. Thus, there are good reasons to *deny* that Descartes could have discovered either a cosine law or a sine law based on medieval perspectival principles.

Costabel regards *Dioptrics* I–II as an execution of the research program prescribed in Rule 8 and as the justification of the law, but he also believes that Descartes, like Mydorge, came to possess the law by purely mathematical means, exploiting the representation of refraction in a prism.[40] Costabel's evidence for the latter claim is based on two sources: Mydorge's representation of refraction in a prism in a document sent to Mersenne in 1631 (at the earliest) and most probably written in 1626/1627, and Descartes's letter to Ferrier on November 13, 1629 (see AT 1:63). There are two significant facts about Descartes's letter to Ferrier. First, Descartes claims in a letter to Mersenne (November 25, 1630, AT 1:180) that a discerning mind could have discovered the sine law by examining letters that he (Descartes) sent to Ferrier describing how to manufacture hyperbolic lenses based on a geometrical representation of refraction in a prism. Second, in his letter to Ferrier, Descartes refers to Mydorge as the one who follows the same path, thereby indicating that he was familiar with Mydorge's representation of refraction in a prism. Costabel concludes that both Mydorge and Descartes discovered the law of refraction by exploiting the representation of refraction in a prism.

Costabel's reconstruction is not without its problems. In his report to Beeckman on October 8, 1628, Descartes only indicates that he (empirically) *explores* the phenomenon of refraction via refraction in a prism, not that he *discovered* the law of refraction by exploiting a geometrical representation of refraction in a prism (see AT 10:335). When one examines the entry in Beeckman's journal (ibid.), one sees very clearly that Descartes introduces the prism, not in order to deduce the law of refraction, but rather in order to obtain "the quantity of the angle of refraction" and then determine all the others by reference to a law that has every appearance of having been independently discovered (see AT 10:335). Had Descartes discovered the law by exploiting the geometrical representation of refraction in a prism, he would have added, in addition to the lines representing the incident and refracted rays, the normal to the refracting surface as well as the parallel and normal component determinations. He does not do that in the prism, but rather in the representation of refraction via a static analogy (the equal-arm balance, to whose arms are attached equal weights, and one of whose arms is submerged in water).[41] The Ferrier letter was written after Descartes had already presented Beeckman with the sine law and a physical rationale based on this static analogy on October 8, 1628, together with a demonstration that the ellipse is an anaclastic. What Descartes did not have in hand is a demonstration that the hyperbola is an anaclastic; he asked Beeckman to demonstrate this, and Beeckman did demonstrate it on February 1, 1629 (see AT 10:341), to Descartes's great satisfaction. It is, therefore, more likely that the material in the Ferrier letter indicates how Descartes learned to apply the sine law to the hyperbola from Beeckman, not how he himself discovered the law.[42]

Unlike previous reconstructions of Descartes's discovery of the law of refraction, my reconstruction is based on Rule 8, where Descartes prescribes an order of research that starts with physics and ends with mathematics. Throughout my reconstruction I will draw on Descartes's early physics and optics in 1619–1628. As we will see, despite a widespread tendency to see an irreducible conflict between Descartes's actual path to the discovery of the sine law and the methodological path he lays out in Rule 8,[43] there is no such conflict: Descartes's discovery of the law of refraction in the 1620s is reflected in his proposed deduction of the anaclastic in Rule 8, and Rule 8 should be read as the culmination of

Descartes's early physics and optical research leading up to his discovery of the law of refraction. The relation between history and method in Descartes's discovery of the sine law is dialectical and mutually reinforcing, not rhetorical and mutually exclusive.[44]

10.3 From Imperfectly Understood Problems to Perfectly Understood Problems: Enumeration$_1$

Before I carry out the deduction proposed in Rule 8, I must discuss how Descartes introduces order into the problem of the anaclastic by reducing it to its simplest component parts via enumeration$_1$. This is not obvious. The problem of the anaclastic—like all problems in natural philosophy—is what Descartes terms an "imperfectly understood problem" in *Rules*. As we have seen in Chapter 3, Section 3.5, unlike perfectly understood problems, imperfectly understood problems are problems in which one or more of the following conditions remains unsatisfied:

> [F]irst, what the criteria are which enable us to recognize what we are looking for when we come upon it; second, what exactly is the basis from which we ought to deduce it; third, how it is to be proved that the two are so mutually dependent that the one cannot alter in any respect without there being a corresponding alteration in the other. So now that we possess all the premises, the only thing that remains to be shown is how the conclusion is to be found (AT 10:429, CSM 1:50–1).

In Rule 8, Descartes has already reduced the problem of the anaclastic to a perfectly understood problem. All of the conditions relevant to the solution of the problem are simply provided for the reader. The process whereby Descartes discovered these conditions is only indicated; it is not explicitly discussed. For those who, unlike Descartes, have not yet reduced the problem of the anaclastic to its simplest component parts, only the search criteria "which enable us to recognize what we are looking for when we come upon it" are known: once the shape of the lens that refracts parallel rays of light toward a common focus has been discovered, then the problem has been solved. The conditions relevant to the solution of the problem (the premises) and, therefore, the relation between the premises and the solution to the problem (the conclusion), are not known. They must be found.

How can the method be employed in order to discover the conditions relevant to the solution of the problem of the anaclastic?

In natural philosophy, imperfectly understood problems can only be reduced to perfectly understood problems by means of enumeration$_1$. As we have seen in Chapter 3, Section 3.4.1, enumeration$_1$ is the operation whereby all of the conditions relevant to the solution of the problem can be determined and ordered "by following Rules 5 and 6," the two rules one must follow "if our choice [of the order in which things are enumerated$_1$] is to be intelligently thought out" (Rule 7, AT 10:391, CSM 1:27). In the case of the anaclastic, the enumeration$_1$ of the conditions relevant to the solution of the problem consists in the reduction of the problem to its simplest component parts as follows:

The search criteria naturally direct the operator of the method to examine "the ratio between the angles of incidence and the angles of refraction" (AT 10:394, CSM 1:29), so it is the search criteria themselves that yield the second problem in the series: "What is the relation between the angle of incidence and the angle of refraction (the law of refraction)?"

The third and fourth problems in the reduction can only be discovered by experiment. One must examine the media themselves in order to determine, not only that they bring about changes in the angles of incidence and refraction, but more importantly how they do so. This directs the operator of the method to examine how "refraction is caused by passing from one medium to another," which is the third problem in the reduction. One learns via experience that light rays seem to pass through transparent media in straight lines and that, depending on the medium, these rays are sometimes refracted toward and sometimes away from the normal. As Descartes puts it in *Dioptrics* I: "[T]hese rays must always be imagined to be exactly straight when they pass through a single transparent body which is uniform throughout. But when they meet certain other bodies, they are liable to be deflected by them, or weakened, in the same way that the movement of a ball or stone thrown into the air is deflected by the bodies it encounters" (AT 6:88–9, CSM 1:155). It is here that one discovers that the angles of incidence and refraction themselves present no determinate ratio in any pair of media, since "the ratio or proportion between these angles varies with all the different inclinations of the rays" (AT 6:101; CSM 1:161). This clearly indicates that the law of refraction cannot be expressed

as a direct ratio between the angles of incidence and refraction themselves, but rather by means of a comparison between some other, associated magnitudes (which magnitudes is not yet clear).

Research on the manner in which light passes through transparent bodies—the fourth problem in the reduction—is perhaps the most experimentally extensive part of the reduction of the problem of the anaclastic. One must conduct research into the *composition* and *shape* of a variety of bodies and surfaces that refract light. Do hard surfaces refract light differently than fluid surfaces? In the case of hard surfaces, are they "even and smooth, or rough and uneven"? If they are "even and smooth," are they "flat or curved"? If they are "uneven," are "they composed of many variously curved parts, each quite smooth in itself," or do they have "many different angles or points, or some parts harder than others, or parts which are moving (their movements being varied in a thousand imaginable ways)" (*Dioptrics* I, AT 6:89, CSM 1:155)? Indeed, while Descartes does not explicitly mention reflection in Rule 8, it is clear that he regards any serious research on refraction to require looking into why some surfaces refract light while others reflect light. It also requires an at least rudimentary classification of how different surfaces reflect or refract light. In the case of reflection, one must observe via experience that when many rays "coming from the same direction meet a body whose surface is completely smooth and even, they are reflected uniformly and in the same order, so that if this surface is completely flat they keep the same distance between them after having met it as they had beforehand" (AT 6, 89–90; CSM 1, 156). Even surfaces reflect rays in the same direction at even distances before and after reflection, while uneven surfaces deflect them in different directions at different distances from one another, "each according to the position of the part of the surface that it touches" (ibid.). We may also consider the color of the surfaces (black surfaces neither reflect nor refract light) as well as whether or not they are highly polished (highly polished mirrors reflect rays more clearly). In the case of refraction, experimental research on the manner in which light passes through transparent bodies proves particularly important vis-à-vis the anaclastic, since it indicates that the shape of the anaclastic must be *curvilinear*, for "if it [the surface] is curved inward or outward they [the light rays] come towards each other or go away from each other in the same order, more or less, *on account of this curvature*" (*Dioptrics* II, AT 6:89–90, CSM 1:155–6; my emphasis). In short, *the*

anaclastic must have the shape of an ellipse, a parabola, and/or an hyperbola, and determining the shape of the anaclastic will require *mathematical knowledge of the properties of the conics.*

The third and fourth problems in the reduction conclude the experimental part of the reduction. Examining the mechanical properties of the relevant media naturally leads to a problem that cannot be experimentally explored: what does the action of light consist in, such that it behaves in the various ways enumerated$_3$ above?[45] This yields the fifth problem in the reduction: "What is the nature of light's action?" Finally, since light is a natural power, knowledge of its action depends on knowledge of "what a natural power in general is," the sixth and most basic problem in the reduction.

In the case of the anaclastic, enumeration$_1$ is an experimental and inferential activity that lies behind, but is not explicitly stated in, the reduction of the problem of the anaclastic to its simplest component parts in Rule 8; only the results of the enumeration$_1$—the ordered problems themselves—are explicitly stated by Descartes. Nevertheless, it is clear that the reduction of the problem of the anaclastic to its simplest component parts via enumeration$_1$ requires a consideration of relevant and irrelevant conditions based on the search criteria (on relevant and irrelevant conditions in problems, see Chapter 3, Section 3.4.1.1). In the case of the anaclastic, experience "would seem to help us see how we might proceed in our investigation by suggesting what further questions it might be helpful for us to look into."[46] Just as Descartes deduces the cause of the rainbow in *Meteorology* VIII by first enumerating$_3$ the conditions under which rainbows are produced,[47] so too does he recommend deducing the anaclastic in Rule 8 by first enumerating$_3$ the conditions under which reflections and refractions are produced. Only the reduction of the problem depends on experiment, not the solution to the problem. The law of refraction may be experimentally confirmed, but it cannot be experimentally discovered, as Descartes explicitly underlines in Rule 8 (AT 10:394, CSM 1:29). Nor can problems about the nature of the action of light and what a natural power is be experimentally solved. Once these problems are solved by means of intuition, one can "follow up the remaining points in due order, until" one "arrives at the anaclastic itself."

10.4 What is a Natural Power (*Potentia Naturalis*)? Descartes's Pre-Metaphysical Physics

The solution to the problem of what a natural power is depends on intuition alone. More specifically, it depends on the intuition of material and common simple natures such as extension; motion and rest; and duration and instant. As we will see below, Descartes's theory of simple natures yields a theory of plenum space as well as the first and third laws of nature in *The World* (the conservation of motion and the conservation of the rectilinear tendency to motion at an instant, respectively). These principles of Descartes's physics play important roles in his deduction of the law of refraction and the shape of the anaclastic lens.

As we have seen in Chapter 7, Descartes's theory of simple natures in *Rules* rests on four pillars, which it may be useful to remind the reader of here:

(1) Descartes divides the simple natures into three classes: intellectual (e.g., knowledge, doubt, ignorance, volition, etc.); material (e.g., extension, shape, motion, etc.); common (e.g., existence, unity, duration, instant, and "common notions," or material inferential principles such as "things that are the same as a third thing are the same as each other") (see AT 10:419, CSM 1:44–5). (2) The simple natures are the most basic notions that can be intuited (clearly and distinctly perceived) by the mind. All other notions or ideas are composed of simple natures. Each simple nature can be intuited entirely on its own either by the intellect or the intellect aided by the imagination. Furthermore, any notion or idea composed of two or three simple natures can also be intuited, provided that the simple natures of which it is composed and the relations between them are intuited (see AT 10:369, 421; CSM 1:14–14, 45–6). (3) Whether a simple nature is an intellectual or material simple nature depends on whether intuiting it requires the intellect alone or the intellect aided by the imagination (which can only conceive extended bodies). The simple natures of extension, shape, and motion are material, since they must be intuited by the intellect aided by the imagination. Finally, (4) all simple natures can be conjoined or connected to one another in one of two ways: necessarily or contingently. There is a necessary conjunction between any two simple natures whenever the intuition of one depends on the intuition of the other. For example, the

intuition of motion depends on the intuition of extension, and so motion is necessarily conjoined with extension. In all other cases, the relevant simple natures are either contingently conjoined or not conjoined at all (see Rule 12, AT 10:418–30, CSM 1:44–51).

With these reminders in place, I return to the first problem that needs to be solved in order to deduce the law of refraction and the shape of the anaclastic lens: what is a natural power? It seems reasonably clear from the very beginning that a natural power is a *force* and that it is closely related to the material simple natures of extension, shape, and motion (especially motion). But its content cannot be determined by any one material simple nature alone; none of the material simple natures individually adds up to force. Force is not included in the intuition of extension or shape (extension and shape are purely geometrical and have no dynamic properties). Motion is closely related to force, but force is the *cause* of motion, which is its *effect.* Consequently, no intuition of motion can by itself deliver an intuition of force, any more than the intuition of an effect can yield an intuition of its cause. At best, motion is necessarily conjoined to or depends on force. Strictly speaking, motion "makes bodies pass from one place to another and successively occupy all the spaces which exist in between" (*The World*, AT 11:40, CSM 1:94). It is "the action by which a body travels from one place to another" (*Principles* I. 24, AT 8A:53, CSM 1:233).[48] These descriptions indicate a relation between force and motion (motion "makes" bodies pass from one place to another; it is an "action"), but they do not clarify the relation. Furthermore, bodies can exert force (pressure) on other bodies without being in motion, and bodies at rest also have force (force of resisting motion).[49] Motion may depend on force, but force does not depend on motion.

The anomalous status of force in Descartes's theory of simple natures makes it difficult to understand what Descartes would have regarded as a solution to the problem of what a natural power is in Rule 8. Nevertheless, force is so closely related to motion that by examining motion on the basis of Descartes's theory of simple natures in *Rules*, one can see how Descartes would have solved the problem of what a natural power is in Rule 8. After showing how Descartes's theory of simple natures yields a theory of plenum space as well as the first and third laws of nature in *The World,* I show that the nature of force only comes to the fore as a presupposition of Descartes's first law: force is the tendency every body has to persist in the

same state (motion or rest). The measure of the quantity of force depends on the material simple natures of extension, motion, and one of the measurable "dimensions" or modes of motion: speed.

In Rule 14 Descartes writes: "By 'extension' we mean whatever has length, breadth, and depth […]" (AT 10:442, CSM 1:59). Since the material simple natures can only be intuited by the intellect aided by the imagination, and since the imagination can only conceive extended bodies, the intellect aided by the imagination cannot distinguish between extension, extended bodies, and space. Descartes repeatedly insists as much throughout Rule 14:

> For although someone may convince himself that it is not self-contradictory for extension per se to exist all on its own even if everything extended in the universe were annihilated, he would not be employing a corporeal idea in conceiving this, but merely an incorrect judgment of the intellect alone. He will admit this himself if he carefully reflects on the image of extension which he tries to form in his imagination. Consequently, whatever our intellect believes about the truth of the matter, these abstract entities are never formed in the imagination in isolation from subjects. […] [H]enceforth we shall not be undertaking anything without the aid of the imagination (AT 10:442–3, CSM 1:59).
>
> [W]e think that there ought to be no difference in conception between extension and that which is extended (AT 10:443, CSM 1:60).
>
> Many […] make the mistake of thinking that extension contains something distinct from that which is extended, in the same way as Paul's wealth is distinct from Paul (AT 10:444, CSM 1:60).
>
> [E]xtension taken in this sense [as distinct from body] cannot be grasped by the imagination. […] Now such an idea necessarily involves the concept of body. So if they [the learned] say that extension so conceived is not body, they are unwittingly snared into saying 'The same thing is at once body and not body.' (AT 10:444–5, CSM 1:60).[50]

Descartes's denial of any distinction between extension, extended things or bodies, and space in Rule 14 explains why he denies that space can be intuited as a void in Rule 12:

> If, say, we conclude that a given space full of air is empty, on the grounds that we do not perceive anything in it, either by sight, touch, or any other sense, then we are incorrectly conjoining the nature of a vacuum with the nature of this space (AT 10:424, CSM 1:48).

In short, the intuition of extension yields no distinction whatever between extension, extended bodies, and space.[51] This means that space is an extended plenum, and that interactions between bodies are interactions in a plenum.[52]

I will now show how the intuition of motion and rest yields Descartes's first law of nature in *The World* (conservation of motion), and how the intuition of an instant of motion yields Descartes's third law of nature in *The World* (conservation of the rectilinear tendency to motion at an instant).

As we have seen in Chapter 7, Sections 7.4 and 7.6–7.7, according to Descartes's theory of simple natures, motion and rest are mutually exclusive simple natures; the intuition of the one in no way depends on the intuition of the other and vice versa (there is no necessary conjunction between motion and rest).[53] This means that there is nothing in the intuition of an extended body in motion that contains or even entails rest (and vice versa). In terms of Descartes's later metaphysics of substance, attribute, and mode, motion and rest are opposite modes, and it is in virtue of their modal opposition that one never tends toward the other (see *Principles* II. 37, AT 8A:62–3, CSM 1:241 and *Principles* II. 44, AT 8A:67; CSM 1:255), "for rest is the opposite of motion, and nothing can by its own nature tend towards its opposite, or towards its own destruction" (*Principles* II. 37, AT 8A:63, CSM 1:241). The mutual exclusion of motion and rest yields Descartes first law of nature in *The World* and *Principles* II. 37: the law of the conservation of motion. A body once in motion "will continue with equal force unless others stop or retard it" (AT 11:38, CSM 1:93), or more simply "what is once in motion always continues to move" (*Principles* II. 37, AT 8A:62, CSM 1:240).[54] The conservation of motion is a conservation of the *force* of motion. This force has a definite quantity (it can increase or diminish), and it is this quantity that is conserved unless some other body affects it in collision. The first law of nature in *The World* and *Principles* is inscribed in the very intuition of the material simple natures of motion and rest in *Rules*, even if it is not metaphysically deduced there.

Displacement in a plenum is necessarily curvilinear, since a body in motion constantly collides with other bodies (including the air), and these bodies deflect its path. However, according to Descartes's third law of nature in *The World*, at every instant of its motion, a body has a tendency to motion in a definite *direction*, which is *rectilinear*. This rectilinear tendency is never realized due to successive collisions with other bodies: "When a body is moving, even though its motion for the most part takes place along a curved path […] yet each of its parts individually tends always to continue moving along a straight line. And so the action of these parts – i.e., the tendency they have to move – is different from their motion" (AT 11:43–44,

CSM 1:96).[55] The tendency to rectilinear motion is instantaneous; at any determinable instant of motion, a body has a definite quantity of force, which can only push the body in a straight line.[56] Why?

The theory of simple natures is behind Descartes's principal argument in support of the third law of nature in *The World*, which is that circular or curvilinear motion *is too complex and, therefore, cannot be conceived in an instant:*

> So it is that of all motions, *only motion in a straight line is entirely simple and has a nature which may be wholly grasped in an instant*. For in order to conceive such motion it suffices to think that a body is in the process of moving in a certain direction, and that this is the case at each determinable instant during the time it is moving. By contrast, in order to conceive a circular motion, or any other possible motion, it is necessary to consider *at least two of its instants, or rather two of its parts, and the relation between them*" (AT 11:45, CSM 1:96–7; my emphasis).

As we have seen in Chapter 7, Section 7.12, in Rule 6, Descartes places "straight" in the class of simple natures because he regards it as simple and absolute, while he regards "oblique" as composite and relative (AT 10:381–2, CSM 1:21). Obliquity presupposes rectilinearity because it can only be conceived relative to rectilinearity: obliquity is rectilinearity *plus a change in direction*.[57] A change in direction is not contained in the intuition of a straight line. To conceive rectilinear motion, it "suffices to think that a body is in the process of moving in a certain direction," whatever the direction, provided that it remains uniform. To conceive curvilinear motion, however, I need multiple directions (at least two).[58] In the passage from *The World* cited above, Descartes effectively argues that the simplicity of rectilinear motion entails that it alone can be "wholly grasped in an instant." To intuit a curvilinear motion, I need to consider at least two instants of motion and the relation between them: the instant before the change in direction, the instant after the change in direction, and the path they trace. This is not needed in the case of rectilinear motion.

It is therefore the very same considerations of simplicity and complexity in *Rules* that determine Descartes's argument in *The World* that only rectilinear motion can be conceived in an instant. In Rule 6, Descartes correlates (1) rectilinearity to simplicity and (2) obliquity to complexity, and in *The World* he places (1) and (2) in the real conditions under which rectilinear and curvilinear lines are produced in nature, further correlating (1) to instantaneity and (2) to duration (both instantaneity and duration are

also simple natures).[59] This yields the content of Descartes's third law of nature in *The World*.[60]

Descartes illustrates the third law of nature in *The World* by means of the famous example of a stone in a sling (see Figure 10.3). As the stone follows the circle ABF described by the sling, it has a tendency to move rectilinearly along BC or FG. What causes this effect? The tendency to rectilinear motion, which exists in the stone at every determinable instant of its motion. Rectilinear tendencies to motion show up in the physical world as the *mechanical pressure one body exerts on another*. The stone places pressure on the sling, and this pressure can be felt in the hand at E as the stone pushes outward at B or F toward C or G. A rectilinear tendency to motion is not a motion, but rather a tendency to motion: it exists and is real (it is not merely a counterfactual motion),[61] but the rectilinear tendency to motion never becomes a rectilinear motion because all local motion in a plenum is necessarily curvilinear.

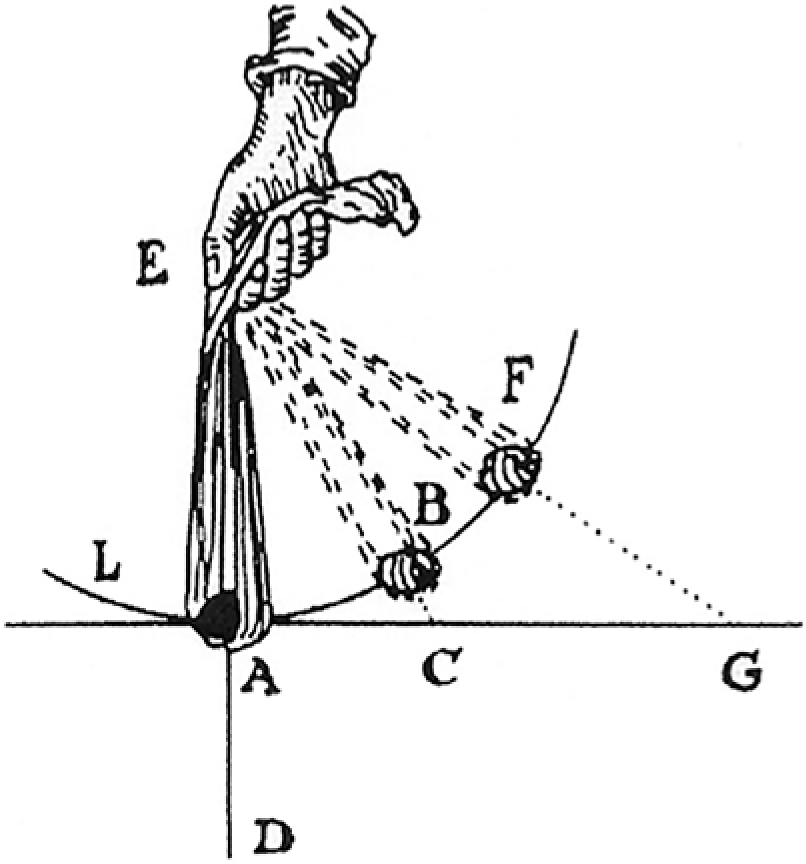


Figure 10.3 Descartes’s illustration of the rectilinear tendency to motion (*Principles* III. 57, AT 8A, 108; CSM 1, 259)

There is no reason to believe that Descartes did not regard the first and third laws of nature in *The World* as contained in his theory of simple natures in *Rules*.[62] As I have already indicated, these laws played a role in Descartes’s physics as early as 1618, in his research on the hydrostatic paradox (where the tendency to motion plays a fundamental role) and in his research on bodies in free fall (where the conservation of motion plays a fundamental role), long before he began his most serious optical research in Paris in the mid-1620s and certainly before he composed Rule 8 around the same period. Furthermore, it is clear that the material simple natures introduced in Rule 12 are the foundation of any systematic mechanical physics, and that whenever Descartes himself elaborates on one or more material simple natures in *Rules*, he immediately introduces theses he would later defend more systematically in *The World* and *Principles*, such as the denial of any real or modal distinction between extension, extended

things, and space (the affirmation of the plenum) in Rule 14. That he does not elaborate on all of the simple natures in *Rules* does not mean that he could not have or that he would not have done so along the lines I have described above. All I have done here is develop the theory of simple natures up to the point where it expresses the very principles of physics Descartes had already been employing since his earliest research with Beeckman, principles he would officially baptize for the first time in *The World* (1630–1633), but that the public would not see until the mid-1640s in *Principles* (1644).[63]

Thus far, I have only discussed the nature of motion and rest, not their cause. When I intuit the nature of motion and rest, I effectively intuit them as effects. I intuit a body in motion from one place to another, but I do not intuit the cause of its motion. To intuit what a natural power or force is, one must turn from a consideration of the *nature* of motion and rest to a consideration of the *cause* of motion and rest. When I intuit motion in relation to its cause, motion becomes a relative simple nature, since in Rule 6 Descartes insists that "cause" is an absolute, while "effect" is a relative, for "if we want to know what the effect is, we must know the cause first, and not vice versa" (AT 10:383, CSM 1:22). What causes motion, the tendency to motion, and rest is clearly force, and as Descartes would later point out in *Principles* II. 43, the "power of any body to act on, or resist the action of, another body […] consists simply in the fact that everything tends, so far as it can, to persist in the same state, as laid down in our first law" (AT 8A:66, CSM 1:243). The nature of force only comes to the fore as a presupposition of Descartes's first law: it is the tendency every body has to persist in the same state. Descartes's first law depends on the fact that forces have an intrinsic conservational tendency.

It is clear in Rule 9 that Descartes distinguishes between the "bare power [*potentiam…nuda*]" (AT 10:402, CSM 1:34) or force in a body, local motion over a duration, and the instantaneous tendency to motion. A stone cannot move from one place to another instantaneously (no motion occurs in an instant), but "a power similar to the one which moves the stone must be transmitted instantaneously if it is to pass, in its bare state, from one object to another" (AT 10:402, CSM 1:34). He then illustrates this via the example of a stick, which he takes up again in *Dioptrics* I: "For example, if I move one end of a stick, however long it may be, I can easily conceive that the power which moves that part of the stick necessarily moves every

other part of it instantaneously, because it is the bare power which is transmitted at that moment, and not the power as it exists in some body, such as a stone which carries it along" (AT 10:402, CSM 1:34). Similarly, the force that I feel in my hand when I press a stick against the ground is a mechanical pressure or tendency to motion, instantaneously communicated from one end of the stick to the other (see *Dioptrics* I, AT 6:84, CSM 1:153). This clearly illustrates that a "natural power can travel instantaneously to a distant place, passing through the whole of intervening space," provided that the bodies across which it is communicated are themselves spatially contiguous (Rule 9, AT 10:402, CSM 1:34).

It is not perfectly clear in *Rules* whether force is a simple notion (a simple nature) or a notion composed of other simple natures. Qua cause, it is in one sense simpler than motion, the tendency to motion, and rest. But as a quantity, it seems to be composed of the material simple natures of extension, motion, and a measurable "dimension" of motion: speed (AT 10:447, CSM 1:62).[64] At least this is how Descartes would later define the quantity of force of motion and rest: the force of motion is the product of the quantity of matter and speed, and the force of rest (or force of resisting) is the product of the quantity of matter in the body at rest and the speed of the body that collides into it (see *Principles* II. 43, AT 8A:66–7, CSM 1:243–4). As Richard Arthur has argued, Beeckman measured the force of motion in precisely this way as early as 1618: the force of motion is measured by the product of the quantity of matter (*corporeitas*) and swiftness or speed (*velocitas*).[65] Since Descartes would later provide an identical measure of the force of motion, there is no reason to believe he did not already have it in view in *Rules*. It can therefore be said that according to Descartes's theory of simple natures in *Rules*, force of motion is causally prior to motion and rest, but that the quantity of force of motion is reducible to the product of extension and speed. In any case, as we will see in Section 10.6, in the case of light, the measure of force is not defined by the measure of the force of motion (light has no motion and, therefore, no speed), but rather as a function of the density of the medium.

To sum up. A *natural power* or *force* is the cause of motion and rest. It has an intrinsic conservational tendency, such that (1) a body in motion conserves its motion unless interfered with, and (2) a body in motion conserves its rectilinear tendency to motion at every determinable instant of its motion. A natural power or force is what Descartes refers to in Rule 9 as

the “bare power” (AT 10:402, CSM 1:34) that can be transmitted from one body to another. When Descartes wrote Rule 8 in the mid- to late-1620s, he already knew how he would solve the problem of what a natural power is because he had already been employing principles that define what a natural power is for nearly a decade.

It may be reasonably objected that Descartes did not arrive at his first and third laws of nature in *The World* via his method, since these laws (or early versions of them) played a role in his science as early as 1618, before he had composed *Rules*. This is true, but not necessarily as relevant as it seems. Descartes accepted the first law (or an early version of it) in 1618 on Beeckman’s authority, and Beeckman claimed that the principle of the conservation of motion, like all principles and explanations in natural philosophy, must be validated by “intuition” (*aanschouwelijkheid*), which for Beeckman meant that only what can be “imagined” by the mind can play an explanatory role in science. Beeckman’s insistence on intuition in natural philosophy dates back to his dissertation (1618): “In philosophy we must not allow a single regulation, or a single prescription or a single rule which has not been approved as absolutely true and completely certain by reason and which does not appear to the mind as open and ‘naked’ as visible things do to our eyes [*Nullum enim statutum, nullum praeceptum, nulla regula in philosophia admittenda quae non sit apodictica et certissima ratione comprobata et intellectui tam aperte et nude objecta atque visibilia oculis objiciuntur*].”[66] Later, in a letter to Mersenne, Beeckman similarly writes: “In philosophy I allow nothing that is not represented to the imagination as if it were observable [*Nihil enim in philosophia admitto quam quod imaginationi velut sensile representatur*].”[67] When Beeckman articulates the principle of inertia, he draws precisely on the methodological principle he would later lay down in his dissertation. It is easier, he claimed, “to imagine that motion in a vacuum never comes to rest because no cause that could change the motion occurs. For nothing changes without some cause of change.”[68] The principle of inertia is here validated by intuition via the imagination. Descartes’s distinction between motion and rest as two distinct simple natures, which must be intuited separately by the intellect aided by the imagination, is continuous with Beeckman’s own methodological principle about intuition. Indeed, when one evacuates Aristotelian self-terminating motions from nature on the basis of their non-intuitability, motion and rest

can no longer be explained "internally," but *must* rather be explained "externally" by the fact that something *else* has either moved a resting body or arrested a moving body. Intuition does real work here. Similarly, in the hydrostatics manuscript, Descartes refers to principles based on the distinction between motion and tendency to motion as "so clear and evident that it can be accepted as a scientific principle" (AT 10:70), precisely the language he employs in *Rules* to describe what any act of intuition consists in. Given Descartes's emphasis on the role played by the imagination in the intuition of the material simple natures in *Rules*, it seems that he, like his teacher Beeckman, had long regarded the authority of the laws of nature as resting on intuition, and that the "picturability" of these laws—the fact that they could in some sense be "seen"—is what both men regarded as the source of these laws's superiority over the purely verbal or discursive principles of Aristotelian natural philosophy.[69] It is, therefore, insufficient to argue that Descartes did not derive the first and third laws of nature methodologically simply because he had not yet *composed* his method, since he seems to have accepted and operated with certain methodological principles long before he decided to write a treatise on method. The *method as a discursively articulated document* in *Rules* is but a reflection and elaboration of *the method as a living practice or* habitus.[70]

10.5 The Action of Light

As we have seen in Section 10.4, the intuition of natural power or force depends on intuiting relations between a series of simple natures: extension, motion and rest, straight and oblique, and instant and duration. The intuition of extension yields the affirmation of a plenum; the intuition of motion and rest as two distinct simple natures yields Descartes's first law of nature (the conservation of motion); and the intuition of an instant of motion leads to Descartes's third law of nature (the conservation of the rectilinear tendency to motion at an instant).

How does the intuition of what a natural power is help solve the problem about the nature of light's action? In Rule 8, Descartes writes that the nature of light's action may be determined analogically by comparison to some other natural powers, or directly, by deduction from the intuition of what a

natural power is. Analogy can be employed if one is "unable to discern at once what the nature of light's action is," and in Rule 9 Descartes will employ the stick analogy in order to confirm that the instantaneous transmission of a natural power is possible (AT 10:395, CSM 1:29). While he does not explicitly connect this analogy to the action of light, he does do so in *Dioptrics* I. This clearly suggests that the stick analogy can play the same role in Rule 8. But what reason would Descartes (or any operator of the method) have to suspect at this point in the deduction that the action of light may be an instantaneously transmitted natural power from a luminous object to the eye? It makes no sense to ask whether light can be instantaneously transmitted unless one already has reason to believe that it might be.

The answer is that *light rays are rectilinear*, which in Descartes's physics indicates that light *is not propagated via local motion over a duration*; local motion in a plenum is necessarily *curvilinear* (see Section 10.4 above). If light rays are rectilinear, and if all local motion is necessarily curvilinear, then either (1) light moves locally in a vacuum, which would explain its rectilinear transmission, but conflict with the intuition that there is no real difference between extension, extended things, and space (see Section 10.4 above), or (2) light does not move locally, which means that it must be a rectilinear tendency to motion in a plenum, i.e., a species of *instantaneous mechanical pressure*. Based on the solution to the problem of what a natural power is in the first step of the deduction, only the second of these two possibilities can be embraced.

The comparisons Descartes employs in *Dioptrics* I fit snugly into the second step of the deduction of the anaclastic in Rule 8:

> In order to derive a comparison from this, I would have you consider the light in bodies we call "luminous" to be nothing other than a certain movement, or very rapid and lively action, which passes to our eyes through the medium of the air and other transparent bodies, just as the movement or resistance of the bodies encountered by a blind man passes to his hand by means of his stick. In the first place this will prevent you from finding it strange that this light can extend its rays instantaneously from the sun to us. For you know that the action by which we move one end of a stick must pass instantaneously to the other end, and that the action of light would have to pass from the heavens to the earth in the same way, even though the distance in this case is much greater than that between the ends of a stick. […] [W]hen a blind man feels bodies, nothing has to issue from the bodies and pass along his stick to his hand […] (AT 6:84–85, CSM 1:153).

There is no local motion of a body from one end of the stick to another, but only a mechanical pressure instantaneously exerted by the ground on the other end of the stick. Thus, by analogy, light can be a mechanical pressure instantaneously exerted on our eyes by luminous bodies. The structure of the comparison is as follows:

Ground: Luminous body (Sun)
Stick: Medium of instantaneous transmission (Air)
Hand: Recipient (Eye)

This comparison makes it very easy to see how a natural power can travel instantaneously without local motion, and facilitates a conception of light according to which a luminous body such as the sun can transmit light to our eyes via the pressure it exerts on the particles in the medium between it and our eyes.[71] As a pressure, light has force, but it does not, strictly speaking, have *force of motion*, as bodies in local motion do. The force of light is the *degree of pressure the particles that produce light in our eyes acquire in the relevant media*. Just as one can distinguish between the greater and lesser pressure a stick can exert on my hand (depending on how hard I press on it), so too can one distinguish between the greater and lesser pressure particles of light exert on the relevant media. This naturally leads to the third problem in the series, “How does light pass through transparent media?”

10.6 How Light Passes through Transparent Media

To understand how the action of light interacts with optical media, the behavior of light in different optical media must be examined. The reduction of the problem of the anaclastic to this point initially required observing how different media reflect and refract light, but the solution to the problem requires more. In the case of refraction especially, one must provide a physical interpretation of the fact that light sometimes bends toward the normal, and sometimes away from it. Does bending toward the normal indicate an *increase* in the force of light, or a *decrease* in the force of light? This problem cannot be solved by observation alone, since one and the same refracted ray can be interpreted by one optical theorist as a

decrease in the force of light (or whatever the relevant physical magnitude is), and by another as an increase in the force of light, depending on the underlying physics.

As early as 1619, Descartes maintained a thesis that ran contrary to the entire history of optics since Ptolemy: in denser media, light acquires more force, and the increase in force causes it to bend toward the normal. All previous optical theorists from Ptolemy to Kepler[72] maintained that in denser media, light loses force, and that the decrease in force causes it to bend toward the normal:

> Since light can only be generated in matter [*Lux quia non nisi in materia potest generari*], where there is more matter it is generated more easily, other things being equal; therefore it penetrates [*penetrat*] more easily through a denser than a rarer medium. Whence it comes about that the refraction is made in the latter away from the perpendicular, in the former towards the perpendicular; therefore, the greatest of all refractions would occur along the perpendicular itself, if the medium were the most dense […] (AT 10, 242–3; translation by Sabra 1981, 105 modified and extended).

In the first sentence, Descartes asserts that *the force of light is directly proportional to the density of the medium.*[73] In the second sentence, *he asserts that the increased force of light in denser media causes light to bend toward the normal*.[74]

Mathematically, the first sentence yields:

$$\mathrm{F}i : \mathrm{F}r = n,$$

where "F*i*" and "F*r*" are the force of the incident and refracted rays, respectively, and "*n*" is a constant defined by the density of the medium.

One may wonder why Descartes asserts in the optical fragment that in denser media, light acquires more force.[75] It seems to me plain that his reasons must be physical. I interpret the physical rationale behind the fragment as follows. In the first part of the first sentence, Descartes asserts that light can only be produced in matter.[76] In the second part of the first sentence, which Descartes regards as logically entailed by the first part, Descartes asserts that light is produced "more easily" in denser media. Why? He does not say. The first part of the first sentence does not by itself entail the second part: from the fact that light can only be produced in matter, it does not follow that it is produced more easily where there is more matter. Descartes's predecessors all agreed that light can only be produced

in matter, but they did not believe that light can be more easily produced where there is more matter. On the contrary, they denied it. There must, then, be some specific property of light, over and above the fact that it can only be produced in matter, that explains why "where there is more matter it is generated more easily." The required entailment follows if one assumes that light is a *mechanical pressure*, such that when it encounters denser transparent media, *it encounters more resistance*, and so *acquires more force*.[77] Thus, the first sentence of the optical fragment is enthymematic; once the unstated minor premise is explicitly asserted, the required entailment between the major premise and the conclusion follows accordingly.

Since the late 70s, Schuster has argued that in the optical fragment, Descartes asserts, not that the force of light is directly proportional to the density of the medium, but only that the force of the *normal component* of the force of light (the tangents of the angles of incidence and refraction, respectively) is directly proportional to the density of the medium, and that if one assumes that the parallel component remains the same, this only yields a law of tangents (a ratio between lines AC (or HB) and BG (or EI) in Figure 10.4), not a sine law. For Schuster,[78] the first sentence in the optical fragment asserts "that the normal component of the force of light is increased in a denser medium." In other words, Descartes is not asserting

$$\mathrm{F}i : \mathrm{F}r = n,$$

but rather

$$\mathrm{F}i\perp : \mathrm{F}r\perp = n.$$

Schuster concludes that there "seems to be no way to proceed directly from the assumptions of 1620 to the sine law of refraction,"[79] and this leads him to search for an alternative path to the discovery of the sine law via the Mydorge document (see Section 10.1 above).[80] However, as Schuster would no doubt acknowledge, Descartes does not explicitly refer to Kepler and he does not resolve light rays into their normal or parallel components in the optical fragment. Descartes does not seem to be reasoning with such components in the optical fragment at all; he focuses exclusively on the relation between the light rays themselves and the relevant media. As I

understand it (and as I mentioned in Section 10.1, I think this is the more natural interpretation), the document provides clear evidence that Descartes already possessed a key premise in his formulation of the sine law: namely, that there is a constant proportion between the force of the incident ray and the force of the refracted ray, and that this proportion is determined by the density of the relevant media. This premise is plainly based on *physical*, not mathematical considerations; the *order of priority* between physics and mathematics required in Rule 8 is respected. The order of research Descartes would later prescribe in Rule 8 seems to *reflect* this fact about his scientific practice in the optical fragment, not to cover it over or conceal it.

Returning to the optical fragment, there also seems to be a missing premise in the second sentence. From the fact that light acquires more force in denser media, it does not follow that it must bend toward the normal. The required premise cannot be found in the optical fragment, but only much later, in *Dioptrics* II, where Descartes asserts that the parallel component determination of the force of light remains the same after refraction because it does not encounter the surface.[81] As can be seen in Figure 10.4, the length of the line representing the parallel determination HX remains the same after refraction as XM. The normal dropped from M yields the point beneath the surface in the denser medium where the ray HE bends toward the normal and intersects the circle at N.

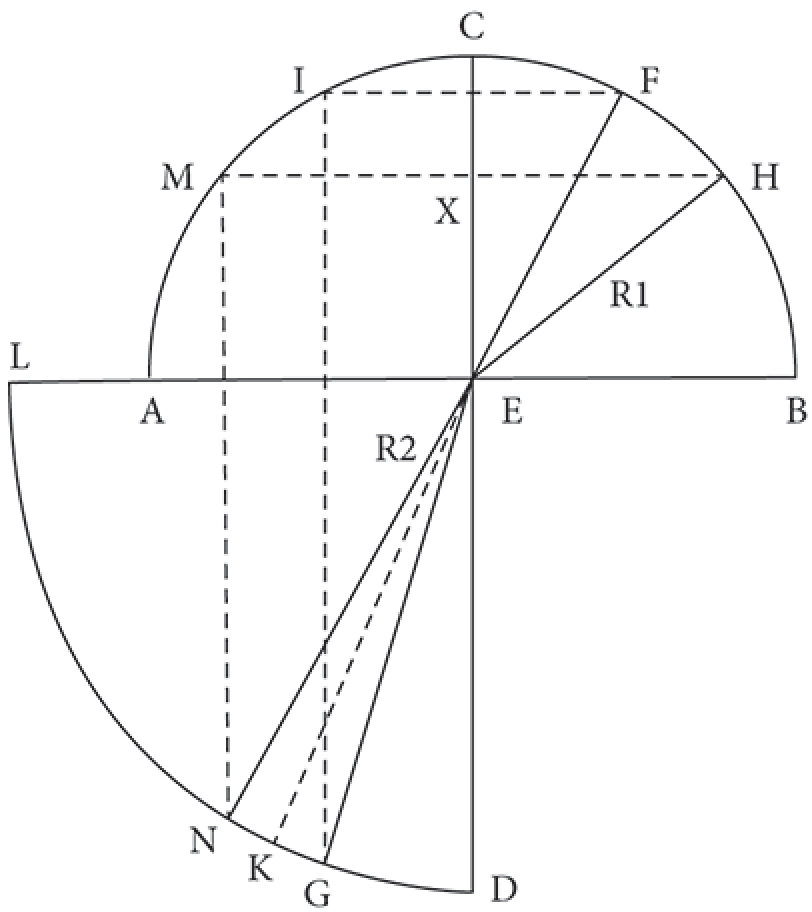


Figure 10.4 Refraction of light from rarer to denser medium (Mydorge) (Mersenne 1932–1988, 1: 405)

The sines of the angles of incidence CEH and refraction NED bear a constant ratio to one another.[82] Mathematically:

$$\mathrm{F}i : \mathrm{F}r = R1 : R2$$
$$\sin\, i = \mathrm{HX}/\mathrm{R1}$$
$$\sin\, r = \mathrm{XM}/\mathrm{R2}.$$
$$\text{But HX} = \text{XM, so } \sin i : \sin r = \mathrm{R1} : \mathrm{R2}.$$

When one combines this with Fi: Fr = n, one obtains the sine law:

$$(\sin i : \sin r) = (\mathrm{F}i : \&\ \mathrm{F}r) = n.$$

How did Descartes arrive at the second premise of the sine law, so as to be able to come up with the construction in Figure 10.4? As I argued above, Descartes does not seem to be reasoning with the normal and parallel components of light rays in the optical fragment. He had arrived at the thesis that there is a direct (rather than an inverse) proportion between the force of light and the density of the medium, but he had not yet mathematically defined the relation between the angles of incidence and refraction. The problem, of course, is that there seems to be no determinable mathematical ratio between the angle of incidence and the angle of refraction (see Section 10.3 above), as there is in the case of reflection (where the angles of incidence and refraction are always equal to one another). No doubt, Descartes presumably knew that optical theorists such as Ibn al-Haytham and Kepler had long employed parallel and normal components when reasoning about the behavior of light rays in reflection and refraction. It is reasonable to suppose that Descartes came to regard any hope for a mathematical expression of the relation between the angles of incidence and refraction as lying in proportional relations, not between the angles of incidence and refraction themselves, but rather between physical magnitudes represented by their respective sines, cosines, and/or tangents. To introduce these components, however, he would need to provide them with a physical interpretation, and he had not yet done so in the optical fragment.

Ibn al-Haytham and Kepler divided the force or motion of light rays into two components: a "normal" component (perpendicular to the surface at the point of reflection or refraction), and a "parallel" component (parallel to the surface).[83] As they understood it, each component has a definite direction and represents an independent part of the motion (*motus*) or force (*vis*) of the ray, and these components behave differently under different conditions.[84] In oblique reflection (see Figure 10.5), the normal component of the incident ray BD encounters resistance at the surface that is proportional to its downward force, so the surface merely redirects it in the opposite direction (from downward to upward), such that the ray is reflected from D to A at an angle equal to the angle of incidence. Only the normal component of the force of light interacts with the surface. The parallel component remains the same before and after reflection. In oblique refraction from a rarer to a denser medium, both the normal and the parallel component forces encounter resistance at the surface, and the ray bends

toward the normal. Finally, in oblique refraction from a denser to a rarer medium, both components acquire more force, and the ray bends away from the normal.

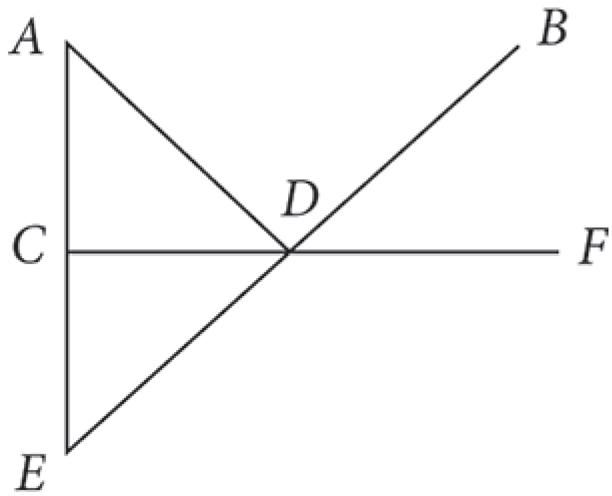


Figure 10.5 Representation of reflection (Kepler 2000, 27)

Ibn al-Haytham and Kepler interpreted the normal and parallel components as representing component *parts of (dynamic) forces*. For Descartes, the parallel and normal components could not be interpreted as parts of forces: the addition of these parts is consistently more than the resultant diagonal ($c < a + b$), but according to Descartes the addition of two forces must be equal to their sum ($c = a + b$),[85] and this is how Descartes adds forces as early as 1618. In his research on free fall, Descartes adds forces *arithmetically* ($a + b = c$), not *geometrically* ($a + b > c$) (see AT 10:75–75). Otherwise, the sum of two forces would be *less than the resultant force they compose*, in which case *the overall quantity of force would not be conserved*. Even in 1619, Descartes could not have interpreted the normal and parallel components as component forces. *A fortiori*, he could not have asserted that a proportion obtains between such component forces. This explains why Descartes only asserts a proportional relation between the force of light and the medium in the optical fragment, and does not resolve the ray into parallel and normal components.

Since Descartes could not have interpreted the parallel and normal components as *forces*, he had to interpret them otherwise. How? Perhaps they represent the *direction* in which a body moves? Direction is not a

physical quantity. The *cause* of the direction—what determines a body to move in a certain direction—*is* a physical quantity. This quantity, moreover, is not identical with the force of motion in a body, since force of motion is a scalar quantity; the same quantity of force in a body can be caused to move in any number of directions. Furthermore, the power that causes a body to move in one direction rather than another can change while the scalar quantity of force remains the same. This further indicates that they must be distinct physical quantities. Given these constraints, Descartes found himself between two ways of interpreting parallel and normal components—either as directions or as components of the force of motion—neither of which proved sufficient. These components, he concluded, represent *directional quantities of force* or "determinations," which "cause" or "determine" a body to exert its force up/down (normal component determination) and/or right/left (parallel component determination). The parallel and normal component determinations compose the *principal determination* of the force of motion (e.g., the physical cause that impels a body downward and to the right along a diagonal).[86] *Each of these components can be weakened or strengthened (*i.e.*, vary in causal efficacy) in physical interactions independently of the other,* depending, like motion itself, on whether it encounters any other bodies.[87] For example, a body may lose none of its determination to the right while it loses all of its downward determination. *Since determinations are not scalar forces, it does not matter that the sum of two component determinations is not arithmetically equal to the resultant determination they compose*. In Descartes's mature ontology, determinations are modes of modes or secondary modes of motion. The force of motion "has" determination as a "mode," which is "in" the motion and "causes" or "determines" the body to exert its force up/down (normal component determination) and/or right/left (parallel component determination).

It is by applying the first law of nature (see Section 10.4 above) to these determinations that Descartes arrives at the missing premise in the optical fragment: the preservation of the parallel component determination—which does not interact with the surface—after refraction. Looking at the case of reflection first, it became clear to Descartes that the parallel component determination must remain the same because it does not interact with the surface. Only the normal component determination does. But then *this must also be the case in refraction*. The parallel component determination must

remain the same after refraction. Descartes could have discovered the second premise anytime in or after 1619, and he certainly could have possessed it before he met Mydorge in Paris in 1626. Indeed, it seems he must have possessed it prior to their encounter, since otherwise it is difficult to see how Mydorge could have come up with the relevant geometrical diagram (see Figure 10.4 above). That the underlying physics is missing in Mydorge's document is not surprising: Mydorge was a mathematician, not a physicist. Descartes did not need him to know—and perhaps did not want him to know—the physics behind the mathematics.

10.7 Deducing the Law of Refraction

We have now covered Problems 1–4 in Descartes's proposed deduction of the law of refraction and the shape of the anaclastic lens in Rule 8. Only now are we in a position to actually deduce the law of refraction and, exploiting the law, deduce the shape of the anaclastic lens. It is here that Descartes's directive in Rule 14 becomes operative: "The problem should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect" (AT 10:438; CSM 1:56). This is exactly what must be done in order to solve Problem 5.

Regarded independently of Descartes's tennis-ball analogy in *Dioptrics* II (about which more below), Descartes's physics enables one to geometrically represent the law of refraction in one of two ways. Refraction can be geometrically represented by means of two unequal semi-circles whose respective radii represent the force differential between the two media (see Figure 10.4 above). The smaller semicircle represents the rarer medium (e.g., air), and the larger semicircle beneath it represents the denser medium (e.g., water). By the first premise, the force of light is increased in the denser medium. Suppose it is doubled. The semicircle beneath the surface must therefore have a radius twice as long. By the second premise, the parallel component determination must be represented by a line that has the same length before and after refraction, as is the case in reflection. Determining the path of the refracted ray takes the form of a locus construction. Take the incident ray HE. Since only the normal component

determination directly encounters the surface AEB, which directly opposes it, only the normal component determination "can be changed in any way" by the surface. By contrast, the surface offers "no opposition at all to the [parallel component] determination" HM, and so after refraction the force of light "loses none of its determination" to advance to the left (AT 6:97–8, CSM 1:159). In refraction, the light ray must intersect the larger semicircle somewhere beneath the surface AEB. By dropping a normal from the unchanged parallel determination at M, one can see that the ray must intersect the semicircle at N. One can do the same for any arbitrarily chosen incident ray. This is how the law of refraction is represented in the Mydorge document.

Another way of representing the law of refraction appears in Descartes's communication of the law to Beeckman in October 1628 (see Figure 10.6).

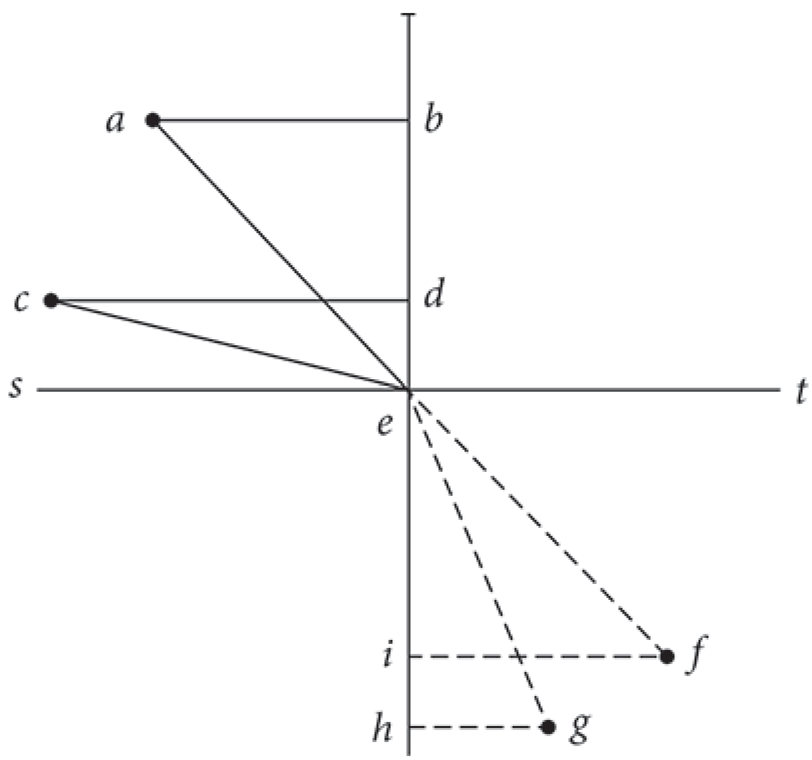


Figure 10.6 Descartes's sine law as communicated to Beeckman in October 1628 (AT 10:336)

Mathematically, this yields:

$$AB : HG = n$$

$$\sin i : \sin r = n.$$

When one combines this with F*i*: F*r* = *n*, as before one obtains:

$$(\mathrm{F}i : \mathrm{F}r) = (\sin i : \sin r) = n.$$

The force ratio between the incident and refracted rays is proportional to the ratio between their respective sines, which is constant in any pair of media.

There are a number of important differences between Figure 10.4 and Figure 10.6 that need to be discussed here. The most striking difference is that there are no circles in Figure 10.6, and that if one adds a circle to it, points *a*, *c*, *g*, and *f all lie on the circumference of one and the same circle*. Furthermore, the lines *ae* and *eg* are radii of this circle and have the *same length*, as do lines *ce* and *ef*. This means that the force differential between the two media (the rarer medium above *st* and the denser medium below *st*) is *not* represented by unequal radii. Finally, the lines *ab* and *hg*—the sines of *bea* and *heg*—do *not* have the same length. They are proportional, but not equal. The preservation of the parallel component determination is not represented by two equal, but rather by two unequal lines before and after refraction. It is clear that this representation is much closer to the canonical representation of the sine law of refraction in *Dioptrics* II.

Why these differences between Figure 10.4 and Figure 10.6? These differences are due to the analogy Descartes employs in explaining the sine law of refraction to Beeckman. Descartes analogizes the behavior of the incident and refracted rays to the behavior of the arms of an equal-arm balance with bodies equal in weight attached to both arms. Below *st*, there lies water, and above *st*, there lies a rarer medium, such as air. The arms *ae* and *eg* are equal to one another, and each one has an equal weight attached to it at *a* and *g*, respectively. The problem is to determine the conditions under which the effective weight of the bodies attached to each arm is equal in different media. Equilibrium can only be maintained when the ratio of the weights is proportional to the ratio of the sines of the angles each balance arm makes with the normal.[88]

Why would Descartes provide such an analogy? Because it illustrates his physical premises quite nicely. (1) Just as the force of light is directly proportional to the density of the relevant media (F*i*: F*r*), so too the effective weight of the bodies attached to each arm is directly proportional

to the density of the relevant media. (2) Just as the parallel component determination is conserved before and after refraction (sin i:sin r), so too are the statical moments on both sides of the fulcrum equal to one another. Furthermore, as Schuster rightly argues, Descartes interpreted both weight and light as tendencies to motion (the weight of a body being equal to the mechanical pressure it exerts on any other body or bodies over a duration), and weight has both a gross magnitude as well as a "directional magnitude."[89] In all these respects, the equal-arm balance analogy is very appropriate.

However, the equal-arm balance analogy is not entirely appropriate—*not when one applies it to the case of reflection*. There is no statical equivalent to reflection in the equal-arm balance, since the arms of an equal-arm balance cannot both be raised *above* the fulcrum. Descartes needed a dynamic analogy similar to the *analogies based on projectile motion* typically employed by optical theorists since Ibn al-Haytham. He decided on the tennis-ball analogy, and this enabled him to preserve the mathematical construction based on the previous equal-arm balance analogy and provide a *unified representation of the law of reflection and the law of refraction*. The incident and refracted rays are represented by the path of the ball before and after it strikes the surface. In Figure 10.7, reflection is represented by the rebound of the ball when it strikes a hard surface at B, and in Figure 10.8, refraction is represented by the deflection of the ball when it penetrates a fluid surface at B. The force differential is analogized to the speed of the ball before and after reflection or refraction in the relevant media (since the radii are equal, the force differential is analogized to the amount of time it will take the ball to cover the same distance).

In Figure 10.7, there is no difference in speed before and after reflection. Since the ball must cover the same distance in the same amount of time, one drops a normal FD equidistant from HB as AC. Since, moreover, the normal component determination AC encounters the surface (which resists it entirely) at B, upon impact at B it is reversed and determines the ball to move upward. The unchanged parallel component determination AH continues to determine the ball to move rightward. The ball intersects the circle above the surface at F.

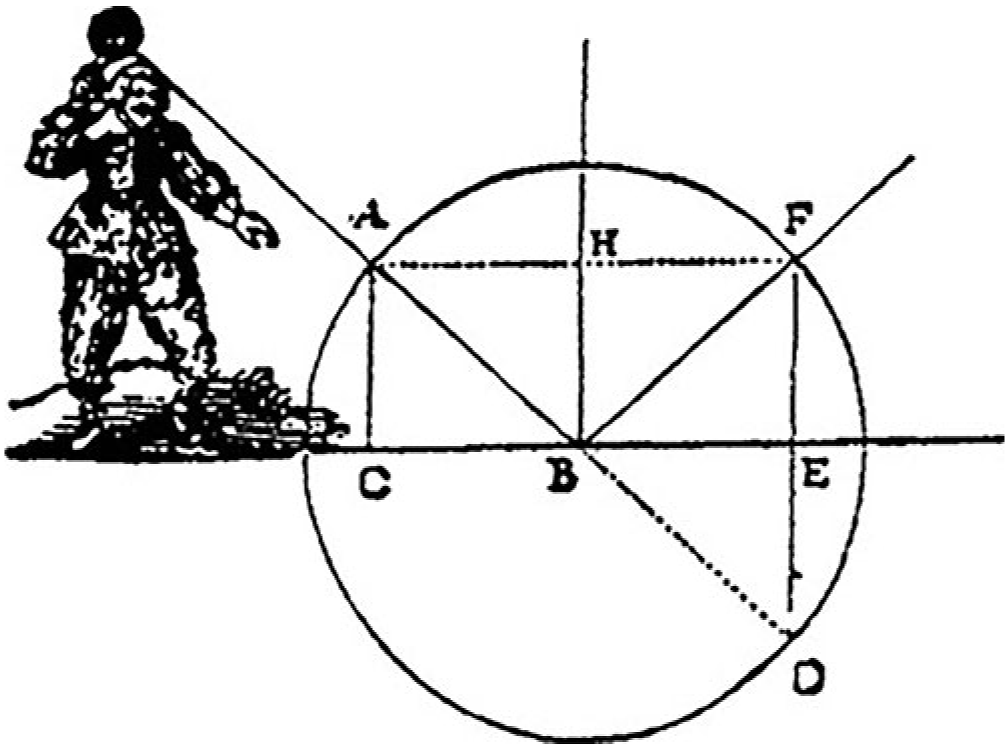


Figure 10.7 The law of reflection in *Dioptrics* II (tennis-ball analogy) (AT 6:93, CSM 1:157)

In Figure 10.8, the ball loses half of its speed after refraction into the denser medium. Since the ball must cover the same distance in double the time, one drops a normal FI twice the distance from HB as AC. Since, moreover, the normal component determination AC is not completely destroyed, but only altered, after impact at B the ball continues to move downward along an altered path. The unchanged parallel component determination AH continues to determine the ball to move rightward. The ball intersects the circle below the surface at I.

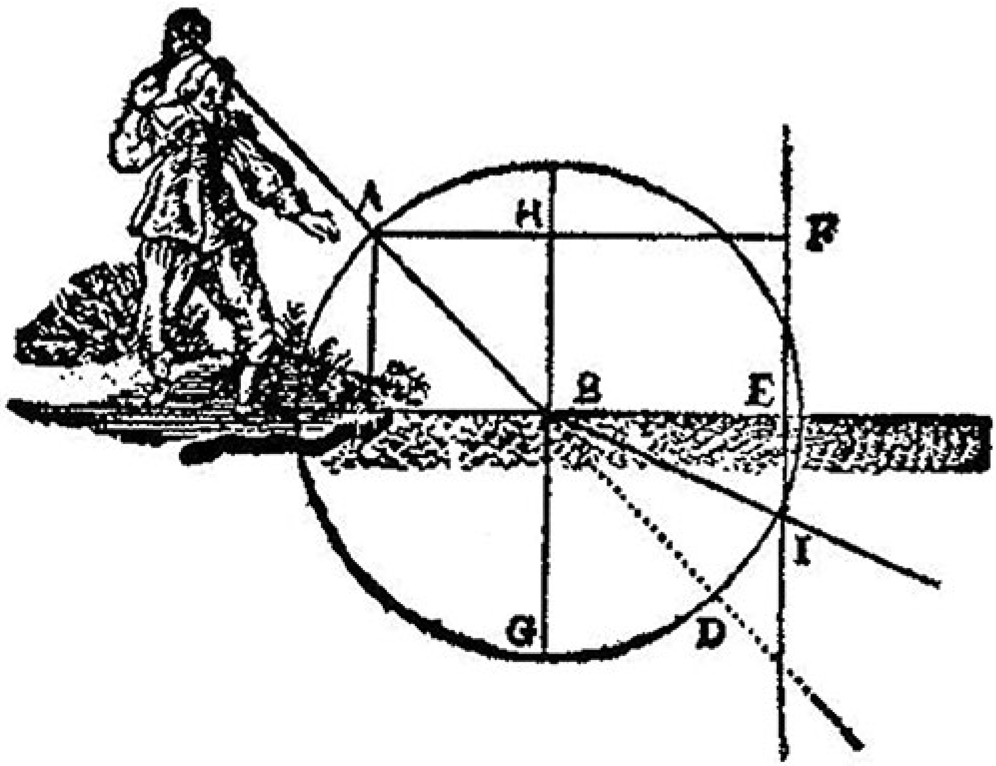


Figure 10.8 The law of refraction in *Dioptrics* II (tennis-ball analogy) (AT 6:98, CSM 1:159)

Unlike tennis balls, which lose speed in denser media, light acquires more force in denser media. Descartes represents the case of light in Figure 10.9, where the incident ray AB acquires double the force in the denser medium. One drops a normal FI half the distance from HB as AC. The normal component determination AC is not completely destroyed, but only altered, and after impact at B the ray continues downward along an altered path. The unchanged parallel component determination AH continues to determine the ray's path rightward. The ray intersects the circle below the surface at I. Descartes arrives at this result by the same premises and procedures employed in the previous two cases.

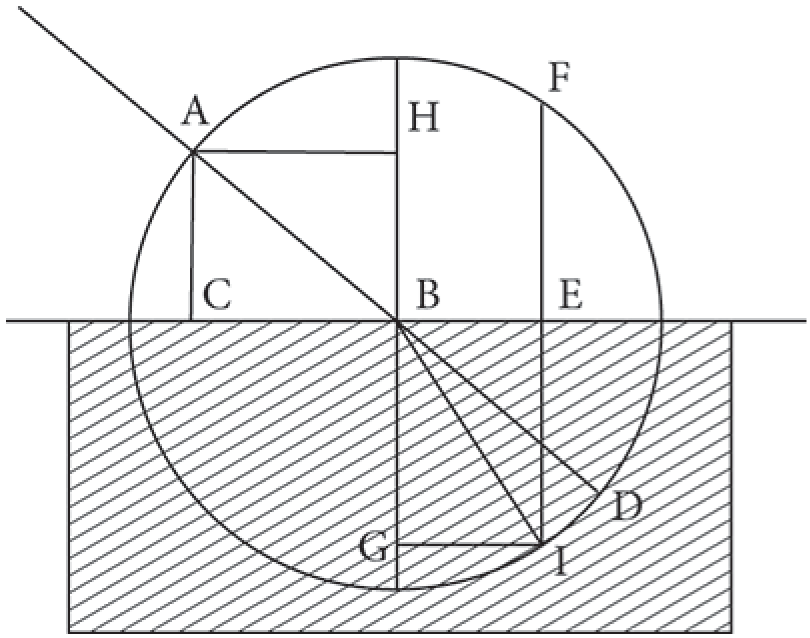


Figure 10.9 The law of refraction in *Dioptrics* II (light) (AT 6:100, CSM 1:161)

To sum up. (1) As the optical fragment indicates, the manner in which light passes through transparent media depends on the density of the medium. The force of light is directly proportional to the density of the medium. (2) Descartes does not arrive at his dynamics of light in the optical fragment by offering an ex post facto physical interpretation of a purely mathematical representation of refraction, but rather by means of physical considerations about the nature of light and the conditions under which it is produced in media of differing density. The optical fragment is not an exercise in "physico-mathematics." On the contrary, (3) the mathematical representation of Descartes's independently-developed dynamics of light in the optical fragment leads very naturally to the sine law.

It has often been argued that the preservation of the parallel component determination after refraction is an ad hoc assumption adopted by Descartes, not because his physics requires it, but rather in order to preserve his locus construction. The assumption seems ad hoc because in denser media (e.g., water), a projectile (e.g., a stone) encounters resistance to its determination to advance to the right as it penetrates further into the depth of the medium. There seems to be no reason why light should be spared this particular dynamic effect.[90] The objection does not succeed because the

transparent media Descartes discusses are *physically homogeneous, so their material constitution is the same at the surface as it is beneath the surface.* Consequently, *whatever mechanical effect light undergoes at the surface of a dense refracting medium is no different than the effect it will undergo beneath the surface.* This explains why Descartes regards refraction as an interface phenomenon. Descartes's insistence on the preservation of the parallel component determination after refraction is not, therefore, ad hoc, but rather based on serious physical considerations. These considerations may be highly idealized, but it does not follow from this that they arise from a desire on Descartes's part to preserve an antecedently-discovered mixed-mathematical law of refraction at all costs. If anything, Descartes's insistence on the preservation of the parallel component determination has more to do with his general tendency to insist on his *physical* principles *even when experience seems to suggest otherwise*. To allow for alteration in the parallel component determination even in the absence of any interaction between the parallel component determination and the surface would fly in the face of Descartes's modal distinction between motion and rest entailed by the theory of simple natures as well as Descartes's later metaphysics built largely on the theory of simple natures (see Section 10.4).

10.8 Deducing the Anaclastic

Descartes's directive in Rule 14 to reduce problems to expressible relations between magnitudes (whose physical meaning has been established via the solution to Problems 1–4) remains operative in the solution to Problem 5: the shape of the anaclastic lens. Furthermore, as we have already seen in Section 10.3, experimental research on the manner in which light passes through transparent bodies indicates that the shape of the anaclastic must be *curvilinear*, for "if it [the surface] is curved inward or outward they [the light rays] come towards each other or go away from each other in the same order, more or less, *on account of this curvature*" (*Dioptrics* II, AT 6:89–90; Descartes CSM 1:155–6; my emphasis). In short, *the anaclastic must have the shape of an ellipse, a parabola, and/or an hyperbola,* and determining the shape of the anaclastic will require *mathematical knowledge of the properties of the conics.* The reduction of the problem to expressible

relations between magnitudes required by Rule 14, together with the experimentally discovered but as yet mathematically indeterminate correlation between the conics and the anaclastic, determines how the solutions to Problems 1–5 are brought to bear on the solution to problem of the shape of the anaclastic lens.

By the time Descartes met with Beeckman in October 1628, he not only already had the sine law in hand, but had also already deduced the shape of the anaclastic in the case of the ellipse. The problem of the anaclastic may be more precisely defined as follows: "Given an ellipse or an hyperbola, on which a ray falls parallel to the focal axis, according to what geometrical condition will the ray be refracted to one of the foci?"[91]

An ellipse is a plane curve surrounding two points or foci such that the sum of the distances of any point on the curve to the two foci is constant, and will always be equal to the transverse axis. Drawing an ellipse, one then constructs lines satisfying the definition of the anaclastic (Figure 10.10).

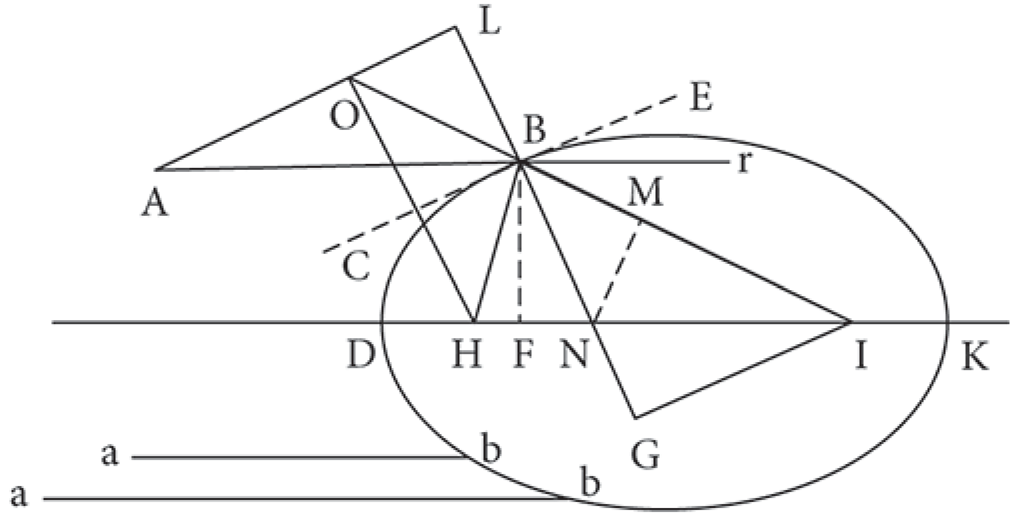


Figure 10.10 The anaclastic in the case of the ellipse (AT 6:168; Descartes 2001, 129)

At B, draw a line, AB, parallel to the transverse axis DK. Let AB represent the incident ray. What condition does the ellipse DBK have to satisfy so that AB and, indeed, any line that intersects the ellipse parallel to the transverse axis, will be refracted toward I? Here, some further lines need to be introduced. Construct the normal LBG perpendicular to the

refracting surface at the point of incidence and add the sines of the angles of incidence and refraction, AL and GI, respectively. By similar triangles, Descartes can easily demonstrate that the sine of the angle of incidence and the sine of the angle of refraction (represented by AL and GI, respectively) are to one another as the transverse axis is to the focal distance (represented by DK and HI, respectively):

$$\mathrm{AL}/\mathrm{IG} = \mathrm{DK}/\mathrm{HI}$$

$$\sin i : \sin r = \text{transverse axis} : \text{focal distance.}$$

Descartes has essentially constructed an ellipse in which the incident and refracted ray satisfy the condition of the anaclastic, such that the incident ray is refracted toward point I. Then, by analysis of these lines and others added to them, he expresses the mathematical conditions under which this line reaches point I as a ratio between the sine law and the defining properties of the ellipse. Finally, since the initial incident ray was chosen arbitrarily, the ratio defines any other line drawn parallel to DHK.[92]

10.9 Order of Research and Order of Exposition

I have argued that Descartes's proposed deduction of the anaclastic in Rule 8 is not only viable, but also plausibly represents Descartes's actual path to the deduction of the sine law and the shape of the anaclastic lens in the case of the ellipse. Descartes's method does not conceal his historical path to discovery—it reveals it. In the end, this is not terribly surprising. Descartes demonstrated the efficacy of his method in both the *Geometry* and the *Meteorology*. Even though many of Descartes's contemporaries (who did not have access to *Rules*) failed to see it,[93] recent scholarship on these texts has demonstrated that the procedures Descartes employs in the *Geometry* and in his deduction of the rainbow in *Meteorology* VIII clearly reflect the procedures recommended by the method.[94] Descartes himself believed the same. He claimed to have provided "a brief sample of [the method] in my account of the rainbow" in *Meteorology* VIII (to Vatier, February 22, 1638, AT 1:559, CSM 1:85), and regarding the *Geometry* he even more

emphatically insists that while in the *Dioptrics* and *Meteorology* he "merely tried to show that my method is better than the usual one; in my *Geometry*, however, I claim to have demonstrated this" (to Mersenne, December 1637, AT 1:478, CSMK 3:77–8). In those parts of the *Essays* where he does not follow the order of research to the letter (including *Dioptrics* II), Descartes underlined that the method "prescribes an order of research which is quite different from the one I thought proper for exposition" (to Vatier, February 22, 1638, AT 1:559, CSMK 3:85). In *Dioptrics* II, Descartes merely presents the results of the method, not the process. For the process, one must turn to Rule 8. That is what I have done here. This distinguishes my reconstruction from most other reconstructions of Descartes's deduction of the sine law, according to which Descartes first deduced a purely mathematical law of refraction, and only later provided the relevant dynamical rationale.[95] By prioritizing mathematics over physics, these reconstructions place the cart before the horse; Descartes clearly insists that deducing the law of refraction and the shape of the anaclastic lens requires both physics and mathematics, with physics playing a privileged role in the solution to the simpler problems in the series. Descartes's method—the Cartesian scientific *habitus*—is not a species of dissimulating rhetoric, but rather a cognitive technology that yields serious scientific results in natural philosophy no less than mathematics, and in solutions to imperfectly understood problems no less than solutions to perfectly understood problems.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0011

[1] The expression "law of refraction" is not Cartesian in origin. Descartes consistently refers to what he terms the "measure" of refraction or the "proportion" between the angles of incidence and refraction (see, e.g., AT 10:394, CSM 1:29 and AT 6:101, CSM 1:161). He reserves the term "law" for the laws of nature. I employ the expression "law of refraction" as a convenience, but it is an anachronism.

[2] Descartes experimentally confirmed the law of refraction in 1626/1627 (see letter to Golius, February 2, 1632, AT 1:240, CSMK 3:36; letter to Huygens, December 1635, AT 1:335–6), and must have discovered it before then, but the relevant documents do not determine the exact date. He first communicated the law of refraction (the sine law) as well as his solution to the problem of the anaclastic (in the case of the ellipse) to Isaac Beeckman on October 8, 1628 (see AT 10:335–9). As I argued in Chapter 4, Section 4.7.1, Rule 8_{CM}, which contains Descartes's discussion of the anaclastic, is likely to have been written no earlier than 1626/1627. See also Appendix.

[3] See Schuster 1977, 1986, 2000, 2013, 265–307 and the discussion in the Introduction and Section 10.2 below.

[4] See Heeffer 2017; Schuster 2013, 167–225; Shea 1991, 149–65; Smith 1987; Sabra 1981, 93–136; Milhaud 1921, 103–24 and 1907, 223–8. Hamou 1999 and Costabel [1982] 2013 are partial exceptions, but see the discussion toward the end of Section 10.2.

[5] Sabra 1981 is an exception, but see the discussion in Section 10.2.

[6] On the important role played by the problem of the anaclastic in Parisian optics in the 1620s, see Shea 1991, 150–1.

[7] See Aristotle 1984, 1:122; Lennox 1986; and Hacking 2014, 146–8.

[8] Schuster 2013, 56–9.

[9] See Schuster 2013, 215–20.

[10] See AT 1:357 (Fermat to Mersenne, April or May 1637) and AT 1:543 (Morin to Descartes, February 22, 1638). See also the discussion in Smith 1987, 29–31.

[11] See Descartes AT 1:358–9 and the discussion in Sabra 1981, 85–136.

[12] Milhaud 1907, 223–8 and 1921, 103–24. For a more detailed discussion of the literature, see Dika 2022.

[13] See Schuster 2013, 617.

[14] Sabra 1981 is an exception, but his reconstruction is based exclusively on *Dioptrics* II. However, as Heeffer (2017, 7) has persuasively argued, Descartes does not claim that his exposition of the law of refraction in *Dioptrics* II reflects the "order of research" whereby he discovered the law, but only the "order of exposition," i.e., the rhetorical order that most effectively facilitates comprehension in the reader. By contrast, Descartes does clam that his deduction of the rainbow in *Meteorology* VIII reflects the "order of research." See the letter to Vatier, February 1638 (AT 1:559–600, CSMK 3:85). The order prescribed in Rule 8 is an order of research, not exposition. Consequently, there is more reason to regard Rule 8 as representative of Descartes's actual path to discovery than there is to regard *Dioptrics* II in this way. For more discussion, see Section 10.9 below.

[15] Milhaud 1907 and 1921.

[16] Schuster 1977 and 2013, Gaukroger 1995.

[17] Shea 1991 and Costabel 1982.

[18] Smith 1987.

[19] See Schuster 2013, 167–224. See letter to Golius, February 2, 1632, AT 1: 240, CSMK 3: 36. Descartes's Parisian period is described in Clarke 2006, 67–97; Gaukroger 1995, 135–87; Rodis-Lewis 1998, 49–73.

[20] See also Gaukroger 1995, 139–46. For a description of the image-location principle, see Schuster 2013, 188–90 and Smith 2014, 110–11.

[21] See Mersenne 1932–1988, 1:404–15. Mersenne copied the document in 1631 (see Shea 1991, 243, n.38), but Schuster (2013, 187) plausibly claims that the material it contains dates from 1626/1627.

[22] See Heeffer 2017, 153. Mydorge never published Book VIII. See Mydorge 1631.

[23] Nevertheless, the diagram used in Proposition 1 of the Mydorge document (see Figure 10.4) represents the law of refraction as Descartes understood it, since in that diagram the lines representing the parallel component determinations are equal in length before and after refraction, as

required by Descartes's principles, whereas the diagrams in *Dioptrics* II represent the law of refraction as interpreted through Descartes's tennis-ball analogy, with differing speeds before and after refraction, so that the lines representing the parallel component determinations have different lengths before and after refraction. On this point, I agree with both Mahoney 1973 and Schuster 2013, even though I disagree with Schuster that this means that Descartes first discovered the law of refraction in a cosecant form based on the image-location principle. For more discussion, see Section 10.6.

24 Heeffer 2017, 161.

25 Mersenne 1932–1988, 1:406. See also the footnote by De Waard.

26 Schuster 2013, 193, n. 58. Schuster argues that Descartes nevertheless employed the image-location principle, and that Rule 8 "seems intended to occlude this fact" (ibid.). This is consistent with his overall view that Rule 8 conceals the path to discovery.

27 See Kepler 1604, cap. 4, sec. 2, 85; Kepler 2000, 101.

28 Heeffer 2017, 156–9. See also Kepler 1604, cap. 4, sec. 2, 85; Kepler 2000, 101 and Descartes AT 10:336.

29 Descartes refers to Kepler as his "first master in optics [*mon 1^er^ maistre en Optique*]" in his letter to Mersenne on March 31, 1638 (AT 2:86), not his "only master in optics," as Heeffer (2017, 155) translates him. Heeffer does provide some very interesting circumstantial evidence, which I encourage the reader to consult.

30 See Smith 1987.

31 See Smith 1987, 57–60.

32 Smith 1987, 58.

33 Ibid.

34 Smith 1987, 59–60.

35 Smith1987, 62.

36 Ibid.

37 Ibid.

38 Smith 1987, 62–4.

39 See Smith 1987, 54–5.

40 See Costabel [1982] 2013, 63–77, 53–9, and 77–87, originally published or delivered in 1965–1966, 1977, and 1978, respectively.

41 I discuss the equal-arm balance analogy in more detail in Section 10.7.

42 For a reconstruction similar to Costabel's see Shea 1991, 149–65.

43 See, e.g., Schuster 2013, 215–20; Gaukroger 1995, 139–46; Shea 1991, 159–63.

44 I am not the first to insist on the importance of Rule 8 as an indicator of Descartes's actual path to the discovery of the law of refraction and the shape of the anaclastic lens. Philippe Hamou has argued that Rule 8 "clearly formulates a methodic program which had to lead to the *Dioptrics* of 1637" (Hamou 1999, 252). He adds that "it does not seem excessive to see in this passage [in Rule 8] the chronical or chronology of Cartesian research on the anaclastic" in the 1620s (ibid.). Hamou shows in detail how the structure of Descartes's *Dioptrics* closely follows the research program laid out in Rule 8 (see the entire discussion in ibid., 239–89, esp. 241–5). In my view, Descartes's exclusive reliance on comparisons and analogies in *Dioptrics* I–II strongly distinguishes it from Rule 8. Thus, while I agree that the order of research prescribed in Rule 8 may in some respects mirror the

order found in *Dioptrics* I–II, I do not regard Descartes as following the order of research prescribed in Rule 8 to the letter to in *Dioptrics* I–II. Costabel [1982] 2013 also insists on the importance of Rule 8. Neither author actually executes Descartes's proposed deduction of the law of refraction and the shape of the anaclastic lens in Rule 8 or concretely shows how Descartes could have discovered the law of refraction and the shape of the anaclastic by its means.

45 Remember that the reduction of the problem to its simplest component parts is enumeration$_1$, while the construction of classes (including classes of effects) is enumeration$_3$. For more discussion, see Chapter 3, Sections 3.4–3.4.3.

46 Garber 2001, 94.

47 See AT 6:250–71; Descartes 2001, 332–45. For detailed reconstructions, see Buchwald 2008 and Garber 2001, 85–111.

48 Descartes's description of motion changes over time (see *Principles* II. 25–5, AT 8A:53–4, CSM 1:233). Since these changes do not directly bear on my argument, I will not discuss them further here. For further discussion, see Garber 1992 and Des Chene 1996.

49 On the force of resisting motion, see Descartes's important letter to Clerselier (February 17, 1645, AT 4:184).

50 All of these passages can be found in the Cambridge manuscript, before Descartes finalized his theory of simple natures in Rule 12$_{AT}$.

51 Garber 1992, 327, n.1 has argued that in Rule 14 Descartes only makes the more restricted claim that "in the imagination, extension is inseparable from its subject, body," but that this is not necessarily so beyond the imagination, in reality. I agree and do not regard this as a problem, since I am only concerned with the role played by extension as intuited by the intellect aided by the imagination here. As so intuited, there can be no void space.

52 Descartes does not *need* a plenum theory of space in order to deduce the law of refraction in Rule 8; he only needs the particles that compose light (the spherical *boules* he would later discuss in more detail in *The World*, AT 11:84–98) to be spatially contiguous with one another, such that the pressure exerted along them can be instantaneously transmitted in a straight line from one end to another (e.g., sun to eye). For more details, see Schuster 2013, 509–14. I introduce the plenum theory here because the Cambridge manuscript suggests (as previous manuscripts did not) that it did indeed play a role in how Descartes was thinking about the relation between space and motion in *Rules* when he was writing Rule 8. Prior to the discovery of the Cambridge manuscript, it was believed that the later parts of *Rules*, including Rule 14, were written around 1629, which would mean that Descartes wrote Rule 14 about three or four years *after* he discovered the law of refraction. This would mean that Rule 14 is not relevant to any historically accurate reconstruction of Descartes's path to the discovery of the law of refraction. However, the Cambridge manuscript changes the chronology. Assuming that the Cambridge manuscript is an earlier draft of *Rules* (see Appendix), it would have to have been written no earlier than 1626, since it contains Descartes's discussion of the anaclastic in Rule 8, and Descartes is known to have worked on lens theory in 1626 with Mydorge in Paris. The Cambridge manuscript contains all of the relevant passages about plenum space in Rule 14. This would mean that Descartes wrote these passages, not in 1629 (as previously believed by Weber 1964, 206), but rather earlier, in or around 1626. The passages on plenum space in Rule 14 do, therefore, seem to be chronologically coeval with the passages on the law of refraction and the shape of the anaclastic lens in Rule 8, and it is likely that Descartes was thinking about motion as motion in a plenum at least as early as 1626—a thesis he would later develop more systematically in *The World*.

[53] See also *The World*, AT 11, 40; CSM 1, 94: "[The] philosophers attribute to the least of these motions a being much more solid and real than they attribute to rest, which they say is nothing but the privation of motion. For my part, I conceive of rest as a quality too, which should be attributed to matter while it remains in one place, just as motion is a quality attributed to matter while it is changing place." For more discussion on rest as a negation (and not a privation) of motion, see Chapter 7, Section 7.6.

[54] The first law of nature in *The World* and *Principles* plays an important role in Descartes's physics as early as his research with Beeckman on bodies in free fall in 1618, before he began writing *Rules* and before his earliest optical research in 1619. It is not, therefore, anachronistic to regard Descartes's first law of nature as operative in *Rules*. See AT 10:78, where Descartes asserts that "*in vacuo quod semel motum est semper movetur*" (see also *Cogitationes privatae* (1619), AT 10:219: "*Quod enim in vacuo movetur, semper moveri*"). For more discussion, see Garber 1992, 213. As early as 1612–1613, Beeckman maintained the conservation principle that "*mota semel nunquam quiescunt nisi impediantur*," a principle he employs throughout the *Journal* (see Beeckman 1939–1953, 1: 24; ibid., 10, 35, 44, 61, 167, 253, etc., cited and discussed in de Buzon 2013, 162–4). In a letter to Mersenne on December 18, 1629, Descartes seems to acknowledge the Beeckmanian origins of the conservation of motion in 1618 (see AT 1:91). On the origins of Descartes's first and third laws of nature in his research on free fall and hydrostatics with Beeckman in 1618–1629, see the extensively documented and rigorously interpreted Schuster 1977; Gaukroger 2000; Gaukroger and Schuster 2002; and Schuster 2013.

[55] The distinction between motion and tendency to motion plays an important role in Descartes's physics as early as his research on the hydrostatics paradox in 1618. It is not, therefore, anachronistic to regard the distinction between motion and tendency to motion as operative in *Rules*. See AT 10:67–74. For a detailed analysis, see the references in the previous note. Beeckman maintained the conservation of rectilinear and curvilinear motion (in celestial bodies) in his *Journal* (see Beeckman 1939–1953, 1:253). See also de Buzon 2013, 164–6. I explain why Descartes rejects the conservation of curvilinear motion below.

[56] To intuit a body at an instant is to intuit an "atemporal stage of the body in motion, a stage in which there is no duration" (Garber 1992, 286). Durations are not composed of instants any more than lines are composed of points; instants are "boundaries of temporally extended durations" (Garber 1992, 287), not atemporal units that constitute time.

[57] See de Buzon 2013, 187. In *Principles* II. 32, Descartes also describes the "straight line" as the "simplest of all" (AT 8A:58, CSM 1:237).

[58] See Des Chene 1996, 283–4.

[59] Gorham 2005, 439 argues that "*a priori* criteria of simplicity are notoriously difficult to justify. Thus, although straight motion is perhaps the simplest in directional terms, circular motion is simpler in having an identical beginning (*terminus a quo*) and end (*terminus ad quem*)." However, Descartes provides very clear criteria of simplicity in *Rules* and *The World* based on the number of parameters that need to be considered by the mind in conceiving rectilinear and curvilinear motion. This criterion clearly excludes any motion besides rectilinear motion from the class of the simplest motions.

[60] In *The World*, Descartes writes that all the laws of nature are based on the same foundation: "God's preserving each thing by a continuous action, and consequently on his preserving it not as it may have been sometime earlier but precisely as it is at the very instant that he preserves it" (AT 11:44–45, CSM 1:96). In *Rules*, Descartes does not consider God's role as the metaphysical cause of motion; he only needs to suppose that the quantity of force in a body and its rectilinear tendency to motion is conserved by *some* cause, immanent or transcendent. The physics contained in Descartes's theory of simple natures in *Rules* is metaphysically neutral.

[61] See Schmaltz 2008, 120–1.

[62] Descartes's discussion of the problem of the anaclastic in Rule 8_{CM} is identical with the same discussion in Rule 8_{AT}, but Descartes's theory of simple natures is not contained in Rule 12_{CM} (see Chapter 8, Section 8.3 and Appendix). Thus, it may be objected that Descartes did not have a clear or ready answer to the problem of what a natural power is when he first composed Rule 8_{CM}. My response is that most of the resources Descartes needs are already contained in Rule 14_{CM}, and that Descartes's theory of simple natures reflects his mind–body dualism, which I have argued in Dika 2020 and Chapter 8 is also contained in the Cambridge manuscript. The intellectual simple natures are acts of the intellect, and the material simple natures are the basic notions of body underlying the mechanized natural philosophy Descartes had been practicing since his earliest research with Beeckman in 1618.

[63] Descartes's deduction of the law of refraction in *Dioptrics* II does not explicitly depend on the third law of nature in *The World*, but since Descartes's proposed deduction of the law of refraction and the shape of the anaclastic lens in Rule 8 depends on a more articulated physics than the deduction based exclusively on comparisons and analogies in *Dioptrics* does, and since Descartes already had the relevant laws on hand when he composed Rule 8, and since, most importantly, the action of light as a rectilinear tendency to motion cannot be physically understood without the third law, the third law cannot be ignored in any reconstruction of Descartes's deduction of the law of refraction and the shape of the anaclastic lens in Rule 8.

[64] "By 'dimension' we mean simply a mode or aspect in respect of which some subject is considered measurable. […] Speed is a dimension – the dimension of motion" (AT 10:447, CSM 1:62).

[65] See Arthur 2007, 13, 23; Beeckman 1939–1953, 1:266; ibid., 3:133–4.

[66] Beeckman 1939–1953, 4:41, cited in van Berkel 2013, 81.

[67] Beeckman 1939–1953, 4:162, cited in van Berkel 2013, 81. Compare Descartes in Rule 14 (which, as we know now, was not written after Rule 8_{CM}, or the parts of Rule 8_{AT} that correspond to Rule 8_{CM}, but is rather coeval with it; see n. 52 above): "[We] generally do not recognize philosophical entities of the sort that are not genuinely imaginable" (AT 10:443, CSM 1:59).

[68] See Beeckman 1939–1953, 1:24–5, cited in van Berkel 2013, 105. See also Beeckman 1939–1953, 1:24, cited in van Berkel 2013, 226. Since there has been some controversy over whether Beeckman discovered "the" principle of inertia (see Koyré 1940, 108–9, n. 3), I employ the expression "principle of inertia" here as a matter of convenience, not in order to make a substantive historiographical claim. See the discussion in van Berkel 2013, 228, n. 13 on the subject.

[69] See the extensive discussion in van Berkel 2013, 81 and 173–85.

[70] See Dika 2020. A deeper analysis of Descartes's early scientific writings would reveal other important connections to his method that I sadly cannot discuss in more detail here.

[71] Buchdahl 1969, 140–1 argues that in Rule 9 Descartes's recourse to analogy means that he "now suddenly changes his method," embracing hypotheses and models, and that "the deductive line is snapped and the original restrictions are relaxed." But there is no change; Descartes clearly regards recourse to analogy, not as a deficiency, but rather as a resource and as *part of the deduction*, and he even identifies the criterion that determines the conditions under which analogies are appropriate: when one does not know how else to proceed. Buchdahl further mischaracterizes both the analogy and the role played by the analogy in the deduction. The analogy is not based on the projectile motion of a stone, but rather the instantaneous transmission of force from one end of a stick to another, and the analogy only confirms that the instantaneous transmission of force is possible, since

it is possible in the most mundane cases. The reasons for believing that the force of light is transmitted instantaneously are not based on the analogy, but rather on the intuition of what a natural power is, together with the observation that the transmission of light is rectilinear. The analogy does not replace the intuition of what a natural power in general is, as Buchdahl seems to believe.

72 See the references in Smith 1987, 47–56.

73 See Sabra 1981, 114. In his interpretation of both the optical fragment and *Dioptrics*, Sabra mistakenly interprets Descartes to be making a claim about the "the velocity of light" as "a characteristic of the medium it is traversing," when the relevant physical magnitude is not velocity (light has no speed and, therefore, no velocity), but rather "penetration" or force.

74 Descartes would later make the same claim in *Dioptrics* II before introducing the law of refraction: "[It] must be said that when its rays pass obliquely from one transparent body into another, which they penetrate more or less easily than the first, they are deflected in such a way that their inclination to the surface between these bodies is always less sharp on the side of the more easily penetrated body, and the degree of this inclination varies exactly in proportion to the varying degrees of penetrability of the respective bodies" (AT 6:100–1, CSM 1:161). As in the optical fragment, in *Dioptrics* II Descartes asserts that the force of light is proportional to the density of the medium, and that in denser media, light acquires more force and bends toward the normal, while in rarer media, light loses force and bends away from the normal toward the refracting surface.

75 Curiously, despite its importance, neither Sabra 1981 nor Schuster 2013 devote serious attention to the physical rationale behind the first sentence.

76 For most optical theorists before Descartes inspired by Aristotelian natural philosophy, luminosity (*lux*) is an immaterial form (quality), which can only be produced in a properly receptive material substratum, where it produces light (*lumen*) in all directions (see the references and discussion in Smith 1987, 32–3). In the optical fragment, Descartes employs the word *lux*, but he only discusses the material conditions under which *lux* produces *lumen*. It is not clear whether he regards *lux* itself as material, but it is likely that he did. In any case, it is clear that he would have regarded it as material in *Rules*.

77 See also *Dioptrics* II AT 6, 103; CSM 1, 163: "…the action of this subtle matter can be impeded much more by the parts of the air (which, being as it were soft and badly joined, do not offer it much resistance) than by those of water, which offer it more resistance; and still more by those of water than by those of glass, or of crystal. Thus, insofar as the minute parts of a transparent body are harder and firmer, the more easily they allow the light to pass," which explains why light acquires more force in water, and why it bends toward the normal after refraction, Descartes believes.

78 See Schuster 2013, 192.

79 Schuster 2013, 192, n.57.

80 Schuster 2013, 153–63 claims that Descartes's principle that in denser media, light acquires more force and bends toward the normal is a "physico-mathematical" interpretation of a geometrical diagram representing refraction in Kepler's *Paralipomena* (see Kepler 1604, cap. 4, sec. 2, 85; Kepler 2000, 101). This is debatable. Descartes does not mention Kepler in the document, and the diagram he employs there is similar to the Kepler diagram Schuster refers to, but far from identical. The optical fragment does not require interpreting Descartes as making a claim about the behavior of the normal components of light rays. Interestingly, while Schuster 2013, 153–64 and 190–3 claims that Descartes could only have derived a law of tangents based on his interpretation of the Kepler diagram, Heeffer 2017, 156–9 has shown that the diagram also could have led to a law of sines.

81 On this point, see Sabra 1981, 106. I discuss Descartes's concept of "determination" in more detail below. Note that I am not importing resources from *Dioptrics* II into my interpretation of the optical fragment, but merely stating the required premise. I offer an account of its origin below.

[82] Schuster 2013, 184–8 and Heeffer 2017, 161 disagree about how to understand this diagram. For Schuster, the unequal radii indicate that it is the constant proportion between the cosecants of the angles of incidence and refraction that is asserted in it. For Heeffer, by contrast, it is the constant proportion between the sines of the angles of incidence and refraction that is asserted in it. Schuster believes that the "cosecant" law is the first form of the law Descartes discovered because (1) that is how others discovered it, and (2) that is how Descartes would have had to discover it by purely mixed-mathematical means using the image-location principle (see Schuster 2013, 188–90). Since the evidence that Descartes employed the image-location principle is conjectural, I am less inclined to regard him as having employed it in the discovery of a "cosecant" law of refraction distinct from the "sine" law of refraction.

[83] See ibn al-Haytham 1572, lib. 7, sec. 8, 241, ibn al-Haytham 2010: 244–7, esp. 246; ibn al-Haytham 1572: lib. 4, sec. 18, 112–13, ibn al-Haytham 2006: 322–3. See also Kepler 1604: cap. 1, prop. 19, 15–15, Kepler 2000: 27 and the discussion in Smith 1987, 47–56; Sabra 1981, 69–78, 93–9; Lindberg 1968.

[84] Ibn al-Haytham differs from Kepler because he believes that light traverses transparent media via local motion (and, therefore, that the local motion of light has a definite speed, so that the normal and parallel components represent force, direction, and speed), whereas Kepler believes that light traverses transparent media instantaneously (so that these components only represent force and direction). See ibn al-Haytham 1572, lib. 7, sec. 21, 37–8, cited in Lindberg 1968, 25; Kepler 1604, cap. 1, prop. 4, 9; Kepler 2000, 21. For my purposes, these differences are not important.

[85] See Miller 2017, 158–61; Damerow et al. 2000, 71–135; McLaughlin 2000, 81–113; Freudenthal 2000.

[86] See Schuster 2013, 171 and 2000, 260.

[87] On Descartes's concept of determination, see Gabbey 1980; Schuster 2013, 1977; McLaughlin 2000; and Damerow et al. 2004.

[88] As Schuster 2013, 203 rightly points out, the analogy "makes no sense if one still has in mind a real balance, with one arm plunged into a real vat of water." See also Costabel [1982] 2013, 71. The reason why the analogy makes no sense if one still has in mind a real balance, with one arm plunged into water, is that in such a case, the arm under water will bend *away* from, rather than *toward* the normal, since the effective weight attached to it will be lighter in water than in air. Thus, to make sense of the analogy, Schuster suggests that "one must abstract from the concrete situation and invoke different media with the appropriate ratio densities" (ibid.). In other words, one has to assume, not only that the medium below is denser, but also that this denser medium bends the arm *toward* the normal, contrary to what actually happens in the case of water in accordance with Archimedean principles. See also Costabel [1982] 2013, 72: "L'analogie proposée ne pouvait conduire à la solution qu'en supposant paradoxalement le rayon réfracté alourdi par la rentrée dans l'eau avant d'être allégé par la poussée d'Archimède." Interestingly, this particular disanalogy between weight in water and light in water remains effective even in Descartes's tennis-ball analogy in *Dioptrics* II, since tennis balls are also deflected away from the normal in water.

[89] Ibid.

[90] See, e.g., Schuster 2013, 178–84 and Smith 1987, 30–1. The objection can be traced back to Bourdin in his letter to Mersenne, December 3, 1640, AT 3, 248–52.

[91] Milhaud 1907, 226.

[92] See Milhaud 1907.

[93] See Introduction and, e.g., letter to Vatier, February 22, 1639, AT 1, 559; CSMK 3, 85. In this letter and many others (letter to an unknown correspondent, end of May 1637, AT 1, 370, CSMK 3,

58; letter to Mersenne, February 27, 1637, AT 1, 349, CSMK 3, 53), Descartes insists that he does not provide a complete exposé of his method in *Discourse on the Method*, but rather only a sketch. (In the letter to an unknown correspondent, CSMK 3, 58 mistranslates "…*une Méthode generale, laquelle veritablement ie n'enseigne pas*" as "I am not actually following the method," when in fact Descartes only asserts that he does not teach the method.)

94 I cannot demonstrate this here, but see Garber 2001 and Bos 2001. For a more critical appraisal of Descartes's application of the method in *Meteorology* VIII, see Buchwald 2008.

95 See Section 10.1 above.

PART V

BEYOND *RULES*

11

Descartes's Method after *Rules*

11.1 Method or Methods?

In Chapters 9–10, I reconstructed Descartes's method in mathematics in Rules 13–21 and his deduction of the law of refraction and the shape of the anaclastic lens in Rule 8. Chapter 10 especially has demonstrated that Descartes's method is an instrumentally efficacious technique or *habitus* that yields discoveries in natural philosophy. In this chapter, I do two things. First, in Sections 11.2–11.3, I examine how problems internal to *Rules* led Descartes to abandon the treatise and turn to metaphysics and systematic natural philosophy in 1629–1630. Second, in Sections 11.4–11.5, I discuss the relation between Descartes's method in *Rules* and Descartes's method post-*Rules*. I examine whether Descartes's method continues to play a role in his philosophy, and I also examine the development of the theory of simple natures during this same period (mostly in *Principles* and related texts and correspondence).

In Section 11.2, I consider the problems that undermine Descartes's methodological project in Rules 13–21 and lead Descartes to abandon *Rules* and condense the presentation of the method in *Discourse* II. Those who have read Chapter 9 have already encountered the immediate reason why Descartes abandons *Rules*: Descartes's geometrical calculus in Rules 18–21, which is based on the manipulation of rectangles and line segments alone, cannot be employed to extract the roots of incommensurate continuous magnitudes (which requires the circle).[1] Descartes does not overcome the distinction between discrete and continuous magnitude; arithmetic and geometry remain irreducibly heterogeneous mathematical sciences in *Rules*. This undermines the unity of science in *Rules*. This means that the method

in *Rules* cannot solve all perfectly understood problems according to the procedure designed to solve them, and this undermines both the unity and the universality of the method. As I will show in more detail in Section 11.2, Descartes responds to these problems after *Rules* by abbreviating his presentation of the method in *Discourse* II, abandoning *mathesis universalis*, and redefining mathematics as the science of extended magnitudes alone.

In Section 11.3, I examine the problems that propel Descartes to turn to metaphysics and systematic natural philosophy in 1629–1630. These problems are not mathematical, and they have little to do with Descartes's alleged encounter with skepticism, as some have argued.[2] The problems that propel Descartes to turn to metaphysics and systematic natural philosophy are *internal* to *Rules*, and include: (1) the problem of demonstrating that reality (i.e., mind and body) is reducible to the simple natures (see Chapters 5–7); (2) the problem of demonstrating that even those properties of extension that are "incomprehensible" because they cannot be intuitively certified by the intellect aided by the imagination can nevertheless exist; (3) the problem of demonstrating that the truths disclosed via intuition have an ontological foundation (i.e., subsist, or continue to exist or have duration) beyond the intellect. All three of these problems show up explicitly in Descartes's correspondence with Gibieuf and Mersenne in July 1629 and April 1630 (respectively), *The World*, and related texts shortly after he abandons *Rules*. These documents are the best evidence on which to base any interpretation of the problems that lead Descartes to abandon *Rules*. Skepticism plays little, if any, substantive role in these documents.

In Section 11.4, I address the argument that Descartes's method became "obsolete" after *Rules*.[3] The argument is that the method in *Rules* is designed to solve particular problems, not to construct a system, and that Descartes decides to construct a system after *Rules*, as early as 1629. My main argument in Section 11.4 is that what Descartes does after *Rules*—ultimately, in *Principles*—is simply invert the order prescribed in Rule 5 and expand it to include the entire system of science. Descartes's system in *Principles* did not render his method obsolete. On the contrary, the method is remarkably well-adapted to Descartes's system, since it is constituted by inverting the order prescribed in Rule 5 and by employing the principal operations of the method.

Finally, in Section 11.5, I examine the development of Descartes's theory of simple natures after *Rules* up to and including *Principles*. Two developments in particular stand out: Descartes's decision to include substance among the common simple natures (see Chapter 8, Sections 8.2–8.3), and his decision to include mind–body union as a separate class of simple nature. These two developments, I argue, require significant revisions in the foundations of the theory of simple natures in *Rules*, but they do not lead to a wholesale rejection of the theory, which continues to play a foundational role in Descartes's later philosophy and science.

11.2 From *Rules* to *Discourse*

As I indicated in Section 11.1, those who have read Chapter 9 will already have encountered the immediate reason why Descartes abandons *Rules*: Descartes's geometrical calculus in Rules 18–21, which is based on the manipulation of rectangles and line segments alone, cannot be employed to extract the roots of continuous incommensurate magnitudes (which requires the circle). Descartes does not overcome the distinction between discrete and continuous magnitude; arithmetic and geometry remain irreducibly heterogeneous mathematical sciences in *Rules*. This undermines the unity of science in *Rules*. This means the method in *Rules* cannot solve all perfectly understood problems according to the procedure designed to solve them, and this undermines both the unity and the universality of the method. Why not simply introduce the circle into the geometrical calculus in *Rules*, as Descartes later would in *Geometry* I? As I argued in Chapter 9, Sections 9.6–9.8, introducing the circle would effectively undermine the unity of mathematics as defined by *mathesis universalis*. In *mathesis universalis*, all problems in mathematics are about relations of order and measure. These relations transcend the distinction between discrete and continuous magnitude. Descartes's geometrical calculus in *Rules*, however, does not. Had Descartes been pursuing geometry alone in *Rules*, there would have been no problem here. Geometry is the science of continuous magnitudes, and so a calculus based on rectangles, lines, and circles is in no way anomalous in geometry. But Descartes does not pursue geometry alone in *Rules*. He hopes to extend the method to all mathematical sciences.

Mathesis universalis subsumes and is more universal than geometry, since order and measure cover both discrete and continuous magnitude. This places unique constraints on Descartes's geometrical calculus in Rules 18–21, since this calculus must be employed in solutions to problems about both discrete and continuous magnitudes. Since the calculus does not transcend the distinction between discrete and continuous magnitude, it is no longer clear that one scientific *habitus* can solve all problems in the sciences. The unity of mathematics is not secured, and this means that the unity of science and the unity and universality of the method are also not secured in *Rules*. The tension between *mathesis universalis* and the method leads, in fact, to a *crisis*.

What is to be done?

It may seem that the appropriate response to the problems raised by Descartes's failure to show how all perfectly understood problems can be solved by means of one method would be to abandon the thesis that one method can solve all problems in the sciences. Descartes's response is different. Instead of abandoning the method, he abandons *mathesis universalis* and *redefines the object of mathematics*. After *Rules*, Descartes focuses on only *one part* of what he terms "pure and abstract mathematics"—*geometry*. In a letter to Ciermans, Descartes expressly indicates that in *Geometry* he does *not* address any problems about order and measure (March 23, 1638, AT 2:70). In *Meditations* V, he *reduces* the object of pure and abstract mathematics to extension *alone* (and, therefore, geometry) (see AT 7:71, CSM 2:49). By reducing pure and abstract mathematics to geometry, the problem of the unity of discrete and continuous magnitude disappears. The purely instrumental role played by number in mathematics in *Rules* (as a symbol representing extended magnitudes) remains, but discrete magnitude is no longer an *object* of mathematics. All problems in mathematics are problems about extension or continuous magnitude, and there is one geometrical calculus appropriate to all such problems—precisely the geometrical calculus that appears in *Geometry* I, where the circle is indeed employed in root extraction. The basic idea of analysis that defines the mathematical application of the method in *Rules* remains in place, and is clearly illustrated in *Geometry*.[4] *Mathesis universalis*, however, is gone, never to return. In short, Descartes had two choices: either abandon the method and retain *mathesis universalis*,

or abandon *mathesis universalis*, redefine mathematics, and retain the method. He chose the latter.

The implications for *Rules* are clear: much of the second part devoted to perfectly understood problems (Rules 13–24) can be transferred to *Geometry*, but without the constraints imposed on the geometrical calculus by *mathesis universalis*. The tripartite division of *Rules* into "simple propositions," "perfectly understood problems," and "imperfectly understood problems" no longer structures Descartes's presentation of the method in *Discourse*. What remains? In *Discourse* II, Descartes provides an abbreviated presentation of the method in four rules, which he describes in as many sentences:

> The first was never to accept anything as true if I did not have evident knowledge of its truth: that is, carefully to avoid precipitate conclusions and preconceptions, and to include nothing more in my judgments than what presented itself to my mind so clearly and so distinctly that I had no occasion to doubt it.
>
> The second, to divide each of the difficulties I examined into as many parts as possible and as may be required in order to resolve them better.
>
> The third, to direct my thoughts in an orderly manner, by beginning with the simplest and most easily known objects in order to ascend little by little, step by step, to knowledge of the most complex, and by supposing some order even among objects that have no natural order of precedence.
>
> And the last, throughout to make enumerations so complete, and reviews so comprehensive, that I could be sure of leaving nothing out (AT 6:18, CSM 1:120).

These four rules can and should be read as a highly abbreviated summary of the method in *Rules*, especially Rules 1–7, which form the procedural "core" of the method in the first part of *Rules*.[5] The objects of the method in Rules 8 and 12—the simple natures—appear obliquely later in *Discourse* II as the "simplest and most easily known things" in the "order required for deducing one thing from another" (AT 6:19, CSM 1:120), and they continue to play a central role in Cartesian science well into *Meditations*, *Principles*, and beyond (see *Principles* I. 47–9, AT 8A:22–4, CSM 1:208–9; letter to Elizabeth, May 21, 1643, AT 3:665–7, CSMK 3:218–19).[6] The role played by *mathesis universalis* in the *culture of the method* is also retained and underlined in *Discourse* II:

> From this [mathematics], […] the only advantage I hoped to gain was to accustom my mind to nourish itself on truths and not to be satisfied with bad reasonings. Nor did I have any intention of trying to learn all the special sciences commonly called "mathematics." For I saw that, despite the diversity of their objects, they agree in considering nothing but the various

> relations or proportions that hold between these objects. And so I thought it best to examine only such proportions in general, supposing them to hold only between such items as would help me to know them more easily (AT 6:19–20, CSM 1:120–1).

As we have seen in Chapter 4, “order and measure” refers, not to any of the “special sciences,” but rather to proportional relations that obtain in all mathematical sciences. Beyond *Rules*, *Discourse* II is the only place where Descartes refers to *mathesis universalis* in his entire corpus—not explicitly, but rather by reference to “relations and proportions.” In *Discourse* II, Descartes describes his past mathematical practice, not a mathematical program or a project he intends to further pursue (and as I have shown above, in his letter to Ciermans, he explicitly denies that he addresses any problems about order and measure in *Geometry*). Finally, Rules 16–17 also make an oblique appearance in *Discourse* II, since Descartes insists on the use of algebra in geometry, and on a geometrical calculus that depends on lines alone (rectilinear and curvilinear; rectangles are not mentioned) (ibid.).

The abandonment of *Rules* may not have led to an abandonment of the method as such, but it did lead to the *abandonment of the thesis that the method can be completely canonized in a theoretical treatise.*[7] As we have seen, in *Rules* even Descartes’s classification of perfectly understood problems remains too general and conceals differences he could not overcome. After *Rules*, there is no *general classification* of problems in Descartes’s *oeuvre*, but rather *local classifications of problems* in *particular sciences*. For example, in *Geometry*, Descartes distinguishes between classes of problem based on complexity (degree) and provides their corresponding equations together with the constructions required in their solution. But beyond geometry, after *Rules* Descartes consistently deals with problems only on a *case by case basis*. The idea of a methodological treatise whose tripartite division corresponds to a complete enumeration$_3$ of problem classes is abandoned. Hence the need to condense the presentation of the method in *Discourse* II. By the late 1630s, Descartes decided to reduce the number of rules and focus on the application of the method rather than on the theory of the method. In an important letter to Mersenne in which he discusses the title of *Discourse*, Descartes insists that the method “is concerned more with practice than with theory” (letter to Mersenne, February 27, 1637, AT 1:349, CSMK 3:53), and he explicitly distinguishes between a “treatise” on method (such as *Rules*) and a

"discourse" on method. A "discourse" on method does not aim at exhaustivity in breadth or depth. It does not "teach" the method; it only "discusses" the method. One can, of course, distinguish between problems in which the conditions relevant to the solution are provided (mathematics) and problems in which these conditions must be methodically discovered (metaphysics and natural philosophy), but one cannot completely enumerate$_3$ these problems or the procedures whereby they may be solved in advance.

This in no way means that Descartes abandoned the method or even that he relegated it to a peripheral role in his philosophy. As I argued toward the end of Chapter 10, however much Descartes's contemporaries may have complained that they could not detect the application of the method in the *Essays*, the *Essays* are not nearly as removed from the method as they seem to be. Descartes illustrates the use of the method throughout *Geometry* and in *Meteorology* VIII, and I have reconstructed how the method can be employed in the deduction of the law of refraction and the shape of the anaclastic lens in Chapter 10. *Dioptrics* I–II as well as *Dioptrics* VIII can therefore find their place in the method no less than *Geometry* and *Meteorology* VIII. These facts suffice to substantiate the claim that Descartes employed the method in the *Essays*, even though he frequently *wrote* according to the "order of exposition" and not the "order of research." Furthermore, as I will argue in Sections 11.3–11.5, the method continues to inform Descartes's philosophy well beyond the *Essays*.

11.3 The Turn to Metaphysics

The problems discussed in Section 11.2 explain why Descartes abandons *Rules*, but they do not explain Descartes's turn to metaphysics and systematic natural philosophy in 1629–1630. As we have seen, Descartes eventually resolves the tension between method and *mathesis universalis* internally by abandoning *mathesis universalis* and redefining the object of mathematics. Metaphysics plays no role in his solution to this problem, nor does it seem to be a necessary part of any such solution. In Section 11.3, I argue that the problems that propel Descartes to turn to metaphysics and systematic natural philosophy are *internal* to *Rules*, and include: (1) the

problem of demonstrating that reality (i.e., mind and body) is reducible to the simple natures, a thesis Descartes supposes, but does not demonstrate in *Rules* (see Chapter 2, Section 2.4; Chapter 5, Section 5.3; Chapter 7, Section 7.12); (2) the problem of demonstrating that even those properties of extension that are "incomprehensible" because they cannot be intuitively certified by the intellect aided by the imagination—i.e., the indefinite extent of extension and the indefinite division of extension into ever smaller parts—can nevertheless exist; (3) the problem of demonstrating that the truths disclosed via intuition have an ontological foundation (i.e., subsist, or continue to exist or have duration) beyond the intellect, and so do not depend on the acts of intuition whereby they are delivered to the mind (acts of intuition are ephemeral, but truths are not).

Immediately after *Rules,* Descartes sets out to establish what he could only suppose in *Rules*: that reality is reducible to the simple natures. This is clear from Descartes's correspondence in 1628–1629. The correspondence indicates that Descartes composed a small metaphysical treatise conceived toward the end of his stay in Paris and written during the first nine months of his stay in Friesland. The treatise has been lost to posterity, but Descartes wrote to Gibieuf and Mersenne about its contents (see letter to Gibieuf, July 18, 1629, AT 1:17, CSMK 3:5; letter to Mersenne, April 15, 1630, AT 1:144, CSMK 3:22 and November 25, 1630, AT 1:181–2, CSMK 3:29), and he mentions it obliquely in *Discourse* and again in a letter to Mersenne in 1637 (see *Discourse* III, AT 6:30, CSM 1:126 and letter to Mersenne, February 27, 1637, AT 1:350, CSMK 3:53). In his letter to Mersenne on November 25, 1630, Descartes explicitly states that in this treatise he demonstrates "*the existence of God and of our souls* when they are separated from the body, from which their immortality follows" (AT 1:182, CSMK 3:29). In an earlier letter to Mersenne, Descartes states that he would "not have been able to discover the foundations of physics" had he not first demonstrated the existence of God and the nature of the self (April 15, 1630, AT 1:144, CSMK 3:22).

The problem of demonstrating that reality is composed of simple natures is what leads Descartes to see metaphysics (God) as the foundation of physics after *Rules*. Since Descartes already endorsed in *Rules* what would later become the real distinction between mind and body (see Chapter 8), it is clear that this distinction in his metaphysical treatise must have rested on the distinction between the intellectual and material simple natures. Thus,

however Descartes may have demonstrated God's existence in this metaphysical treatise, it is clear that God's function must have consisted in creating the world *according to the simple natures*. God creates the soul according to the intellectual simple natures, which entails (1) the reduction of the soul to the mind, and (2) the incorporeality (and, therefore, immortality) of the soul. God creates body according to the material simple natures, which establishes the possibility of a purely mechanical physics based on extension, shape, and motion alone. Finally, as we have seen in Chapter 10, Section 10.4, in *Rules* the first and third laws of nature are based on the intuition of relations between the material and common simple natures. Consequently, God's creating body according to the material simple natures is equivalent to his creating the world according the laws based on the simple natures and the relations between them as they appear in intuition.

Thus, the mind–body dualism and physics articulated via the theory of simple natures in *Rules* become properly metaphysical as early as 1628–1629. The order of the sciences, in which metaphysics is the foundation of physics, would never have arisen had Descartes not set out to demonstrate his supposition in Rule 12 that reality is reducible to the simple natures. The imperative to demonstrate this supposition explains both Descartes's turn to metaphysics after *Rules* as well as the specific place metaphysics comes to occupy in the order of the sciences. The priority of metaphysics over physics in the system of science is nowhere to be found in *Rules*. Indeed, it seems that any conception of a hierarchy between the sciences is *proscribed* by Descartes's theory of simple natures in *Rules*: "Lastly, we should not regard some branches of our knowledge of things as more obscure than others, since they are all of the same nature and consist simply in the putting together of self-evident facts" (AT 10:427–8, CSM 1:50). Only after *Rules*, once Descartes decides that he must demonstrate that reality is reducible to the simple natures, does the place of metaphysics in the order of sciences become clearly defined.

However, problems immediately begin to arise as Descartes further develops his theory of simple natures:

(1) Descartes begins to consider some properties of extension that he did not consider in *Rules*: the indefinite extent of extension and the indefinite division of extension into ever smaller parts. Before he communicates the

theory of eternal truths to Mersenne, he writes to ask him "whether there is anything definite in religion concerning the extension of created things, that is, *whether it is finite or infinite* […]" (letter to Mersenne, December 18, 1629, AT 1:86, CSMK 3:14; my emphasis). Descartes had not yet addressed this problem in *Rules*, but now that he had embarked on a treatise devoted to "the whole of physics" (i.e., *The World*), he could no longer avoid it (letter to Mersenne, November 13, 1629, AT 1:70, CSM 1:7). This led to the realization that extension cannot be intuited in the manner Descartes requires in *Rules*. As we have seen in Chapter 7, the simple natures cannot "contain anything beyond what we intuit or reach in our thinking," but it is clear that the material simple nature of extension *does* contain something beyond what we intuit or reach in our thinking. In *The World*, extension in general has no determinable boundary; any boundary I assign to extension always has more extension on the other side, such that "it fills spaces much greater than all those we have imagined" (see AT 11:32–3, CSM 1:90; see also *Principles* II. 21, AT 8A:52, CSM 1:232). Furthermore, extension is "divided into as many parts having as many shapes as we can imagine," and as Descartes would later clarify in *Principles* II. 34, "the number of particles into which matter is divided is in fact indefinite, although it is beyond our power to grasp them all" (AT 8A:59, CSM 1:239). Extension exceeds the boundaries of the imagination both in its indefinite extent and indefinite division, and as Descartes points out, the indefinite "is beyond our power to grasp [*incomprehensibiles*]." The simple natures are not supposed to be incomprehensible! Regarding extension, in Rule 14 Descartes clearly insists that whatever is not comprehended by the imagination *cannot* be ascribed to extension (see AT 10:444, CSM 1:60). It is clear that in both extent and division, extension *exceeds the limits of what can be intuited by the intellect aided by the imagination*. Here, God's function includes more than merely creating the world according to the simple natures. God's function also consists in creating extension such that *the reality of its indefinite extent and division can be secured beyond the limited scope of what can be intuitively certified by the intellect aided by the imagination*. God creates extension such that those properties of extension that are *incomprehensible* to the mind are nevertheless *real*: "In general we can assert that [God] can do everything that is within our grasp but not that he cannot do what is beyond our grasp" (letter to Mersenne, April 15, 1630, AT 1:146, CSMK 3:23). This means that the concept of intuition operative

in *Rules* must be *broadened*. As early as 1630, Descartes consistently distinguishes between intuition as *knowing* that something is the case and intuition as *conceiving* or *grasping* it (ibid.). On the basis of this distinction, Descartes can assert that the indefinite extent and division of extension can be known to be the case, even if it cannot be conceived or grasped.[8] He is forced to admit that some properties of reality exceed what can be conceived or grasped by the human mind.

(2) The theory of eternal truths provides truths with an ontological foundation they do not have in *Rules*. In *Rules*, Descartes restricts himself to establishing *the properties of the act (intuition) whereby truths are known by the human mind* (see Chapter 3, Section 3.2–3.2.1). The mode of existence of truths—the ontology of truth—is not discussed. In the letters to Mersenne of April 1630, it becomes clear that truths are created by God, and that they remain in existence (i.e., subsist or have duration) because the act whereby God creates or causes them and the act whereby he conserves them are one and the same act.[9] Furthermore, God places these truths in the human mind, and it is their innateness in the mind that disposes the mind to intuit them when it is correctly employed. Every act by which this class of innate dispositions (*habitus*) is appropriately actualized is an act of intuition. Truths known via intuition have a foundation beyond the intellect in God, who creates them and places them in the human mind as innate dispositions.[10]

Readers familiar with existing accounts of Descartes's transition to metaphysics after *Rules* may be surprised that I have not mentioned Descartes's alleged encounter with skepticism in the late 1620s here. On some prominent interpretations, Descartes came to regard intuition as requiring a metaphysical foundation after *Rules*, in the sense that Descartes could no longer rely on intuition as an intrinsically credible cognitive act.[11] Descartes's encounter with skepticism led him to realize that his position in *Rules* remained excessively dogmatic. This interpretation is problematic because there is no indication in Descartes's correspondence immediately after he abandoned *Rules* that skepticism is much on his mind. He does confide to Mersenne that he has "found how to prove metaphysical truths in a manner which is more evident than the proofs of geometry" (April 15, 1630, AT 1:144, CSMK 3:22), which obliquely suggests that mathematical demonstrations must in some sense or under some conditions be dubitable. But this is consistent with Descartes's maintaining that mathematical

demonstrations are only dubitable *when they are no longer objects of an act of intuition*, and this is the position he takes up in *Meditations* V and many related texts (see AT 7:69–70, CSM 2:48; AT 7:58–9, CSM 2:41; AT 7:65, CSM 2:45; AT 7:140–141, CSM 2:100–1; AT 7:144–6, CSM 2:103–4; AT 9A:205–6, CSM 2:271; AT 7:460–2, CSM 2:309–10; AT 3:64, CSMK 3:147; AT 8A:21, CSM 1:207; AT 4:115–16, CSMK 3:233). Descartes's theory of eternal truths does not require that mathematical demonstrations be dubitable when they *are* objects of an act of intuition.

Indeed, the interpretation according to which Descartes regards intuition as requiring a metaphysical foundation after *Rules* rests on the questionable assumption that Descartes came to believe that without God acts of intuition (or clear and distinct perception) are not known to yield truth. On this interpretation, God's role is to establish a *connection* between clear and distinct perception, on the one hand, and truth, on the other—a connection that Descartes merely assumed to be there in *Rules*. However, neither in *Rules* nor in *Meditations* does Descartes ever separate intuition and truth. On the contrary, he consistently maintains that acts of intuition *always yield knowledge of truth when they are being performed* (see citations above). To be sure, any intuition can be doubted *afterwards*, when I entertain the possibility that my nature may be defective (ibid.). However, doubts such as these are second-order acts directed at non-occurrent first order acts.[12] I can doubt *any* intuition, but *not when I am performing it.* When I doubt whether I know that a square has four sides or that two and three make five (as Descartes does in *Meditations* I), I am not counting the sides of a square or adding two and three; I am regarding them *generally in an enumeration$_3$ as members of the class of things that are evidently known by me*. I am not performing *any* of the acts contained in this class. If I *were* performing any of these acts, then not only could I not possibly doubt them, I would also know that the contents of these acts are true. What is new in *Meditations* is not that present acts of intuition require a metaphysical foundation (they do not), but rather the problem of establishing their reliability *after they have been performed*.

Descartes does not address this problem in *Rules*, but he certainly could have formulated it on the basis of his theory of intuition in *Rules*. In both *Rules* and *Meditations*, the problem with intuition or clear and distinct perception is that intuition is an ephemeral act that transpires in time, so that I cannot know that a past intuition of a notion or proposition is true

unless I intuit it. The problem of intuition extends to deduction too: when the premises of a conclusion are no longer present to mind, then the conclusion does not have the character of evidence. In *Meditations*, God is needed so that I do not have to endlessly refresh the experience of evidence in both intuition and deduction. In *Rules*, Descartes requires that the evidence of intuition and deduction be refreshed whenever the need arises. In the case of deduction, this must be done by means of enumeration$_2$, which is meant to reduce deduction to intuition (see Chapter 3, Section 3.4.2). In deduction, the conclusion must be intuited together with *the totality of its logical conditions*, otherwise the conclusion produces neither certainty nor, therefore, evidence. Conclusions severed from the totality of their logical conditions are propositions effectively dead to science. As we have seen in Chapter 3, enumeration$_2$ is the supplemental operation Descartes introduces in *Rules* in order to shore up the limitations of intuition, but this operation suffers from the same limitation as intuition itself: however much it may expand the scope of intuition, it remains irreducibly *finite*. There is a limit here beyond which enumeration$_2$ simply cannot cross. Many deductions are simply too complex to permit reduction to intuition. This means that not every deduction can be reduced to intuition. But *any deduction irreducible to intuition is neither a deduction* nor, therefore, a case of *science* at all. In *Meditations*, God is the ontological foundation of truth as well as of the acts by which truths are known, such that once any truth is intuited, the goodness of the act by which it is intuited can never be reasonably doubted again, since I know that God created me and is not a deceiver.

Thus, when it comes to the intrinsic credibility of present acts of intuition, Descartes doubts their credibility neither in nor after *Rules*. In fact, *there is no skeptical problem about (past) intuitions after* Rules *that Descartes could not have generated entirely on his own on the basis of materials already contained in* Rules *itself*. Descartes did not have to encounter some variety of seventeenth-century Pyrrhonean skepticism in order to generate the doubts about intuition rehearsed in *Meditations*. His radical intuitionism in *Rules* suffices to produce doubt about past intuitions and deductions on its own.

11.4 Method and System after *Rules*

As I argued in Section 11.3, after *Rules*, Descartes needed to establish that reality is reducible to the simple natures. This need, together with the other problems discussed in Section 11.3, motivated his turn to metaphysics in 1629–1630. Thereafter, problems in metaphysics came to have priority over problems in physics in the order of problems that need to be solved in science. Only under these conditions could the unity of science come to have an arborescent form, as Descartes famously describes it in the Preface to the French edition of *Principles*: “Thus the whole of philosophy is like a tree. The roots are metaphysics, the trunk is physics, and the branches emerging from the trunk are all the other sciences […]” (AT 9B:14, CSM 1:186).

Is Descartes’s method in *Rules* equipped to constitute such a system? It may seem that it is not. The method, Garber has argued, is only equipped to solve particular problems. Indeed, according to Garber, Descartes already regarded the sciences as having the systematic unity described in the Preface to the French edition of *Principles* in *Rules* (based on Descartes’s claims about the interconnectedness of the sciences in Rule 1, discussed in Chapter 2, Sections 2.5–2.6 and Section 11.3 above). It is precisely the systematic unity of the sciences, identically understood by Descartes in both *Rules* and *Principles*, that both “makes the method possible and leads to its demise” post-*Rules*. Garber’s reasoning is as follows:

> For if all knowledge is interconnected, then what we should be doing is not solving individual problems, but constructing the complete system of knowledge, the interconnected body of knowledge that starts from intuition and comes to encompass everything capable of being known. Though he may have recognized this implication from the start, in 1619, it will be ten years before he begins such a system, in 1629 with the first metaphysics, unfortunately lost, followed immediately by the composition of the *World*.[13]

In this passage, Garber assumes that the interconnection of the sciences asserted in Rule 1 is identical to the “complete system of knowledge” most clearly articulated in the Preface to the French edition of *Principles*. This assumption, in my view, is false: the interconnection of the sciences asserted in Rule 1 is not terribly clear, and by Rule 12, it becomes clear that it *cannot* constitute a system, since the latter requires a *hierarchical ordering of sciences* that is in principle ruled out in *Rules* due to the fact that the simple natures are *equally* knowable, such that “we should not

regard some branches of our knowledge as more obscure than others" (AT 10:427, CSM 1:50) (see my discussion in Chapter 2, Section 2.5). Thus, the alleged "demise" of the method cannot be due to Descartes's failure to "construct the complete system of knowledge" in *Rules*, for the simple reason that he had not yet defined knowledge as such a system, and could not have done so until *after* Rules.

Nevertheless, even if Garber's interpretation of the unity of science in *Rules* is rejected, his underlying suggestion that Descartes's method in *Rules* is not suited to the constitution of the complete system of knowledge does not lose all of its force. Another separate, but no less important feature of Garber's interpretation of Descartes's method in *Rules* is that the method is only suited to the solution of particular problems, while the constitution of the system of science requires intuiting the principles of science and deducing all other propositions from these principles:

> Descartes's strategy [after *Rules*] is to start, not with individual questions, but to start at the beginning, with the intuitively graspable first principles that ground the rest, and progress step by step from there downward to more particular matters. No longer a mere problem solver, Descartes has become a system builder.
>
> But as a system builder, what role can he find for a method whose goal is the solution of individual problems? With this crucial change in Descartes's conception of scientific activity, a change motivated by the same doctrine of the interconnection of knowledge that motivated his method, the method becomes obsolete; or if not obsolete, at very least it is less central than it once had been.[14]

I will address Garber's claims in the order in which he makes them in the two passages above. First, I do not see in Descartes a rigid distinction between "particular problems," on the one hand, and "system," on the other, since *the system can only be constituted by solving a series of particular problems in the right order*. All problems are "particular," i.e., no one problem is every problem, and so the expression "particular problem(s)" is both redundant and misleading, insofar as it suggests that the construction of a system consists in some activity other than solving particular problems in the right order by means of the method. In this sense, Descartes never really abandoned a "problem-solving" conception of science. What changed was merely his understanding of the ordered relations between problems and, therefore, the ordered relations between the sciences in which these problems are found, with metaphysics having priority over physics. This

new understanding can indeed be traced back to 1629, and it is most explicitly articulated in *Principles* (1644).

Second, given his new understanding of the ordered relations between problems and sciences after *Rules*, what Descartes does after *Rules*—and, ultimately, in *Principles*—is simply invert the order prescribed in Rule 5 and expand it to include the entire system of science.[15] Recall that in Rule 5, he recommends reducing complex problems to their simplest component parts, solving the simplest problem first (via intuition), and then solving the remaining, more complex problems (via deduction based on the initial intuition) (see Chapter 3, Section 3.4.1). After *Rules*, Descartes begins by solving the simplest problems first (via intuition), and then solving the remaining, more complex problems (via deduction based on intuition). He isolates and solves the simplest or most basic problems upon which the solution to all other problems in philosophy depends. Given his realization that problems in metaphysics have priority over problems in physics in the order of problems that need to be solved in science, the most basic problems upon which the solution to all other problems in philosophy depends are problems in metaphysics. But to solve problems in this order is precisely to do what Descartes does in *Principles*. Consequently, it does not seem to me that Descartes's system after *Rules* rendered his method "obsolete." On the contrary, the method is remarkably well-adapted to Descartes's system, since it is constituted by inverting the order prescribed in Rule 5 and employing the principal operations of the method.[16]

But the argument that the method of *Rules* suffers from a "demise" has textual grounds in addition to philosophical grounds. While Descartes's texts and correspondence leading up to 1637 emphasize the centrality of the method, it has been argued that subsequent texts relegate the method to the periphery, perhaps as a mere propaedeutic to science. For example, Descartes does not mention the method all that much in *Meditations* and *Principles*. This too has motivated arguments in the literature that Descartes abandoned the method described in *Rules* after *Rules*.[17]

It does not seem to me that Descartes relegates the method to the periphery after *Rules,* not even on purely textual grounds. First, in *Meditations*, Descartes very clearly underlines in the Preface that he wrote *Meditations* partly in order to demonstrate the use of the method in metaphysics (see AT 7:3, CSM 2:4). This clearly means that he intended the reader to regard *Meditations* as an *application* of the method. Thus, it is

clear that Descartes did not relegate the method to the periphery in *Meditations*, at least not by his own lights. Second, in the Preface to the French edition of *Principles*, not only does Descartes demand that the reader learn the method, he even requires that the reader learn the method in the same order he required in *Rules*, first by "practicing the rules on very easy and simple questions like those of mathematics," and then, once they have "acquired some skill [*habitude*] in finding the truth on these questions," they should "tackle true philosophy in earnest" (AT 9B:13–14, CSM 1:186). As I argued in Chapter 4 and subsequent chapters, in *Rules* Descartes requires that the learner acquire the Cartesian scientific *habitus* by degrees, first by solving problems in elementary mathematics (Chapter 4), then by solving the problem of the limits of knowledge (Chapters 5–7), followed by particular problems in the sciences, both perfectly understood problems and imperfectly understood problems (Chapters 9–10). The difference between *Rules* and *Principles* is that after elementary mathematics, one tackles "true philosophy" in *Principles*, which begins with metaphysics and, as such, replaces the problem of the limits of knowledge in *Rules*. This difference is due to the turn to metaphysics in 1629–1630 and the systematic structure of science it enabled Descartes to see. That the method should be learned by solving problems in the simplest sciences before solving problems in the more advanced sciences, however, is not an innovation in *Principles*; it is already Descartes's considered doctrine in *Rules*, one that he reiterates in the Preface to the French edition of *Principles* (i.e., in 1647, only three years prior to his death). In this respect, at least, there is perfect continuity between *Rules* and *Principles* on the method and its place in science. The difference here lies more in *which complex science* one must learn *after* having acquired the first degree of the Cartesian scientific *habitus* in elementary mathematics. Given Descartes's systematization of science after the turn to metaphysics in 1629–1630, it is no surprise that he replaces the problem of the limits of knowledge with the problem of the first principles of human knowledge, i.e., metaphysics. The latter problem encompasses the former and provides both the ontological and the logical foundations for the system of science, a system whose hierarchical structure Descartes could not have seen in *Rules*, but which he would constitute employing that very same method, as I argued above. There is no suggestion in *Principles* that Descartes does *not* employ the method when constituting the system of science. On the contrary, Descartes

quite explicitly suggests that one must learn the method *in order to* constitute the system.

Two years later, in 1649, Descartes publishes *Passions of the Soul*, where a number of important remarks about the method are made in two anonymous letters published in the Preface.[18] None of these remarks suggest that the method has been relegated to the periphery of his philosophy. The upshot of these letters, which I cannot examine in more detail here, is that the method is sufficient to "finding everything that can be found in physics" and, indeed, "everything the human mind can [find]" in general (AT 9:315, 317, Descartes 1989 10, 12), but that its efficacy in *medicine* specifically requires experiments the cost of which exceeds the expenditure of any one individual and requires state support. These letters clearly indicate that not only the roots and the trunk of philosophy (metaphysics and physics, respectively) are to be constituted by the method, but also its branches (medicine, mechanics, and morals). This hardly suggests that the method has been relegated to the periphery of Descartes's philosophy. On the contrary, it reasserts, vis-à-vis the entire system of philosophy, what Descartes had already written in his definition of the method in Rule 4: the method enables one to "arrive at a true understanding of everything within one's capacity" (AT 10:372, CSM 1:16).

11.5 Simple Natures and Simple/Primitive Notions after *Rules*

In *Principles* I. 47, Descartes introduces the "simple notions" (he already called the simple natures "notions" in *Rules*, see AT 10:417, CSM 1:43) "which are the basic components of our thoughts" in order to "correct the preconceived opinions of our early childhood" (AT 8A:22, CSM 1:208). The enumeration$_3$ provided in the following article reproduces the intellectual, material, and common simple natures found in Rule 12, but not without some important modifications.

First, among the common simple natures (Descartes now refers to them as "the most general items which we regard as things"), Descartes includes, not only *order* and *number*, but also *substance*.[19] The inclusion of substance among the common simple natures determines how Descartes presents the intellectual and material simple natures in *Principles*. He now introduces

them straightaway as substances: “[F]irst, intellectual or thinking things, i.e., those which pertain to mind or thinking substance; and secondly, material things, i.e., those which pertain to extended substance or body” (AT 8A:22, CSM 1:208).

Second, Descartes also adds *another class* of simple nature: the union between mind and body. In *Principles* I. 48, he adds “certain other things which must not be referred [*referri*] either to the mind alone or to the body alone” (AT 8A:23, CSM 1:209). These other things—appetites, emotions or passions, and sensations—“arise […] from the close and intimate union of our mind with the body” (ibid.). While he does not explicitly baptize the simple nature to which these things must be referred here, he does do so in a famous letter to Elizabeth (May 21, 1643). There, he refers to the simple natures of thought and extension as “primitive notions,” and he refers to the union as another such primitive notion, in addition to thought and extension (AT 3:665–6; CSMK 3:218).

Can these revisions in the theory of simple natures be regarded as mere extensions of the list of simple natures, or must they rather be regarded as indications that the criteria of cognitive indivisibility and univocity underlying the theory of simple natures in *Rules* have themselves been modified (see Chapter 7, Section 7.2)?

Regarding substance, Descartes explicitly *excludes* substance from his theory of simple natures in *Rules* (see AT 10:381, CSM 1:21). Why? Because, as I argued in Chapter 8, Section 8.2, the Aristotelian category of substance as a hylomorphic compound of matter and form is ontologically ambiguous between the intellectual simple natures and the material simple natures, and so it cannot be intuited. Descartes’s theory of simple natures in *Rules* excludes the possibility of confusing these two classes for one another. Insofar as the category of substance requires precisely such a confusion, then it must be rejected as unscientific. Furthermore, substance, defined as a hylomorphic compound, is arguably not *simple*, but rather *composite*, not only ontologically, but also epistemically. This violates the criterion of cognitive simplicity.

What changed between *Rules* and subsequent texts and treatises is the turn to metaphysics in 1629–1630. Descartes’s metaphysical turn placed demands on the theory of simple natures that he did not previously have to confront, given his limited ambitions in *Rules*. To demonstrate the immortality of the soul and the existence of God required more resources

than the theory of simple natures in *Rules* could provide. It required constructing a category of substance that (1) does not illicitly confuse the intellectual simple natures and the material simple natures; (2) does not violate the criterion of cognitive indivisibility; (3) can be an object of intuition. These constraints led Descartes to *define substance as identical with its principal attribute* (see *Principles* I.62, AT 8A:30, CSM 1:214–15), which in the case of the soul or the mind is *thought*, a purely intellectual simple nature, and in the case of body is *extension*, a purely material simple nature. In this way, the category of substance ceases to be ontologically ambiguous between the intellectual simple natures and the material simple natures. Furthermore, since the principal attribute of each substance is a simple nature, substance does not violate the criterion of cognitive indivisibility. Finally, substance can be intuited, since the simple nature that is its principal attribute is perfectly intuitable. And so it is that Descartes eventually managed to integrate the category of substance into his theory of simple natures after *Rules*.

Sadly, matters are more complicated than I have suggested. After all, Descartes applied the category of substance, not only to mind and body, but also to God, whose existence he also claims to have demonstrated in the treatise on metaphysics in 1629. In order to apply the category of substance to God, however, Descartes needed to distinguish between *finite* and *infinite* substance. Now, as we know from *Principles* I.51, “the term ‘substance’ does not apply *univocally*, as they say in the Schools, to God and to other things; that is, there is no distinctly intelligible meaning of the term which is common to God and his creatures” (AT 8A:24, CSM 1:210). Thus, on the one hand, substance is a simple nature, but on the other, it is not univocal. This clearly violates the criterion of univocity.

In *Rules*, the theory of simple natures simply excluded any of the problems relating to analogy. In virtue of the univocity criterion, no non-univocal category satisfied the criteria needed for inclusion in any class of simple natures. Not even the common simple natures violated the criterion of univocity, as I have shown in Chapter 7, Section 7.5. The category of substance clearly belongs to this class of simple natures, and it is to this class that Descartes assigns it in *Principles*. Substance is common to both thinking substance and to extended substance, and in both cases it means the same thing: “[T]hings that need only the concurrence of God in order to exist” (*Principles* I. 52, AT 8A:25, CSM 1:210). The problem here is that

there is another, more radical community of substance: the community, not only of thinking substance and extended substance, but also of finite and infinite substance. The distinction between finite and infinite substance is more radical than the distinction between any two finite substances. Here, univocity simply cannot obtain; to regard the category of substance as univocally applied to both finite and infinite substance is to undermine the very possibility of any distinction between finite and infinite substance, at least according to Descartes. And so two competing requirements generated a serious dilemma in Descartes's theory of simple natures post-*Rules*: on the one hand, Descartes needed God and, therefore, not only substance, but also analogy, while on the other hand, his theory of simple natures excluded the category of substance, in part because the latter, due to its analogical application, fails to satisfy the criterion of univocity.

There seem to be only two ways out of this dilemma: either affirm that substance is, in fact, univocal (a possibility Descartes does not pursue) or deny that its being analogical undermines its *cognitive indivisibility* and the possibility of its being *intuited*. Descartes endorses the latter disjunct. How, exactly? Is it not rather the case that when one intuits substance, the content of one's intuition hovers indecisively between finite substance or infinite substance, and that, therefore, substance cannot have a distinct content? What, then, does it mean to intuit substance as a simple nature? What is the content of this intuition?

In *Principles* I. 51, Descartes writes: "By *substance* we can understand nothing other than a thing which exists in such a way as to depend on no other thing for its existence" (AT 8A:24, CSM 1:210). He then adds: "And there is only one substance which can be understood to depend on no other thing whatsoever, namely God" (ibid.). Later, in a letter to Clerselier (April 23, 1649), Descartes writes: "By 'infinite substance' I mean a substance which has actually infinite and immense, real and true perfections. This is not an accident added to the notion of substance, *but the very essence of substance taken absolutely and bounded by no defects*" (AT 5: 355–6, CSMK 3:377; my emphasis). He then adds that "these defects, in respect of substance, are accidents; but infinity or infinitude is not" (ibid.; my emphasis). The notion of substance, "taken absolutely," contains *only infinite substance,* and this is "the very essence of substance." In the same letter, Descartes traces this doctrine back to *Meditations* III. He clarifies what he means in *Meditations* III when he argues that "the notion of the

infinite is in me before that of the finite." The reason, he argues, is that "by the mere fact that I conceive being, or that which is, without thinking whether it is finite or infinite, *what I conceive is infinite being*," whereas "in order to conceive a finite being, *I have to take away something from this general notion of being, which must accordingly be there first*" (AT 5: 356, CSMK 3:377; my emphasis). Once again, being or substance does not have a distinct content prior to the distinction between finite and infinite substance (it would have to have such a content in order to be univocal), but rather only within the distinction. Taken absolutely, "substance," as a common simple nature, contains only infinite substance, or God.

The problem, then, is not that the notion of substance has no distinct content *tout court*, since taken absolutely, this notion contains only infinite substance. This, however, does not mean that substance is a univocal category; the "univocity" or "analogy" of a category depends on whether it has the same content when it is applied to *multiple* entities. When substance, as a common simple nature, is intuited "without thinking whether it is finite or infinite, what I conceive is infinite being," as Descartes puts it in the letter to Clerselier. When, however, it is intuited as the subject of either the principal attribute of (finite) thought or the principal attribute of extension, then something must be "taken away" from "this general notion of being," as he also puts it in the same letter. Thus, substance, as a common simple nature, must be conjoined to another simple nature, *nothingness* or *negation*, in order to be applicable to finite substances. In the language of Descartes's theory of simple natures, this conjunction is contingent, not necessary; otherwise, substance and finite substance would be identical.

How do these considerations establish the cognitive indivisibility and intuitability of substance as a simple nature? The notion of infinite substance is indivisible: it cannot be compounded of any notion of finite substance because any such notion presupposes or depends on the notion of infinite substance, as Descartes argues in both *Meditations* III and the letter to Clerselier cited above. Furthermore, the notion of infinite substance is clearly and distinctly perceivable or intuitable:

> We can also have a clear and distinct idea of uncreated and independent thinking substance, that is of God. Here we must simply avoid supposing that the idea adequately represents everything which is to be found in God; and we must not invent any additional features, but

> concentrate only on what is really contained in the idea and on what we clearly perceive to belong to the nature of a supremely perfect being (AT 8A:26, CSM 1:211).

Substance is, therefore, analogically applied, but as a common simple nature it is entirely *distinct*, and what is entirely distinct can be *intuited*. In this sense, Descartes preserves the theory of simple natures by modifying the criteria of inclusion so that univocity is no longer required. But he *retains cognitive indivisibility*, only he sees that in the case of substance, *cognitive indivisibility no longer uniformly yields univocity*, but does yield *intuitability*.

What about the union between mind and body? Descartes's decision to add of the union among the simple natures raises problems of its own.

First, it seems that the notion of an entity composed of two substances is itself composed of two primitive notions, in which case it is *not* a *primitive* notion. The union, it seems, violates the criterion of cognitive indivisibility, and without this criterion, it is not clear what, if anything, remains of the foundations of Descartes's theory of simple natures in *Rules*. As we have seen above, Descartes's decision to include substance among the simple natures required severing the link between cognitive indivisibility and univocity. To abandon, not only the univocity criterion, but also the criterion of cognitive indivisibility, amounts to abandoning whatever remains of Descartes's theory of simple natures in *Rules*.

Second, in an important letter to Elizabeth, Descartes seems to argue that the union *cannot* be clearly and distinctly understood or intuited according to either of the two faculty configurations whereby the simple natures may be intuited in *Rules*: "[W]hat belongs to the union of the soul and the body is known only obscurely by the intellect alone or even by the intellect aided by the imagination […]" (June 28, 1643, AT 3:691–2, CSMK 3:227). He then adds that the union may be "known very clearly by the senses" (ibid.). Since the senses, regarded independently of the intellect, cannot know anything, Descartes presumably means that the union can be known very clearly by the *intellect aided by the senses*. This is a new faculty configuration, which Descartes does not correlate to any class of simple natures in *Rules*, but which does have important implications. In *Principles* I. 46, Descartes writes that a sensation (e.g., pain) may be very clearly perceived without being distinctly perceived. In the above-cited letter to Elizabeth, he only writes that the union can be very clearly perceived. He does not add that it can be distinctly perceived. Thus, it seems that the

union, which can be very clearly perceived, but perhaps not distinctly perceived, cannot be intuited, since intuition requires distinctness in addition to clarity. In *Rules*, the simple natures can be intuited, but in *Principles* and related texts, the union, which is a simple nature, cannot be intuited. The union, therefore, violates the requirement of intuitability. How, then, can the union be included in Descartes's enumeration$_3$ of simple natures or primitive notions in *Principles* I. 48? Again, it is not clear what remains of the foundations of Descartes's theory of simple natures here.

Third, given these problems, *why* even include the union among the simple natures at all? Would it not have been sufficient to refer the appetites, emotions or passions, and sensations to an entity composed of both intellectual and material simple natures, *without having to regard the notion of this entity as a simple nature*? In *Principles* and related texts, the union appears to be a *gratuitous* addition to Descartes's enumeration$_3$ of simple natures.

Because the solution to the third problem contains the elements required to solve the first two problems, I must solve the third problem first.

What philosophical motivations underpin Descartes's inclusion of the union among the simple natures in *Principles* and related texts? Below, I argue that Descartes's motivations, which are diverse, can nevertheless be boiled down to one: the union cannot be understood according to either the primitive notion whereby the mind is understood (thought), nor the primitive notion whereby body is understood (extension), nor can it be understood according to their combination. The properties of mind and body in the union are different than the properties of the mind and body regarded according to their respective primitive notions.

First, and perhaps most obviously, there is a class of modes—appetites, sensations, and passions—that cannot be referred to either mind alone or to body alone. This is suggested in *Meditations* VI (see AT 7:82–3, CSM 2:56–7) and explicitly asserted in *Principles* I. 48: "But we also experience within ourselves certain other things which must not be referred either to the mind alone or to the body alone. These [modes] arise […] from the close and intimate union of our mind with our body [*arcta et intima mentis nostrae cum corpore unione*]" (AT 8A:23, CSM 1:209). Descartes regards the union between the mind and body as an "intimate" union (see also *Meditations* VI, AT 7:72, CSM 2:50; *Sixth Replies*, AT 7:437, CSM 2:295).

In what does this intimacy consist? As he famously writes in *Meditations* VI:

> Nature also teaches me, by these sensations of pain, hunger, thirst, and so on, that I am not merely present in my body as a sailor is present in a ship, but that I am most closely joined and, as it were, intermingled with it [*arctissime esse conjunctum et quasi permixtum*], so much so that I and the body compose one thing [*adeo ut unum quid cum illo componam*]. If this were not so, I, who am nothing but a thinking thing, would not feel pain when the body was hurt, but would perceive the damage purely by the intellect, just as a sailor perceives by sight if anything in his ship is broken (AT 7:81, CSM 1:56; translation modified).

Thus, while Descartes asserts that the human being is a composite entity (see also *Comments on a Certain Broadsheet*, AT 7B:351, CSM 1:299; AT 7:423–4, CSM 2:286), he distinguishes between two species of unity this composite entity might be regarded as having. One such species, which Descartes decidedly rejects in the above-cited passage, does not require anything more than that sensations, appetites, and emotions or passions be perceived "purely by the intellect, just as a sailor perceives by sight if anything in his ship is broken" (AT 7:81, CSM 1:56). This latter species of unity is one in which the relation between mind and body is the relation between an *operator* and an *instrument*. As Descartes indicates in the previous sentence of the same passage, this species of unity is not consistent with what nature teaches about the *phenomenology* of sensations, appetites, and emotions or passions. These latter are one and all *felt* or *suffered*. Descartes sometimes characterizes the species of unity he asserts the mind and body to have in *Meditations* VI as "substantial unity" or "substantial union" (see *Fourth Replies*, AT 7: 219, CSM 2:155 and AT 7:228; to Regius, January 1642, AT 3:508, CSMK 3:209; to Mesland, February 9, 1645, AT 4:166, CSMK 3:243). However these terms are best understood,[20] it is clear that the union, which Descartes, in language evocative of the theory of simple natures in *Rules*, describes as a *conjunction* in *Meditations* VI, *cannot be understood as the conjunction (necessary or contingent) between the intellectual simple natures and the material simple natures in one entity*, since the conjunction between these simple natures remains at best *equivocal* between the two species of unity discussed above, and at worst *insufficient* to constitute the "intimacy" of the union. Indeed, it seems that the latter is the case, since the intellectual simple natures include only such modes as knowledge, doubt, ignorance, volition (terminating only in a purely intellectual act), etc. (see AT 10:419, CSM 1:44), so that only a

perception of the body "purely by the intellect" is permitted by these modes when they are conjoined to the body via the material simple natures. Thus, Descartes adds the union to the simple natures (1) because appetites, sensations, and emotions or passions cannot be referred either to the mind alone or to the body alone, and (2) the conjunction between the mind and body, to which these modes are referred, is more "intimate" than the conjunction of the intellectual simple natures and the material simple natures in one entity seems to allow. This certainly warrants introducing the union as a distinct simple nature or primitive notion.

Second, the body in the union is *not identical* to the body as understood according to the principal attribute of *extension*. On the contrary, between the human body and the body as understood according to the principal attribute of extension, there is an ontological *equivocity* that must henceforth be distributed between *two notions of body*, neither of which should be confused with the other. In an important letter to Mesland (February 9, 1645), Descartes writes that "this word 'body' is very equivocal [*equivoque*]" (AT 4:166, CSMK 3:242; translation modified). He continues:

> When we speak of a body in general, we mean a determinate part of matter, a part of the quantity of which the universe is composed. In this sense, even if the smallest amount of that quantity were removed, we would judge without more ado that the body was smaller and no longer complete; and if any particle of the matter were changed, we would at once think that the body was no longer quite the same, no longer numerically the same [*idem numero*].
>
> But when we speak of the body of a man, we do not mean a determinate part of matter, or one that has a determinate size; *we mean simply the whole of the matter which is united with the soul of that man*. And so, *even though the matter changes, and its quantity increases or decreases, we still believe that it is the same body, numerically the same body* [*idem numero*], *so long as it remains joined and substantially united with the same soul*; and we think that *this body is whole and entire so long as it has in itself all the dispositions required to preserve that union*. Nobody denies that we have the same bodies as we had in our infancy, although their quantity has much increased and, according to the common opinion of doctors, which is doubtless true, there is no longer in them any part of the matter which then belonged to them, and even though they no longer have the same shape; so that they are numerically the same [*eadem numero*] only because they are informed by the same soul.
>
> Personally, I go further. I have examined the circulation of the blood, and I believe that nutrition takes place by a continual expulsion of parts of our body, which are driven from their place by the arrival of others. Consequently *I do not think that there is any particle of our bodies which remains numerically the same* [*la même* numero] *for a single moment, although our body, qua human body, remains always numerically the same* [*la même* numero] *so long as it is united to the same soul*. In that sense, it can even be called *indivisible*; because if an arm or a leg of a man is amputated, we think that it is only in the first sense of "body" that his

> body is divided – we do not think that a man who has lost an arm or a leg is less a man than any other.
>
> Altogether then, provided that a body is united with the same rational soul, we always take it as the body of the same man, whatever matter it may be and whatever quantity or shape it may have; and we count it as the whole and entire body, provided that it needs no additional matter in order to remain joined to this soul (AT 4:166–7, CSMK 3:242–3; my emphasis throughout).

The basic thesis Descartes develops in this letter is that *none of the properties of body as understood according to the principal attribute of extension alone are properties that constitute the (completeness and numerical identity of the) human body*. A non-human body is understood as a determinate quantity of matter, such that any diminution in this quantity means that the body is "smaller and no longer complete," and indeed such that any variation in this quantity means that it is no longer "numerically the same." A human body, by contrast, *cannot be understood as such a quantity*; however much its quantity may increase or decrease, it remains "numerically the same body," and indeed it remains complete or "whole and entire so long as it has in itself all the dispositions required to preserve that union" between soul and body. Furthermore, unlike the non-human body, changes in the *shape* and *size* of the human body in no way undermine its numerical identity, so long as it remains *dispositionally fit* to preserve the union. It is precisely this *dispositional fitness of the whole* (i.e., the organization and operation of its parts), rather than any determinate quantity of matter or its shape or size, that constitutes the numerical identity of the human body. While this dispositional fitness is material, it is not tied to any determinate quantity of matter, since any such quantity may change in shape or size and may even be replaced (and indeed, is always being replaced in nutrition) without in the least bit undermining the numerical identity of the human body. Regarded according to extension alone, the quantity of matter of which the human body is composed is *never* numerically identical, but regarded according to the union, this very same quantity is *always* numerically identical, so long as it remains dispositionally fit to preserve the union. Indeed, Descartes argues that, unlike other extended bodies, the human body is in this sense *indivisible*. Regarded according to the union, then, the soul *"communicates" the property of indivisibility to the human body*, thereby enabling the body regarded according to the attribute of extension alone to "*lose*" *the property of divisibility* it had prior to the union.

Were the body in the union to be regarded according to the attribute of extension alone, then no body of mine would be the same body, since the latter can undergo *radical modifications* that nevertheless do not undermine its numerical identity. In short, the human body *cannot even be understood* apart from the mind or the soul. This is certainly a very strong reason to regard the union as *primitive*; to regard the union according to the two antecedently intuitable primitive notions of thought and extension would be to regard the human body according to the attribute of extension alone, and that, as we have seen, cannot yield a properly *human* body at all.

Third, not only does the soul "communicate" the property of indivisibility to the human body in the union, but *the body also "communicates" the property of extension to the soul* in the union. As Descartes famously puts it in *Sixth Replies*, the mind is "coextensive [*coextensam*] with the body – the whole mind in the whole body and the whole mind in any one of its parts" (AT 7:442, CSM 2:298). Two years later, in 1643, Descartes writes to Elizabeth:

> Your Highness observes that it is easier to attribute matter and extension to the soul than to attribute to it the capacity to move and be moved by the body without having such matter and extension. I beg her to feel free to attribute this matter and extension to the soul because that is simply to conceive it as united to the body. And once she has formed a proper conception of this and experienced it in herself, it will be easy for her to consider that the matter she has attributed to the thought is not thought itself, and that the extension of this matter is of a different nature from the extension of the thought, because the former has a determinate location, such that it thereby excludes all other bodily extension, which is not the case with the latter (June 28, 1643, AT 3:694–5, CSMK 3:228).

Clearly, as described in this letter, the extension of the mind cannot have the same ontological sense that it has in the case of body regarded according to the attribute of extension alone. The co-extensivity of the mind and the body is not such that the mind has shape, size, motion, or is in any way divisible. Paradoxically, in the union, the mind has extension *without any of the modes of extension.* Indeed, Descartes writes that the *whole* mind is coextensive with the whole body. This means that the extension of the mind is such that it is in some sense *in the same place as the whole body*. *No* two bodies, by contrast, can be co-extensive with one another; every body has a "determinate location, such that it thereby excludes all other bodily extension," as Descartes puts it in the letter. Co-extensivity can *only* obtain between mind and body. Descartes also writes that the *whole mind is coextensive with the parts of the body* (e.g., the whole mind is coextensive

with the hand, or the foot, etc.). This means that the mind is coextensive with every part of the body, not by being divided into as many parts as there are parts of the body, but rather by remaining whole in each such part.[21]

In short, Descartes's holenmerism leads him to introduce the union as a third primitive notion because mind and body cannot be holenmerically understood according to either of the other two primitive notions or their composition. Consequently, mind and body can only be holenmerically understood according to an altogether different primitive notion: namely, the union. In this respect, it is perhaps not unimportant that, chronologically, Descartes introduces holenmerism in *Meditations* VI and *Sixth Replies* (1641), whereas he does not introduce the third primitive notion until 1643. It only gradually dawned on Descartes that his holenmerism required making the union a separate primitive notion.[22]

In sum, Descartes introduces the union as a primitive notion because neither the mind nor the body in the union can be understood according to the other two primitive notions (thought and extension), neither in isolation nor in combination. This means that the notion of the union is indeed simple, and not composite. The union does not, therefore, violate the criterion of cognitive indivisibility. On the contrary, the notion of the union perfectly respects the criterion of cognitive indivisibility.

But can the union be intuited? As we have seen in Chapter 7, while intuition is an intellectual act, it is not the case that all objects can be intuited by the intellect *alone*. The intellectual simple natures must be intuited by the pure intellect alone, but the material simple natures must be intuited by the intellect aided by the imagination, while the common simple natures can be intuited according to either of these faculty configurations. Thus, Descartes's claim that the union cannot be understood *by the intellect alone* does not mean that it cannot be intuited *tout court*, but rather that the pure intellect can only be employed in the intuition of the intellectual simple natures. Similarly, the union cannot be intuited according to the intellect aided by the imagination; while I must indeed have a body in order to imagine anything at all, when I imagine anything at all, including my own body, I imagine it according to the material and common simple natures alone. The body in the union, by contrast, cannot be understood according to the material simple natures. It can only be understood by the intellect aided by the senses: "[W]hat belongs to the union of the soul and the body is known only obscurely by the intellect alone or even by the

intellect aided by the imagination, but it is known very clearly by the senses" (AT 3:691–2, CSMK 3:227). Pre-philosophically, I "have no doubt that the soul moves the body and that the body moves the soul," and indeed I even regard the soul and the body "as a single thing," i.e., I "conceive their union; because to conceive the union between two things is to conceive them as one single thing" (AT 3:692, CSMK 3:227). Descartes's point is not that the pre-philosophical manner in which the union is conceived is dubious, but rather that the *philosophical* manner in which the union is conceived is dubious, precisely because the doubts about the union are based on the inappropriate faculty configurations:

> Metaphysical thoughts, which exercise the pure intellect, help to familiarize us with the notion of the soul; and the study of mathematics, which exercises mainly the imagination in the consideration of shapes and motions, accustoms us to form very distinct notions of body. But it is the ordinary course of life and conversation, and abstention from meditation and from the study of the things which exercise the imagination, that teaches us how to conceive the union of the soul and the body (ibid.).

The addition of the union as a distinct primitive notion no doubt required the addition of a correspondingly distinct faculty configuration whereby it may be "very clearly known," viz., the intellect aided by the senses. But how clearly is "very" clearly, and is the degree of clarity here sufficient to constitute a case of *intuition*? It is certainly notable that, in his letter to Elizabeth, Descartes does not say "clearly," but rather "very clearly." In *Principles* I. 46–7, Descartes defines a clear and distinct perception as a perception that "contains within itself only what is clear" (AT 8A:22, CSM 1:208). Between a clear and distinct perception and a perception that is *entirely* clear there is no difference. The only question, then, is whether by "very" in "very clearly" Descartes means that the union is known by the intellect aided by the senses in such a manner that it contains "only what is clear."

Prima facie, the verdict seems to be negative: a perception can be very clear without being distinct. In *Principles* I. 46, Descartes writes: "[W]hen someone feels an intense pain, the perception he has of it is indeed *very clear, but is not always distinct*" (ibid.; my emphasis). Clarity requires only a sufficient degree of mental presence; it does not require distinguishing what is mentally present from all other things. But note that here, Descartes only writes that the perception is not *always* distinct. This leaves open the possibility that it is *sometimes* distinct. The reason why it is not always

distinct is that it can be combined with a *judgment* about the location of the pain in a part of the body (e.g., in the foot); the pain is regarded as a mode of the body. There is, therefore, nothing about the feeling of pain as such that renders it irreducibly confused. On the contrary, here as elsewhere, the problem is that the judgment refers the pain to the wrong primitive notion. Descartes's suggestion is clearly that, were the pain to have been referred to the *right* primitive notion, the perception would not only be very clear, but *also* distinct. Which primitive notion should the pain be referred to? Descartes provides the answer in *Principles* I. 48, where he writes that appetites, emotions or passions, and sensations "must not be referred either to the mind alone or to the body alone," but rather to "the close and intimate union of our mind with the body" (AT 8A:23, CSM 1:209). Pain is a sensation. When one combines what Descartes says in *Principles* I. 48 with what he says in *Principles* I. 46, it is not at all implausible to read him as arguing that it is possible to perceive sensations, not only very clearly, but also distinctly, provided that the judgment about the pain does not refer it to the body alone or to the mind alone, but rather to the union between the mind and the body. The preconceived opinion that Descartes combats in *Principles* I. 46 does indeed arise in the course of pre-philosophical experience, and to that extent, the experience *is* philosophically dubious. But that is only because the perception under consideration includes the erroneous judgment that the pain is a mode of the body. Descartes does not argue that this judgment is a necessary constituent or even a necessary consequence of the perception of pain.

Analogously, the union can be perceived not only very clearly, but also distinctly. In this case, it simply requires not referring the union to any other primitive notion or combination of primitive notions. To perceive pain very clearly, I need only perceive it at all. To perceive it, not only very clearly, but also distinctly, I must not judge that it is a mode of the body. To perceive the union very clearly, I need only perceive an appetite, an emotion or passion, or a sensation at all, as I do "in the ordinary course of life." To perceive the union, not only very clearly, but also distinctly, I must not judge that it is any of the other primitive notions, nor yet a combination of them. When I do so refrain from judging, the union is entirely distinguished from the other primitive notions, and so it is entirely distinct, since there are no remaining notions with which it might be confused. Thus, it seems to me very plausible that Descartes did not regard the union as recalcitrant to

intuition. The union does not violate Descartes's requirement that the simple natures be intuitable.[23] On the contrary, in a letter to Arnauld, Descartes writes: "That the mind, which is incorporeal, can set the body in motion is something which is shown to us not by any reasoning or comparison with other matters, but by the most certain and evident everyday experience [*certissima et evidentissima experientia quotidie*]" (July 29, 1648, AT 5:222, CSMK 3:358). There are only two species of certain and evident experience: intuition and deduction. Clearly, deduction is not relevant here. That leaves intuition via the intellect aided by the senses.

To conclude. Descartes's addition of substance and the union to his enumeration$_3$ of simple natures in *Principles* and related texts did indeed lead to a substantial revision in the theory of simple natures as developed in *Rules*. Substance satisfies the criterion of cognitive indivisibility, but not univocity. The univocity criterion is, therefore, decisively rejected in favor of cognitive indivisibility alone. As for the union, it too satisfies the criterion of cognitive indivisibility, and it can be clearly and distinctly perceived or intuited by the intellect aided by the senses. Thus, while the theory of simple natures becomes richer, it is not abandoned. It is only "updated" to reflect metaphysical decisions Descartes had not yet undertaken in *Rules*. Furthermore, while these changes concern the criteria behind the theory (cognitive indivisibility and univocity) as well as the content of the theory (the enumeration$_3$ of the simple natures themselves), the *function* of the theory remains the same throughout: to ensure that every object can be known only via those simple natures or primitive notions on which it ultimately depends, so that no object, however composed, exceeds the scope of the certain and evident cognition (*scientia*) that intuition and deduction alone yield.

11.6 Simple Natures and Descartes's Ontology of Substance, Attribute, and Mode

In Chapter 7, Section 7.7, I argued that Descartes's definition and examples of necessary conjunctions between simple natures are ambiguous between what he would later term a rational distinction ("two-way inseparability," in

which a substance cannot be intuited without its attributes and vice versa) and a modal distinction ("one-way inseparability," in which the substance can be intuited without the mode, but not vice versa). Indeed, Descartes's definition of necessary conjunction, regarded entirely on its own, independently of Descartes's examples, corresponds to the rational distinction between a substance and its attributes. As for contingent conjunction, all Descartes writes is that "the union between such things […] is contingent when the relation conjoining them is not an inseparable one" (AT 10:421, CSM 1:46). This is ambiguous between what Descartes would later term a modal distinction ("one-way inseparability") and the real distinction ("two-way separability," in which a substance can be intuited without another substance and vice versa).[24] Ultimately, the distinction between necessary conjunction and contingent conjunction proved too crude to articulate the diversity of relations that obtain between simple natures.

After the turn to metaphysics in 1629–1630, Descartes needed to satisfy two interrelated desiderata. First, he needed a theory of distinctions that could unambiguously articulate the diversity of relations that obtain between simple natures. Second, he needed the internal hierarchical relations that obtain between simple natures to be perspicuously expressible. Recall the distinction drawn in Chapter 7, Section 7.7 between simple natures that are absolutely simple (e.g., extension) and simple natures that are only relatively simple (e.g., motion). The sources of this distinction can be traced back to Rule 6, where Descartes writes: "[W]e should note that there are very few pure and simple natures which we can intuit straight off and *per se* (independently of any others)" (AT 10:383, CSM 1:22). He terms these simple natures "simple in the highest degree" (ibid.). Descartes's distinction between necessary conjunction and contingent conjunction can be productively combined with his distinction between absolutely simple natures and relatively simple natures to yield a rudimentary theory of distinctions, but it is not a theory that he himself canonized in *Rules* (or, for that matter, any text prior to *Principles*), nor is it clear that these two distinctions (necessary conjunction/contingent conjunction, absolutely simple nature/relatively simple nature) could overcome the ambiguities discussed above.

Not until *Principles* did Descartes fully come to terms with these problems. The bipartite distinction in *Rules* between necessary conjunction

and contingent conjunction becomes a tripartite distinction between the real distinction, the modal distinction, and the rational distinction, while the distinction between absolutely simple natures and relatively simple natures becomes the distinction between substance, attribute, and mode. The necessary conjunction in *Rules* between body and extension becomes a rational distinction between a substance and its principal attribute, while the necessary conjunction in *Rules* between extension and motion becomes a modal distinction between a substance and one of its modes. The absolutely simple natures become principal attributes of substance (extension and thought), while the relatively simple natures (e.g., motion and doubt) become modes of these substances. The disambiguation of Descartes's first theory of distinctions in *Rules*—a disambiguation ultimately required by his turn to metaphysics in 1629–1630, whether he knew it at that time or not—leads to his mature theory of distinctions in *Principles*.[25]

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0012

[1] See Bos 2009 and Rabouin 2009, 333. Schuster 2013, 343–4.

[2] See, e.g., Garber 1992, 56–7 and Curley 1978, 38. See also Popkin 2003. Garber regards the encounter with skepticism as *one* reason for Descartes's abandonment of *Rules*. He discusses others. Curley focuses on skepticism exclusively.

[3] See Garber 2001, 48–9.

[4] See Bos 2009, 2001; Rabouin 2009; and de Buzon 2013, 2019.

[5] Gilson (Descartes 1987a, 196–214) compellingly draws out the parallels in considerable detail.

[6] See Marion 1992 and Section 11.5 below.

[7] See Dika and Kambouchner forthcoming and Kambouchner 2019.

[8] See Rabouin 2009, 337–46. Rabouin (ibid.: 343) claims that Descartes did not necessarily have to introduce God in order to establish the reality of the indefinite extent and division of extension. Descartes could have simply appealed to the Aristotelian notion of "potential infinity" (see also Koyré 1957; Curley 1978; Ariew 1987; Nelson and Nolan 2006; Janiak 2015). But it is not clear that this is so. The division of matter is not merely a potential division (i.e., a division that *can* be continued indefinitely), but rather one that has *already been effectuated* and is therefore *already actual*. See Schmaltz 2020.

[9] See Rabouin 2009, 344 and Beyssade 1979, 61–74.

[10] As Rabouin 2009, 344 and Beyssade 1979, 101–12 point out, the theory of eternal truths is really about the ontology of truth, and less about the metaphysical foundations of intuition or clear and distinct perception. The theory of eternal truths responds, not to a problem about the reliability of intuition, but rather to the problem of inconceivable/ungraspable truths and the problem of the mode of existence truths have when they are not intuited by the intellect.

[11] See, e.g., Garber 1992, 56–7 and Curley 1978, 38. See also Popkin 2003.

[12] See Carriero 2009, 337–58.

[13] Garber 2001, 48. See also Garber 1992, 49–50.

[14] See Garber 2001, 48–9.

[15] While I do not believe that Descartes's method is identical to the procedure recommended in Rule 5 (see Chapter 3, Section 3.7 and Chapter 5, Section 5.2), Garber's argument turns on whether Descartes continues to rely on that procedure post-*Rules*, and I think there is a sense in which he does.

[16] A complete demonstration of this thesis would require a systematic interpretation of the role played by the method in *Meditations* and *Principles*, which requires a book in its own right. My purpose in this section is to show that there need be no *obstacle* to pursuing such an interpretation.

[17] See Garber 1992, 48–50.

[18] I suspend judgment about the true author of these letters (see AT 9:301–22, 324–5). For my purposes, it is sufficient that when Descartes pens his response to these letters in his own name, he does not contest any of the claims about the method made by the anonymous author to whom he responds. Furthermore, the claims the anonymous author makes about the method are so reminiscent of Descartes's own characterizations of the method in other texts that either the author is paraphrasing Descartes's texts or the author is Descartes himself. If the author is not Descartes, then Descartes's agreement to publish the letters, together with his lack of disagreement with those parts of the letters in which the role played by the method in his philosophy is described, is surely relevant. If the author is Descartes, then the same is true.

[19] For reasons of space, I will not discuss order and number here.

[20] There is some debate in the scholarly literature about whether Descartes regards the union as a third substance, distinct from both mind and body (in which case Descartes would be, not a dualist, but rather a "trialist"), or whether he regards it as a hylomorphic unity (along Aristotelian lines), or whether he regards it as neither of these. See, e.g., Cottingham 1985; Hoffman 1986; and Rozemond 1998, respectively. See also my discussion in Chapter 8, where I agree with Rozemond that Descartes does not endorse hylomorphism about the union. For my purposes, which are limited to laying out Descartes's motivations for adding the union to the simple natures, I need not enter further into this debate here.

[21] As Marleen Rozemond aptly puts it, the mind "is not in the body by having parts of the soul in parts of the body but by being whole in the parts." See Rozemond 2003, 354.

[22] Note that I am not trying to explain *why* Descartes adopts holenmerism. Nor am I trying to explain whether holenmerism can do the tasks Descartes needs it to do. For a discussion of these issues, see Rozemond 2003. Rozemond compellingly argues that Descartes adopts holenmerism (1) to characterize how the mind can act on the body (the mind can act anywhere in the body because it is "extended" throughout the whole body), and (2) to characterize the union between mind and body (even when a body part is removed, the mind remains whole in the whole body and whole in the parts). I am trying to explain why Descartes introduces the union *as a primitive notion* (which only occurs in 1643, *after* he adopts holenmerism in *Meditations* VI and, more explicitly, *Sixth Replies*). For my purposes, it suffices that Descartes *does* adopt holenmerism, and that his adoption of it yields an ontological sense of extension that cannot be found in the other two primitive notions, and that must, therefore, be understood on the basis of another primitive notion: the union.

[23] For a different interpretation, see Marion 2013, 160–4. Marion argues that sensations can never be distinctly perceived, and that, consequently, the union, which is experienced through sensation,

can never be distinctly, but only clearly perceived. His argument ignores that Descartes only writes in *Principles* I. 46 that sensations are not *always* distinct, not that they are *never* distinct.

24 As I suggested in Chapter 7, Section 7.7, these ambiguities are likely behind Descartes's failure to distinguish between the rational distinction and the modal distinction in *First Replies* (to Caterus), as Descartes himself would later admit in *Principles* I. 62 (AT 8A:30, CSM 1:214–15).

25 Sadly, I cannot reconstruct the entire genealogy of Descartes's theory of distinctions from *Rules* to *Principles* here. The purpose of this section is simply to establish the continuities and discontinuities between Descartes's theory of conjunction/distinctions in *Rules* and *Principles*, and to locate their underlying motivation in Descartes's turn to metaphysics in 1629–30.

Conclusion

Reassessing the Meaning of Method in Descartes

I have argued that Descartes's method is a *habitus* that can be acquired in degrees and efficaciously deployed in solutions to problems in the sciences. The acquisition of the method is a complex, temporally extended process, and the application of the method can differ from one case to another because it is dictated by the parameters of the problem. I have also argued that, so understood, Descartes need not be regarded as having abandoned the method post-*Rules*, and I have done so while remaining maximally sensitive to problems that generate important transformations in Descartes's intellectual development after *Rules*. While it is indeed universal in scope, Descartes's method is not uniform in application. Consequently, variations in the application of the method need not be interpreted as different methods. Each application deploys one or more operations of the method—intuition, deduction, and enumeration—but the manner in which it does so (the number and order of operations) neither can nor need be defined in advance. It depends on the problem.

In its primary sense, science is not a propositional system or "order of reasons." It is rather the *habitus* whereby the propositional order of science is first produced. The concept of science as an *habitus* did not come to an abrupt end in Descartes. Rather, it was transformed. Unlike his predecessors, Descartes does not have an explicit "theory" of *habitus*, nor did he ever intend to write one. Be that as it may, Descartes continued to conceive science as an *habitus*, and rather than develop a theory of *habitus* (scientific or other), in *Rules* he prescribes practices that actually *produce* one. In this respect, Descartes's *Rules* should be understood as a practical manual whose prescriptions, once appropriately executed, form or produce the subject or agent of science. Scholastic theories of *habitus* never aimed

to do that; they do not prescribe acts whose execution actually terminates in any one *habitus*, but rather make *habitus* an object of theory. Descartes's *Rules* belongs more to a literary tradition of *regulae* dating back to fourth- and fifth- century monastic rules (e.g., the *Regula Sancti Augustini*) and well into the seventeenth century (e.g., the *Regulae Societas Jesu*) designed to transform subjects (via prescribed practices and the acquisition of the corresponding virtues or *habitus*) for some definite end rather than to a tradition of theory which has no prescriptive content and does not prescribe acts to anyone.[1] In Descartes's case, the end is not religious, but rather decidedly scientific. Practice in the method enables the human *ingenium* to perfect the natural operations of the mind (intuition, deduction, and enumeration) (Chapter 3–4); determine the limits of its own cognitive capacity (Chapters 5–8); and solve problems in the sciences (Chapters 9–11). Thus, while Descartes is indeed participating in the scholastic debate about scientific *habitus*, the manner in which he does so is not by developing an alternative *theory* of scientific *habitus*, but rather by developing an alternative *cognitive practice* that *produces* a *habitus* of its own. Descartes's underlying intent throughout is to direct the mind so that it may *acquire* the *habitus* needed to solve problems in the sciences.

That the Cartesian scientific *habitus* actually produces concrete results in the sciences, and is not merely an attractive category through which to understand Descartes's method in isolation from the practice of Cartesian science, is also something I have also sought to demonstrate in this book. As I mentioned in the Introduction, Descartes's contemporaries complained that he had not provided enough examples of the method in the *Essays*. It is only a short step from here to the argument that Descartes did not provide enough examples of the method in the *Essays* because he did not use it to arrive at most of the relevant discoveries presented there. The textual structure of Descartes's scientific treatises in the *Essays* must not, however, mislead us into believing that he did not make use of his method in the *Essays*. These treatises obey the order of exposition, and with the exception of *Geometry* and *Meteorology* VIII, they do not always reflect the order of research. Descartes says as much in a letter to Vatier. In those parts of the *Essays* where he does not follow the order of research (including *Dioptrics* II), Descartes underlined that the method "prescribes an order of research which is quite different from the one I thought proper for exposition" (to Vatier, February 22, 1638, AT 1:559, CSMK 3:85). The decisions that

determine the textual presentation of Cartesian science are rhetorical in the (non-pejorative) sense that they are made on the basis of diverse considerations about how best to persuade the intended reader(s) of the treatises and navigate a complex and oftentimes fraught scientific and theologico-political terrain. But the *material* presented in these texts and the *cognitive practice* whereby it is *produced* should be distinguished from the *text* in which this material is presented. Not only do these texts integrate research from diverse periods in Descartes's intellectual development from 1619 to 1637, but by Descartes's own admission they do not by themselves reveal the order of research he followed in making the relevant discoveries. In the case of the law of refraction and the shape of the anaclastic lens, I have shown how one can fruitfully distinguish between the order of research and the order of exposition in Cartesian science, since in this case we have evidence detailing both orders, albeit located in different texts: Rule 8, which describes the order of research, and *Dioptrics* II, which follows the order of exposition. These two levels are in many respects heterogeneous strata in Cartesian science, and to regard them as the same or the latter as indicative of the former would, it seems to me, be a grave error. Another error would be to argue that there is no room for a distinction between method and rhetoric in Descartes,[2] as if Descartes's scientific practice were distinct *both* from the order of research prescribed by the method *and* the order of exposition provided in the texts. The role played by rhetoric in Cartesian science is enormously important, but it is not the case that rhetoric constitutes the meaning of method in Descartes. On the contrary, rhetoric is deployed at the level of the order of exposition in order to present the results of the method.

I will conclude with some very brief remarks about how Descartes's method, as I have interpreted it, may also require reassessing the meaning of method as it has come to be understood by historians and philosophers of science since the late eighteenth century. The idea of scientific method is no longer understood today the way it was understood by Descartes. When contemporary historians and philosophers of science criticize "the" scientific method, they are reacting, not to Descartes, but rather to "theories" of scientific method from d'Alembert to Popper.[3] These theories were principally intended to serve *philosophical* and *historiographical* purposes in definitions and defenses of science as a distinct (and particularly privileged) cognitive practice and in reconstructions of the

history of science as determined by a single, underlying method the repeated application of which runs throughout each "epoch" in the history of science and connects them to one another, effectively constituting the "unity" of this history and, therefore, its underlying sense or meaning. These historians and philosophers believed that the "progress" of science from Descartes to Newton and beyond could only be explained as the byproduct of an especially successful, trans-historical and uniformly applied method. This conception of method informed the historiography of science up to the twentieth century, when it became the object of serious critique by Koyré, Kuhn, and Feyerabend discussed in the Introduction.

However, it is not self-evident that these critiques rest on an historically adequate concept of scientific method(s) any more than the theories of scientific method they criticize. What is striking is that friends and foes of "the" scientific method alike *share a common concept of scientific method*, which one camp defends and the other camp criticizes. The immediate target of Kuhn's criticism is not early modern methodologists such as Descartes, but rather historians of science who explain the history of science as a series of successful applications of "the" scientific method. As I argued above, the "myth of method," i.e., the myth that "scientists discover truth by the quasi-mechanical (and perhaps not very interesting) application of scientific method," is first and foremost a *philosophical and historiographical* myth, and it is not the case that Descartes ever embraced either a "mechanical" (Bachelard) or even a "quasi-mechanical" (Kuhn), let alone a "firm, unchanging, and absolutely binding" (Feyerabend), conception of method.[4] This is precisely the conception of method that Descartes *rejected*. Descartes would have *agreed* with Putnam's thesis that the "hope for a formal method, capable of being isolated from actual human judgments about the content of science (that is, about the nature of the world), and from human values seems to have evaporated."[5] Descartes never entertained the ambitions Putnam describes here. On the contrary, Descartes openly insists that the method cannot be applied *without the seasoned judgment of the operator of the method*. Furthermore, the method contains an ontology (the simple natures), which *defines the nature of the world insofar as it can be known by the human intellect and, therefore, science*. Finally, Descartes never accepted the ideal of "value-free science." Indeed, he would have found such an ideal bizarre. The aim of science, he writes in Rule 1, is to "increase the natural light of […] reason" so that "the

intellect should show […] [the] will what decision it ought to make in each of life's contingencies" (AT 10:361, CSM 1:10). Science *is* a value, and its value is rooted in how it may meet the demands of *human life* in its practical, physiological, and moral being. That science should yield results in mechanics, morals, and medicine is one of Descartes's most enduring normative theses about science (see *Principles*, AT 8A:14, CSM 1:186). In the end, Descartes's conception of science and method is not inductive enough for the likes of Mill, not anti-metaphysical enough for the likes of Carnap, not falsifiable enough for the likes of Popper, and not rigid enough for the likes of Feyerabend. Descartes's method eludes any general theory of scientific method and their associated categories. The heart of science lies in the *subject* of science, in the formation, by means of prescribed practices, of a subject capable of responding to problems in a supple, sensitive way—a subject capable of perfecting their cognitive endowment such that they may finally have a relation to truth. This is a conception of science and method as *virtue* or *habitus*.

Descartes's Method: The Formation of the Subject of Science. Tarek R. Dika, Oxford University Press. © Tarek R. Dika 2023. DOI: 10.1093/oso/9780192869869.003.0013

1 Augustine 1967 and Societas Jesu 1616. Ignatius' *Exercitia spiritualia* also contains many passages similar in structure to Descartes's *Rules*. See Costabel and Marion in Descartes 1977, 86–8. For a history of medieval monasticism(s), see Beach and Cochelin 2020.

2 See, most recently, Schuster 2013, 265–303. See also Schuster and Yeo 1986.

3 See d'Alembert [1750] 1995, 25: "…the single true method of philosophizing as physical scientists consists either in the application of mathematical analysis to experiments, or in observation alone, enlightened by the spirit of method […]." Condorcet 1794, 319: "La découverte des vraies méthodes dans toutes les sciences […] suffisent pour nous répondre qu'aucune d'elles ne peut descender désormais au-dessous du point où elle a été portée."; Comte 1830, 3: "…l'esprit humain, par sa nature, emploie successivement dans chacune de ses recherches trois méthodes de philosopher, dont le caractère est essentiellement différent et même radicalement oppose: d'abord la méthode théologique, ensuite la méthode métaphysique, et enfin la méthode positive."; ibid., viii: "[…] la philosophie positive […] désigne une manière *uniforme* de raisonner applicable à tous les sujets sur lesquels l'esprit humain peut s'exercer" (my emphasis); Whewell [1840] 2014, 1:ix: "The progress of such [physical] science during the last three centuries has given us the means of inquiring, with advantages which former generations did not possess, what that Organ, or intellectual method, is, by which solid truth is to be extracted from the observations of nature."; Whewell ibid., 3, where he defines the principal end of the philosophy of science as "the complete insight into the essence and conditions of all real knowledge, and an exposition of the best methods for the discovery of new truths. […] We are most likely to learn the best methods of discovering truth, by examining how truths, now universally recognized, have really been discovered."; Sarton 1921, 1924, 26; Popper [1935] 2002, 27–9: "Epistemology, or the logic of scientific discovery, should be identified with the

theory of scientific method. […] Thus I shall try to establish the rules, or if you will the norms, by which the scientist is guided when he is engaged in research or discovery […].”

[4] Bachelard 1934, 165–6; Kuhn 1977, 137; Feyerabend 1975, 14. See also Koyré 1956, 15.

[5] Putnam 1981, 192.

APPENDIX

Descartes's *Rules*: Manuscripts, Dates, and Title(s)

THE CIRCULATION AND AVAILABLE MANUSCRIPTS OF *RULES*. After his death in Stockholm in 1650, Descartes's papers and letters were entrusted to his friend Chanut,[1] who included *Rules* in his inventory of Descartes's papers, which he appears to have completed in 1653–1654.[2] Chanut eventually passed these papers on to Clerselier (his brother-in-law),[3] who appears to have shared the original manuscript of *Rules* with Arnauld and Nicole, Baillet, and Poisson (see AT 10:351–2 and 476). To date, there are two available manuscript copies of *Rules* and two printed editions: (1) the Hanover manuscript (a copy obtained by Leibniz in July 1683 from Tschirnhaus's copy of Clerselier's original, also in 1683);[4] (2) a Dutch translation published in 1684 as *Regelen van de Bestieringe des Verstants*,[5] also most likely based on Tschirnhaus's copy;[6] (3) the Latin Amsterdam printed edition (published in Descartes 1701), also most likely based on the Tschirnhaus' copy;[7] and (4) most recently, the Cambridge manuscript (discovered by Richard Serjeanston at Cambridge University Library),[8] which is independent of the Hanover manuscript and the Dutch and Latin editions and may very well have been copied from the original manuscript of *Rules* during a certain period of its composition. (None of the manuscript copies are in Descartes's hand.) The original manuscript of *Rules* appears to have been lost.

PRINCIPAL DIFFERENCES BETWEEN DIFFERENT VERSIONS OF *RULES*. The principal difference between the Amsterdam and Hanover manuscripts is that the part of Rule 4 devoted to *mathesis universalis* appears at the end of the treatise in the Hanover manuscript, whereas it appears in the body of Rule 4 in the Amsterdam manuscript. Of these three manuscripts, the Cambridge manuscript appears to be the earliest. Much of the evidence for this view is based on the fact that the Cambridge manuscript is considerably shorter than either of the other two. On the whole, the Cambridge manuscript contains only about 40 percent of the other two manuscripts, with significantly shorter versions of Rules 1, 3, 4, 6, 7, 8, 12, and 13. The Cambridge manuscript contains no theory of simple natures in Rule 12. Rules 14–16$_{\mathrm{CM}}$ are more or less identical to Rules 14–16 in the other two manuscripts. The Cambridge manuscript ends at Rule 16 (the other manuscripts/editions end at Rule 21). Other evidence that the Cambridge manuscript represents an earlier draft of *Rules* depends on detailed comparisons between the Cambridge manuscript and the Hanover manuscript and Amsterdam edition compellingly interpreted by Richard Serjeanston and Michael Edwards. They argue that the order of composition goes from *Rules*$_{\mathrm{CM}}$ (earliest version) to the Hanover manuscript (intermediate version) to the Amsterdam edition (final version). (The order obtains between the material these documents contain, not the physical documents themselves. On dates, see below.)

Regarding the date of the Cambridge manuscript, the manuscript contains Descartes's discussion of how to deduce the shape of the anaclastic lens in Rule 8, and it is known that Descartes conducted research in dioptrics with Mydorge in Paris in 1626/1627. Mydorge would later write a letter to Mersenne in 1631 about his collaboration with Descartes years earlier in 1626/1627.[9] Thus, Rule

8_{CM} is likely to have been drafted no earlier than 1626/1627.[10] Other parts of the Cambridge manuscript may have been written earlier (although as we will see below, this is unlikely), but probably not later.

DATES OF COMPOSITION OF *RULES*. Due to the fact that none of the available manuscripts of *Rules* indicate when the treatise was composed, there have been many conjectures as to when Descartes composed *Rules*. According to Adam and Tannery, Descartes composed the text around 1628 (see AT 10:486–8). According to Weber, Descartes composed the text over a period of nine years (1619–1628) (Weber 1964, 204–6). According to van de Pitte, most of *Rules* had been written in or by 1628, except for the part of Rule 4 devoted to *mathesis universalis*, which he regards as having been written as late as 1639–1640.[11] Evidence from the Cambridge manuscript suggests that the material contained in it was written by Descartes no later than 1628, and that some of the material contained in the subsequent manuscripts may have been written after Descartes left Paris for the Netherlands in 1628.

Descartes was publicly known to have had a method in his possession as early as March 1626,[12] but whether he had written any of it down in a treatise remains unknown. He appears to have illustrated his method during a famous meeting at the Papal Nuncio in Paris in 1627. Descartes later explicitly referred to his illustration of the method on that day in a letter to Villebressieu in Summer 1631,[13] but again, it is not clear that he had written anything down by that time. In fact, in his letter to Villebressieu, Descartes indicates that as of the meeting in 1627 he had *not* yet begun writing a treatise on method: "You were as convinced as everybody else, and you were all good enough *to beg me to put them in writing and publish them*" (Summer 1631, AT 1:213, CSMK 3:32). Based on this letter, it seems reasonable to assume that Descartes likely began writing *Rules* in or after 1627. This suggests that the Cambridge manuscript arguably represents the earliest phase in the composition of *Rules* in or after 1627 (see above).

There remains the question of when Descartes stopped working on *Rules*. In a famous letter to Mersenne written on April 15, 1630, Descartes writes: "Perhaps you find it strange that I have not persevered with some other treatises I began while I was in Paris" (AT 1:137, CSMK 3:21). If *Rules* is one of these treatises, then this letter suggests that Descartes stopped working on *Rules* after he left Paris in Fall 1628. An interesting consideration is Descartes's algebraic notation in Rule 16. Descartes visited Beeckman in October 1628, and Beeckman recorded "A Certain Specimen of Descartes's Algebra." Beeckman copied it in cossic notation. There are two possibilities here: either Beeckman preferred the more standard cossic notation to Descartes's new notation, or Descartes himself employed cossic notation in the document, either because he himself was still using it at the time or because he felt that Beeckman would not understand his (Descartes's) new notation. If the latter is the case, it could mean that Rule 16, which contains Descartes's new notation in all known manuscripts and editions of *Rules*, had not yet been written, and that, therefore, Descartes continued working on *Rules* after his visit to Beeckman in October 1628. There is no way to decide the matter, as we do not know whether the notation in the document is due to Beeckman or Descartes.

The strongest evidence that Descartes may have continued working on parts of *Rules* until late 1629 is the appearance of metaphysical propositions in Rule 3 and Rule 12 in the Hanover manuscript and the Dutch and Latin editions. In Rule 3, Descartes writes that "everyone can mentally intuit that he exists, that he is thinking," (AT 10:368, CSM 1:14), and in Rule 12, he writes that the propositions, "I am, therefore God exists" and "I understand, therefore I have a mind distinct from body" are necessary propositions (AT 10:422, CSM 1:46). These propositions are not contained in the Cambridge manuscript. When might Descartes have added them?

In a letter to Mersenne written in Friesland in April 1630, Descartes writes that "all those to whom God has given the use of […] reason have an obligation to employ it principally in the endeavor to know him and to know themselves. […] *During my first nine months in this country I*

worked on nothing else" (letter to Mersenne, April 15, 1630, AT 1:144, CSM 1:22; AT 1:144, CSMK 3:22; my emphasis). Descartes's first nine months in the Netherlands were spent in Friesland in March 1629–November 1629.[14] Seven months later, in another letter to Mersenne, Descartes, now in Leiden, explicitly indicates that he began working on a metaphysical treatise in Friesland: "Perhaps I may someday complete a little treatise on Metaphysics, which I began when in Friesland, in which I set out to principally prove *the existence of God and of our souls* when they are separate from the body, from which their immortality follows" (letter to Mersenne, November 25, 1630, AT 1:182, CSMK 3:29). Descartes had earlier written to Gibieuf about the treatise, describing it as a "little treatise *I am starting*" (letter to Gibieuf, July 18, 1629, AT 1:17, CSM 1:5; my emphasis). Indeed, Descartes had expressed a desire to write something on the subject of "divinity" as early as Summer 1628 (see Baillet 1691, 1:170–1). In the letter to Mersenne (April 15, 1630, cited above), Descartes wrote that "I think that you heard me speak once before of my plan to write something on the topic," presumably while he (Descartes) was still in Paris. He further intimates that "I would not have been able to discover the foundations of physics if I had not looked for them along that road" and claims to have in hand a way to "prove metaphysical truths in a manner which is more evident than the proofs of geometry" (ibid.).

There appears to be some overlap between the contents of the little treatise on metaphysics in 1629 and the metaphysical propositions in Rule 3 and 12 mentioned above. Since these propositions do not appear in the Cambridge manuscript, Descartes may have inserted these propositions in 1629 as he began working on metaphysics. However, one must be careful here. In *Rules*, Descartes nowhere states that metaphysical truths can be proven in a manner which is more evident that the proofs of geometry, whereas he does seem to have stated this in the little metaphysical treatise begun in 1629. To be sure, Descartes does mention in Rule 3 that "what has been revealed by God is more certain than any knowledge," but this only refers to the revealed truths themselves, not to their demonstration by natural reason. When Descartes adds that they may be demonstrated by natural reason, he does not expressly indicate that the manner in which they may be demonstrated is superior to the proofs of geometry, and when he discusses the proposition in Rule 12 that "I am, therefore God exists," he does not state or even suggest that it is more evident than any other truth.

All other potential evidence suggested by comparisons between the Cambridge manuscript and other manuscripts and editions that Descartes continued working on *Rules* after 1629 does not seem to *require* dating any of the material in any of the available manuscripts and editions beyond 1629. For example, the appearance of *mathesis universalis* in the Amsterdam edition and the Hanover manuscript does not require pushing the date of composition to the 1640s, contrary to what van de Pitte has claimed.[15] Van de Pitte's argument rests on the fact that Descartes only employs the term *mathesis* and its variations in the 1640s in *Meditations*, *Principles*, and select correspondence. However, none of these uses are uses of *mathesis universalis*; *mathesis universalis* is distinct from the uses Descartes makes of the term *mathesis* in these texts.[16] As for how Descartes came across the term *mathesis universalis*, it is quite possible that he encountered it in Hardy's translation of Euclid's *Data*,[17] cited by Mersenne in *La Vérité des Sciences*.[18] Thus, the appearance of *mathesis universalis* in the Amsterdam edition and the Hanover manuscript does not require dating the part of Rule 4 devoted to *mathesis universalis* beyond the 1620s. In conclusion, it seems that the Cambridge manuscript likely contains material that dates to 1627 at the earliest, while the Amsterdam edition and Hanover manuscripts contain material that dates to 1629 at the latest. In other words, Descartes appears to have worked on *Rules* for a period of no more than three years and to have definitively abandoned it in 1629.

DIVERSE TITLES OF *RULES*. *Rules* has appeared under many titles and descriptions. Chanut's Stockholm inventory has the title as *Règles utiles & claires pour la direction de l'esprit en la recherche de la vérité* (cited in AT 10:351). A Latin translation of the Stockholm inventory by Pierre Borel in 1656 has the title *Regulis utilibus & claris ad ingenij directionem in veritatis inquisitione*

(ibid.). The Dutch translation published in 1684 bears the title *Regelen van de Bestieringe des Verstants* (the letter to the reader has the title as *Regelen om't Verstant in't Onderzoek der Waarheit te bestieren*).[19] The Amsterdam Latin edition in 1701 bears the title *Regulae ad directionem ingenii, ut et inquisitio veritatis per lumen naturale*.[20] The Hanover manuscript bears the title *Renati Cartesii de inquirenda veritate*,[21] and the Cambridge manuscript has no title at all. Other possible titles and/or descriptions include those enumerated by Leibniz in various loci: "…un discours…de la recherche de la vérité" (cited in AT 10:208); *Methodus inquirendae veritatis* (cited in AT 10:354); and *Regulae veritatis inquirendae* (cited in AT 10:355). Other titles can be found in Baillet, including: *Règles pour la direction de l'Esprit dans la recherche de la Vérité*;[22] *Règles pour conduire notre esprit dans la recherche de la Vérité*;[23] *…la direction de l'esprit pour rechercher la Vérité* (Baillet 1691, 2:487); and *Règles pour la direction de l'esprit*.[24] I will not speculate on the original title, but only insist that none of these titles betrays Descartes's intention in *Rules*. Based on convergences between the titles, Costabel and Marion have opted for *Règles pour la direction de l'esprit en la recherche de la vérité*.[25]

[1] See Baillet 1691, 2:428.

[2] See the detailed discussion in Bos et al. in Descartes 2002, xvi–xxi.

[3] See Baillet 1691, 2:428.

[4] See Breger 1983 and Bos 1999. The Hanover manuscript (MS IV 308, Gottfried Wilhelm Leibniz Bibliothek, Hannover) was discovered in the 1850s by Louis-Alexandre Foucher de Careil and published in his edition of Descartes's *Oeuvres inédites*. See Descartes 1859, iv–v.

[5] See Descartes 1684 and Crapulli's critical edition in Descartes 1966.

[6] See Breger 1983 and Bos 1999.

[7] Ibid.

[8] See Descartes 1626/1627?

[9] Mydorge's letter to Mersenne can be found in Mersenne 1932–1988, 1:404–15.

[10] See Schuster 2013, 388–9, n. 73.

[11] See van de Pitte 1991.

[12] See Cornier's letter to Mersenne in Mersenne 1932–1988, 1:429.

[13] See AT 1:213–17, CSMK 3:32–3 and, on the date of the debate at the Papal Nuncio, Rodis-Lewis 1998, 67.

[14] See Beeckman 1939–1953, 3:112, 114. Beeckman notes that Descartes briefly visited him in Dordrecht on his way to Amsterdam and then Franeker, where he enrolled as a student at the University of Franeker.

[15] See van de Pitte 1991.

[16] See Rabouin 2009, 251–69; de Buzon 2013; and my discussion in Chapter 4, Section 4.7.

[17] See Hardy 1625.

[18] Mersenne 1625, 749 (on this possible source and other important details, see Rabouin 2009, 244–9).

[19] Descartes 1684.

[20] Descartes 1701.
[21] Cited in Descartes 1859, v.
[22] Baillet 1691, 1:282.
[23] Baillet 1691, 2:404.
[24] Baillet 1691, 2:471.
[25] Descartes 1977.

Bibliography

Adams, Marilyn McCord. 1976. "Ockham on Identity and Distinction." *Franciscan Studies* 36: 5–74.

Adams, Marilyn McCord. 1987. *William Ockham*. 2 vols. Notre Dame: University of Notre Dame Press.

Adams, Marilyn McCord. 2008. "The Metaphysics of the Trinity in Some Fourteenth Century Franciscans." *Franciscan Studies* 66: 101–68.

Adriaenssen, Han Thomas. 2017. *Representation and Scepticism from Aquinas to Descartes*. Cambridge: Cambridge University Press.

Aertsen, Jan. 2012. *Medieval Philosophy as Transcendental Thought: From Philip the Chancellor (*ca. *1225) to Francisco Suárez*. Leiden: Brill.

Agostini, Igor. 2020. "*Ingenium* between Descartes and the Scholastics." In *Descartes and the* Ingenium*: The Embodied Soul in Cartesianism*, edited by Raphaële Garrod and Alexander Marr, 139–62. Leiden: Brill.

Alanen, Lilli. 1999. "Intuition, Assent and Necessity: The Question of Descartes's Psychologism." In *Acta Philosophica Fennica: Norms and Modes of Thinking in Descartes*, edited by Tuomo Aho and Mikko Yrjönsuuri, 99–125. Helsinki: Philosophical Society of Finland.

Alanen, Lilli. 2003. *Descartes's Concept of Mind*. Cambridge: Harvard University Press.

Alanen, Lilli and Mikko Yrjönsuuri. 1997. "Intuition, jugement et évidence chez Ockham et Descartes." In *Descartes et le Moyen Âge*, edited by Joël Biard and Rashed Roshdi, 155–75. Paris: Vrin.

Albert the Great. 1890–1899. *Opera omnia*. 38 vols., ed. Borgnet E. Vives. Paris.

Alexander of Hales. 1622. *Universae theologiae summa*. 4 vols. Cologne: Ioannis Gymnici.

Allard, Jean-Louis. 1963. *Le mathématisme de Descartes*. Ottawa: Éditions de l'Université d'Ottawa.

Alquié, Ferdinand. 1950. *La découverte métaphysique de l'homme chez Descartes*. Paris: Presses Universitaires de France.

Alsted, Johann Heinrich. 1613. *Methodus admirandorum mathematicorum complectens novem libros matheseos universae*. Herbron: G. Corvini.

Aquinas, Thomas. 1882–. *Opera omnia iussu Leonis XIII P.M. edita, cura et studio fratrum ordinis praedicatorum*. 50 vols. Rome: Ex Typographia Polyglotta S. C. de Propaganda Fide.

Aquinas, Thomas. 1929–1956. *Scriptum super libros Sententiarum Magistri Petri Lombardi Episcopi Parisiensis.* edited by R. P. Mandonnet. Paris: Lethielleux.

Aquinas, Thomas. 1945. *Basic Writings of Saint Thomas Aquinas*, edited by Anton C. Pegis. 2 vols. Indianapolis: Hackett.

Aquinas, Thomas. 1965. *Expositio super librum Boethii De trinitate*, edited by Bruno Decker. Leiden: E. J. Brill.

Aquinas, Thomas. 1970. *Commentary on the* Posterior Analytics *of Aristotle*. Translated by Fr. Larcher, O. P. Albany: Magi Books, Inc.

Aquinas, Thomas. 1986. *The Division and Methods of the Sciences: Questions V and VI of His Commentary on the* De Trinitate *of Boethius*, edited by Armand Maurer. Toronto: Pontifical Institute of Medieval Studies.

Aquinas, Thomas. 1998. *Selected Writings*, edited by Ralph McInerny. New York: Penguin.

Aquinas, Thomas. 2002. *The Treatise on Human Nature.* Summa Theologiae *1a 75–89*. Translated by Robert Pasnau. Indianapolis: Hackett Publishing Company.

Aquinas, Thomas. 2007. *Summa Theologica Part III, First Section*. Translated by Fathers of the English Dominican Province. New York: Cosimo Classics.

Araujo, Marcelo de. 2003. *Scepticism, Freedom and Autonomy: A Study of the Moral Foundations of Descartes's Theory of Knowledge*. Berlin: De Gruyter.

Arbini, R. 1983. "Did Descartes Have a Philosophical Theory of Sense Perception?"*Journal of the History of Philosophy* 21 (3): 317.

Ariew, Roger. 1987. "The Infinite in Descartes's Conversation with Burman."*Archiv für Geschichte der Philosophie* 69 (2): 140–63.

Ariew, Roger. 1990. "Christopher Clavius and the Classification of Sciences." *Synthese* 83 (2): 293–300.

Ariew, Roger. 2012. "Descartes and Leibniz as Readers of Suárez: Theory of Distinctions and Principle of Individuation." In *The Philosophy of Francisco Suárez*, edited by Benjamin Hill and Henrik Lagerlund, 28–54. Oxford: Oxford University Press.

Ariew, Roger. 2014. *Descartes and the First Cartesians*. New York: Oxford.

Ariew, Roger. 2016. "The Mathematization of Nature in Descartes and the First Cartesians." In *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*, edited by Geoffrey Gorham et al., 112–33. Minneapolis: University of Minnesota Press.

Ariew, Roger. 2020. "Descartes and Logic: Perfecting the *Ingenium*." In *Descartes and the* Ingenium*: The Embodied Soul in Cartesianism*, edited by Raphaële Garrod and Alexander Marr, 31–46. Leiden: Brill.

Ariew, Roger and Marjorie Grene. 1995. "Ideas, in and before Descartes."*Journal of the History of Ideas* 56 (1): 87–106.

Aristotle. 1984. *The Complete Works of Aristotle*, edited by Jonathan Barnes. 2 vols. Princeton: Princeton University Press.

Armogathe, Jean Robert and Jean-Luc Marion. 1976. *Index des* Regulae ad directionem ingenii *de René Descartes, avec des listes de leçons et conjectures établies par G. Crapulli*. Roma: Ateneo.

Arnauld, Antoine and Pierre Nicole. 1664. *La logique ou l'art de penser*. 2nd ed. Paris: C. Savreux.

Arndt, Hans W. 1971. *Methodo scientifica Pertractatum: Mos Geometricus und Kalkülbegriff in der Philosophischen Theorienbildung des 17. und 18. Jahrhunderts*. Berlin: de Gruyter.

Arthur, Richard. 2007. "Beeckman, Descartes, and the Force of Motion." *Journal of the History of Philosophy* 45 (1): 1–28.

Augustine. 1844–1864. *Patrologia Latina*, edited by J. P. Migne. Vol. 32–47. Paris.

Augustine. 1967. *La Règle de Saint Augustin*, edited by L. Verheijen. 2 vols. Études Augustiniennes: Paris.

Augustine. 1968. *De trinitate*, edited by W. J. Mountain. 2 vols. Thurnholti: Brepols.

Augustine. 2002. *On the Trinity. Books 8–15*, edited by Gareth B. Matthews and Stephen McKenna. Cambridge: Cambridge University Press.

Averbach, Bonnieand and Orin Chein. 2000. *Problem Solving through Recreational Mathematics*. Mineola: Dover Publications.

Averröes (ibn Rushd). 1550–1552. *Aristotelis Stagiritae libri omnes […] cum Averrois Cordubensis variis in eosdem commentariis*. 11 vols. Venice: Juntas.

Ayers, Michael. 1998. “Ideas and Objective Being.” In *Cambridge History of Seventeenth-Century Philosophy*, edited by Michael Ayers and Daniel Garber. Vol. 2, 1062–1107. Cambridge: Cambridge University Press.

Azouvi, F. and D. Kambouchner, eds. 1991. *Le sensible: transformations du sens commun d’Aristote à Reid*. *Revue de Métaphysique et de Morale* 96 (4).

Bachelard, Gaston. 1934. *Le nouvel esprit scientifique*. Paris: Presses Universitaires de France.

Bachelard, Gaston. 1972. *La formation de l’esprit scientifique*. Paris: Vrin.

Baillet, Adrien. 1691. *La vie de Monsieur Descartes*. 2 vols. Paris.

Baker, Gordon and Katherine J. Morris. 1996. *Descartes’ Dualism*. London: Routledge.

Beach, Alison I. and Isabelle Cochelin. 2020. *The Cambridge History of Medieval Monasticism in the Latin West*. 2 vols. Cambridge: Cambridge University Press.

Beck, John Leslie. 1952. *The Method of Descartes: A Study of the* Regulae. Oxford: Clarendon Press.

Beck, John Leslie. 1965. *The Metaphysics of Descartes: A Study of the* Meditations. Oxford: Clarendon Press.

Beeckman, Isaac. 1939–1953. *Journal tenu par Isaac Beeckman de 1604 à 1634*, edited by C. de Waard. 4 vols. The Hague: Martinus Nijhoff.

Belgioioso, Giulia. 2009. “The Hyperbolic Way to the Truth from Balzac to Descartes: ‘Toute hyperbole tend là, de nous amener à la vérité par l’excès de la vérité, c’est-à-dire par la mensonge.” In *Skepticism in the Modern Age: Building on the Work of Richard Popkin*, edited by José R. Maia Neto, Gianni Paganini, and John Christian Laursen, 269–97. Leiden: Brill.

Berger, Gaston. 1972. *The Cogito in Husserl’s Philosophy*. Evanston: Northwestern University Press.

Bermon, Pascale. 2018. “‘*Tot Scibilia Quot Scientiae*?’ Are there as Many Sciences as Objects of Science? The Format of Scientific Habits from Thomas Aquinas to Gregory of Rimini.” In *The Ontology, Psychology and Axiology of Habits (*Habitus*) in Medieval Philosophy*, edited by Nicholas Faucher and Magali Roques, 301–21. Cham: Springer.

Beyssade, Jean-Marie. 1979. *La philosophie première de Descartes*. Paris: Flammarion.

Beyssade, Jean-Marie. 1997. “‘L’étendue n’est pas le corps’ (*Regulae* XIV, AT X, 444, l. 18). Genres d’être et façons de parler dans les *Regulae*.” *Laval théologique et philosophique* 53 (3): 755–66.

Biard, Joël. 1997. “La conception cartésienne de l’étendue et les débats médiévaux sur la quantité.” In *Descartes et le Moyen Âge*, edited by Joël Biard and Roshdi Rashed, 349–62. Paris: Vrin.

Biard, Joël. 2011. “Intuitive and Abstractive Cognition.” In *Encyclopedia of Medieval Philosophy: Philosophy between 500 and 1500*, edited by Henrik Lagerlund, 568–71. Dordrecht: Springer.

Blander, Josh. 2020. “Same as It Never Was: John Duns Scotus’s Paris *Reportatio* Account of Identity and Distinction.” *British Journal for the History of Philosophy* 28 (2): 231–50.

Bos, Erik-Jan. 1999. “La première publication de *La Recherche de la Vérité* en 1684: Onderzoek der Waarheit door’t Naturelijk Licht.” *Nouvelles de la République des Lettres* 1: 13–26.

Bos, H. J. M. 2001. *Redefining Geometrical Exactness: Descartes’s Transformation of the Early Modern Concept of Construction*. New York: Springer.

Bos, H. J. M. 2009. “Descartes’s Attempt, in the *Regulae*, to Base the Certainty of Algebra on Mental Vision—a Conjectural Reconstruction.” In *Logic, Methodology and Philosophy of Science: Proceedings of the Thirteenth International Conference*, edited by Glymour et al., 4–23. London: King’s College Publications.

Boutroux, Pierre. 1900. *L’imagination et les mathématiques selon Descartes*. Paris: F. Alcan.

Boyce-Gibson, Alexander. 1896. “La ‘Géométrie’ de Descartes au point de vue de sa méthode.” *Revue de Métaphysique et de Morale* 4 (4): 386–98.

Boyce-Gibson, Alexander. 1898a. “The *Regulae* of Descartes I.” *Mind* 7 (26): 145–58.

Boyce-Gibson, Alexander. 1898b. “The *Regulae* of Descartes II.” *Mind* 7 (27): 332–63.

Boyce-Gibson, Alexander. [1932] 1967. *The Philosophy of Descartes*. New York: Russell & Russell.

Breger, Herbert. 1983. "Über die Hannoversche Handschrift der Descartesschen *Regulae*." *Studia Leibnitiana* 15 (1): 108–14.

Broughton, Janet. 2002. *Descartes's Method of Doubt*. Princeton: Princeton University Press.

Brower-Toland, Susan. 2012. "Medieval Approaches to Consciousness: Ockham and Chatton." *Philosophers' Imprint* 12 (17): 1–29.

Bruyère, Nelly. 1984. *Méthode et dialectique dans l'œuvre de la Ramée. Renaissance et âge classique*. Paris: Vrin.

Buchdahl, Gerd. 1969. *Metaphysics and the Philosophy of Science*. Cambridge: MIT Press.

Buchwald, Jed Z. 2008. "Descartes's Experimental Journey Past the Prism and Through the Invisible World to the Rainbow." *Annals of Science* 65 (1): 1–46.

Budnik, Clarisse. 2018. "Plaisir et récréations mathématiques en France au XVIIIe siècle." *Hypothèses: Travaux de l'école doctorale d'histoire* 21 (1): 57–67.

Cajori, Florian. 1993. *A History of Mathematical Notations*. 2 vols. New York: Dover Publications.

Capreolus, Johannes. [1409–1432] 1899–1908. *Defensiones theologiae divi Thomae Aquinatis*, edited by C. Paban, T. Pègues. Turin: Cattier.

Carleton, Thomas Compton. 1649. *Philosophica universa*. Antwerp: J. Meursius.

Carraud, Vincent. 2002. *Causa sive ratio: la raison de la cause de Suárez à Leibniz*. Paris: Presses Universitaires de France.

Carriero, John. 2009. *Between Two Worlds: A Reading of Descartes's* Meditations. Princeton: Princeton University Press.

Cassan, Elodie. 2015. *Les chemins cartésiens du jugement*. Paris: Honoré Champion.

Cassirer, Ernst. 1902. *Leibniz' System in seinen wissenschaftlichen Grundlagen*. Marburg: Ekwert.

Cassirer, Ernst. 1937. "Descartes et l'idée de l'unité de la science." *Revue de synthèse* 14 (7): 7–28.

Cassirer, Ernst. 1942. *Zur Logik der Kulturwissenschaften: Fünf Studien*. Göteborg: Wettergren & Krebers Förlag.

Chabaud, Gilles. 1994. "Sciences en jeux: Les récréations mathématiques et physiques en France du XVIIe au XVIIIe siècle." Ph.D. Dissertation. EHESS.

Charrak, André. 2005. "La critique du syllogisme dans Bacon et Descartes." *Les Études Philosophiques* 75 (4): 469–84.

Chenu, M. D. 1957. *La théologie au XIIe siècle*. Paris: Vrin.

Chenu, M. D. 1967. *La théologie comme science au XIIIe siècle*. Paris: Vrin.

Chomsky, Noam. 1976. "Problems and Mysteries in the Study of Human Language." In *Language in Focus: Foundations, Methods and Systems*, edited by Asa Kasher, 281–359. Dordrecht: Reidel.

Cicero, Marcus Tullius. 1942. *De oratore*, edited by E. W. Sutton and H. Rackham. 2 vols. Cambridge: Harvard University Press.

Cicero, Marcus Tullius. 1949. *De inventione*, edited by H. M. Hubbell. Cambridge: Harvard University Press.

Clarke, Desmond. 1982. *Descartes' Philosophy of Science*. Manchester: Manchester University Press.

Clarke, Desmond. 2003. *Descartes's Theory of Mind*. Oxford: Oxford University Press.

Clarke, Desmond. 2006. *Descartes: A Biography*. Cambridge: Cambridge University Press.

Clauberg, Johann. 1652. *Defensio cartesiana*. Amsterdam: Elzevir.

Clavius, Christopher. 1611–1612. *Opera omnia*. 5 vols. Mainz.

Comte, Auguste. 1830. *Cours de philosophie positive I*. Paris: Bachelier.

Condorcet, Jean-Antoine-Nicolas de Caritat. 1794. *Esquisse d'un tableau historique des progrès de l'esprit humain*. Paris: Agasse.

Conimbricenses. 1593. *In Libros Aristotelis, qui Parva Naturalia apellantur*. S. Lopes: Lisboa.

Conimbricenses. 1597. *In duos libros De Generatione et Corruptione Aristotelis Stagiritae*. Coimbra: A. Mariz.

Conimbricenses. [1598] 1604. *In tres libros De anima Aristotelis Stagaritae*. Cologne: L. Zetzneri.

Conimbricenses. [1606] 1611. *Commentarii in universam dialecticam Aristotelis*. Cologne: B. Gualtherium.

Cook, Monte. 1987. "Descartes' Alleged Representationalism." *History of Philosophy Quarterly* 4 (2): 179–95.

Costa, Michael J. 1983. "What Cartesian Ideas are Not." *Journal of the History of Philosophy* 21 (4): 537–49.

Costabel, Pierre. [1982] 2013. *Démarches originales de Descartes savant*. Paris: Vrin.

Cottingham, John. 1985. "Cartesian Trialism." *Mind* 94 (374): 218–30.

Cottingham, John. 1998. "Descartes's Treatment of Animals." In *Descartes*, edited by John Cottingham, 225–33. Oxford: Oxford University Press.

Cottingham, John. 2017. "Context, History, and Interpretation: The Religious Dimension in Descartes's Metaphysics." In *Descartes and Cartesianism: Essays in Honour of Desmond Clarke*, edited by Stephen Gaukroger and Catherine Wilson, 42–57. Oxford: Oxford University Press.

Courtine, Jean-François. 1990. *Suárez et le système de la métaphysique*. Paris: Presses Universitaires de France.

Crapulli, Giovanni. 1969. *Mathesis universalis: genesi di una* idea *nel XVI secolo*. Rome: Ateneo.

Cronin, Timothy J. 1966. *Objective Being in Descartes and in Suarez*. Rome: Gregorian University Press.

Cross, Richard. 1999. *Duns Scotus*. Oxford: Oxford University Press.

Cross, Richard. 2004. "Scotus's Parisian Teaching on Divine Simplicity." In *Duns Scot à Paris (1302–2002)*, edited by O. Boulnois et al., 519–62. Turnhaut: Brepols.

Cross, Richard. 2014. *Duns Scotus's Theory of Cognition*. Oxford: Oxford University Press.

Curd, Patricia and Richard D. McKirahan. 2011. *A Presocratics Reader: Selected Fragments and Testimonia*. Indianapolis: Hackett.

Curley, E.M. 1978. *Descartes Against the Skeptics*. Cambridge: Harvard University Press.

d'Alembert, Jean. [1750] 1995. *Preliminary Discourse to the Encyclopedia of Diderot*. Translated by Richard N. Schwab. Chicago: University of Chicago Press.

Dambska, I. 1957. "Sur certains principes méthodologiques dans les *Principia philosophiæ* de Descartes." *Revue de Métaphysique et de Morale* 62 (1): 57–66.

Damerow, Peter, et al. 2004. *Exploring the Limits of Preclassical Mechanics. A Study of Conceptual Development in Early Modern Science: Free Fall and Compounded Motion in the Work of Descartes, Galileo, and Beeckman*. New York: Springer.

Davenport, Anne Ashley. 2006. *Descartes's Theory of Action*. Leiden: Brill.

de Buzon, Frédéric. 2013. *La science cartésienne et son objet: Mathesis et phénomène*. Paris: Honoré Champion.

de Buzon, Frédéric. 2019. "Métaphysique, géométrie et réalité: sur la connaissance des corps chez Descartes. Réponse à Daniel Garber." In *Mathesis, Grund, Vernunft: Die Philosophische Identität Europas zwischen Deutschem Idealismus und Phänomenologie*, edited by Fausto Fraisopi, 73–90. Baden-Baden: Ergon-Verlag.

Delauncey, Leo. 2017 (29 December). Untitled image of Tantalus cup in "Why the Romans are to blame for our cheeky humour: Incredible 4th Century Tanatalus bowl that soaked unsuspecting drinkers in wine could be the world's first practical joke," by Phoebe Weston. *Daily Mail.*

de Méziriac, Claude-Gaspard Bachet. 1612. *Problèmes plaisans et délectables, qui se font par les nombres*. Lyon: P. Rigaud.

Dear, Peter. 1995. *Discipline & Experience: The Mathematical Way in the Scientific Revolution*. Chicago: University of Chicago Press.

Dear, Peter. 2000. “Method and the Study of Nature.” In *The Cambridge History of Seventeenth-Century Philosophy*, edited by Daniel Garber and Michael Ayers. Vol. 1, 147–77. Cambridge: Cambridge University Press.

Dellsén, Finnur. 2017. “Certainty and Explanation in Descartes’s Philosophy of Science.” *HOPOS: The Journal of the International Society for the History of Philosophy of Science* 7 (2): 302–27.

Demange, Dominique. 2004. “Objet premier d’inclusion virtuelle. Introduction à la théorie de la science de Jean Duns Scot.” In *Duns Scot à Paris (1302–2002)*, edited by O. Boulnois et al. 89–116. Turnhaut: Brepols.

Demange, Dominique. 2009a. “Structure et unité de la science selon Duns Scot.” *Itinerarium* LV: 329–56.

Demange, Dominique. 2009b. “La théologie est-elle une science? La réponse de Duns Scot à Godefroid de Fontaines dans le prologue des *Reportata Parisiensia*.” *Documenti e studi sulla tradizione filosofica medieval* 20: 1–17.

Deprun, J. 1983. “Descartes et le ‘genie’ de Socrate.” In *La passion de la raison. Hommage à Ferdinand Alquié*, edited by Jean-Luc Marion, 145–58. Paris: Presses Universitaires de France.

Des Chene, Dennis. 1996. *Physiologia: Natural Philosophy in Late Aristotelian and Cartesian Thought*. Ithaca: Cornell University Press.

Des Chene, Dennis. 2000. *Life’s Form: Late Aristotelian Conceptions of the Soul*. Ithaca: Cornell University Press.

Des Chene, Dennis. 2006. “Review Essay: *Descartes’s Theory of Mind*, by Desmond Clarke.” In *Oxford Studies in Early Modern Philosophy*, edited by Daniel Garber and Steven Nadler. Vol. 3, 315–41. Oxford: Clarendon Press.

Descartes, René, Forthcoming. Regulae ad directionem ingenii: *An Early Manuscript Version*, edited by Richard Serjeanston and Michael Edwards. Oxford: Oxford University Press.

Descartes, René. 1626/1627? The Cambridge manuscript. MS Dd. 14.26.6. Cambridge University Library.

Descartes, René. 1683? The Hanover manuscript. MS IV 308, Gottfried Wilhelm Leibniz Bibliothek, Hannover.

Descartes, René. 1684. *Regelen van de Bestieringe des Verstants*, edited by J. H. Glasemaaker. Amsterdam: Jan Rieuwertsz.

Descartes, René. 1701. *Opuscula posthuma, physica et mathematica*. Amsterdam: P. & J. Blaeu.

Descartes, René. 1824–1826. *Oeuvres de Descartes*, edited by Victor Cousin. 11 vols. Paris: F. G. Levrault.

Descartes, René. 1859. *Oeuvres inédites de Descartes*, edited by Foucher de Careil. Paris: Auguste Durand.

Descartes, René. 1933. *Règles pour la direction de l’esprit*. Translated by George Le Roy. Paris: Boivin.

Descartes, René. 1951. *Règles pour la direction de l’esprit*. Translated by J. Sirven. Paris: Vrin.

Descartes, René. 1963. *Œuvres philosophiques*, edited by Ferdinand Alquié. 3 vols. Paris: Garnier.

Descartes, René. [1963] 1997. *Régles pour la direction de l’esprit*. Translated by Jacques Brunschwig. Paris: Vrin.

Descartes, René. 1966. *Regulae ad directionem ingenii*, edited by G. Crapulli. The Hague: Martinus Nijhoff.

Descartes, René. 1977. *Règles utiles et claires pour la direction de l'esprit et la recherche de la vérité*, edited by Pierre Costabel and Jean-Luc Marion. The Hague: Martinus Nijnoff.

Descartes, René. 1982. *Descartes on Polyhedra: A Study of the* De Solidorum Elementis, edited by P. J. Federico. New York: Springer.

Descartes, René. 1985–1991. *The Philosophical Writings of Descartes*, edited by John Cottingham, Robert Stoothoff, and Dugald Murdoch. 3 vols. Cambridge: Cambridge University Press.

Descartes, René. 1987a. *Discours de la Méthode: Texte et commentaire par Etienne Gilson*. Paris: Vrin.

Descartes, René. 1987b. *Exercices pour les éléments des solides*, edited by Pierre Costabel. Paris: Presses Universitaires de France.

Descartes, René. 1989. *The Passions of the Soul*. Translated by Stephen Voss. Indianapolis: Hackett.

Descartes, René. 1996. *Oeuvres de Descartes*, edited by Charles Adam and Paul Tanner. 11 vols. Paris: Vrin.

Descartes, René. 2001. *Discourse on Method, Dioptrics, Geometry, and Meteorology*. Translated by Paul J. Olscamp. Indianapolis: Hackett.

Descartes, René. 2002. *La recherche de la vérité par la lumière naturelle de René Descartes*, ed. Ettore Lojacano et al. Milan: Franco-Angeli.

Descartes, René. 2013. *Éude du bon sens, La recherche de la vérité, et autres écrits de jeunesse: 1616–1631*, edited by Vincent Carraud and Gilles Olivo. Paris: Presses Universitaires de France.

Descartes, René. 2016. *Oeuvres complètes I*, edited by Jean-Marie Beyssade and Denis Kambouchner. Paris: Gallimard.

Descartes, René and Martin Schoock. 1988. *La Querelle d'Utrecht*, edited by Theo Verbeek. Paris: Les Impressions Nouvelles.

Dika, Tarek R. 2018. "Extrinsic Denomination and the Origins of Early Modern Metaphysics: The Scholastic Context of Descartes's Regulae." In *The Ontology, Psychology, and Axiology of Habits (*Habitus*) in Medieval Philosophy*, edited by Nicolas Faucher and Magali Roques, 385–403. Cham: Springer.

Dika, Tarek R. 2020. "The Origins of Cartesian Dualism." *Journal of the American Philosophical Association* 6 (3): 335–52.

Dika, Tarek R. and Denis Kambouchner. Forthcoming. "Descartes's Method." In *The Cartesian Mind*, edited by Jorge Secada and Cecilia Wee. New York: Routledge.

Dika, Tarek R. 2022. "Descartes' Deduction of the Law of Refraction and the Shape of the Anaclastic Lens." *HOPOS: The Journal of the International Society for the History of Philosophy of Science* 12 (2): 395–436.

Dika, Tarek R. 2015. "Method, Practice, and the Unity of *Scientia* in Descartes's *Regulae*," *Journal of Early Modern Studies* 4 (2): 93–110.

Doračić, Damir et al. 2017. "The Vinkovci Treasure of Late Roman Silver Plate: Preliminary Report." *Journal of Roman Archaeology* 30.

Douglas, Alexander. 2017. "Descartes's Critique of the Syllogistic." *History of Philosophy Quarterly* 34 (4): 331–50.

Doyle, John P. 1984. "Prolegomena to a Study of Extrinsic Denomination in the Work of Francis Suárez, S. J." *Vivarium* 22 (2): 121–56.

Doyle, John P. 1990. "Extrinsic Cognoscibility: A Seventeenth-Century Supertranscendental Notion." *The Modern Schoolman* 68 (1): 57–80.

Doyle, John P. 1991. "Suárez on the Unity of a Scientific Habit." *American Catholic Philosophical Quarterly* 65 (3): 311–34.

Doyle, John P. 1997. "Between Transcendental and Transcendental: The Missing Link?" *The Review of Metaphysics* 50 (4): 783–815.

Doyle, John P. 2012. *On the Borders of Being and Knowing: Some Late Scholastic Thoughts on Supertranscendental Being*, edited by Victor M. Sallas. Leuven: Leuven University Press.

Dreyfus, Hubert L. and Charles Taylor. 2015. *Retrieving Realism*. Cambridge: Harvard University Press.

Dubouclez, Olivier. 2013. *Descartes et la voie de l'analyse*. Paris: Presses Universitaires de France.

Duhem, Pierre. 1954. *The Aim and Structure of Physical Theory*. Translated by Philip P. Wiener. Princeton: Princeton University Press.

Dumont, Camille, S. J. 1961. "La réflexion sur la méthode théologique." *Nouvelle Revue Théologique* 83 (10): 1034–50.

Dumont, Camille, S. J. 1962. "La réflexion sur la méthode théologique II." *Nouvelle Revue Théologique* 84 (1): 17–35.

Dumont, Stephen. 1992. "The *Propositio Famosa Scoti*: Duns Scotus and Ockham on the Possibility of a Science of Theology." *Dialogue* 31: 415–29.

Dumont, Stephen. 2005. "Duns Scotus's Parisian Question on the Formal Distinction." *Vivarium* 43 (1): 7–62.

Edwards, Sandra. 1977. "Some Medieval Views on Identity." *New Scholasticism* 51 (1): 62–74.

Euclid. 1908. *The Thirteen Books of Euclid's Elements*, edited by Thomas Heath. 3 vols. Cambridge: Cambridge University Press.

Eustachius. 1609. *Summa philosophiae quadripartita*. Paris.

Faucher, Nicolas and Magali Roques. 2018. "The Many Virtues of Second Nature: Habitus in Latin Medieval Philosophy." In *The Ontology, Psychology and Axiology of Habits (*Habitus*) in Medieval Philosophy*, edited by Nicholas Faucher and Magali Roques, 1–25. Cham: Springer.

Ferejohn, Michael T. 2013. *Formal Causes: Definition, Explanation, and Primacy in Socratic and Aristotelian Thought*. Oxford: Oxford University Press.

Feyerabend, Paul K. 1975. *Against Method: Outline of an Anarchistic Theory of Knowledge*. London: New Left Books.

Fichant, Michel. 1998. *Science et métaphysique dans Descartes et Leibniz*. Paris: Presses Universitaires de France.

Flores, Juan Carlos. 2000. "Accidental and Essential Causality in John Duns Scotus' Treatise *On the First Principle*." *Recherches de théologie et philosophie médiévales* 67 (1): 96–113.

Fonseca, Petrus. 1599. *In libros Metaphysicorum Aristotelis Stagaritae*. 2 vols. I. Schönwetter and L. Zetzner.

Foucault, Michel. [1961] 2006. *History of Madness*. Translated by Jean Khalfa. London: Routledge.

Foucault, Michel. 1966. *Les mots et les choses*. Paris: Gaillmard.

Foucault, Michel. 1970. *The Order of Things: An Archaeology of the Human Sciences*. London: Routledge.

Fowler, C. F. 1999. *Descartes on the Human Soul: Philosophy and the Demands of Christian Doctrine*. Dordrecht: Kluwer Academic Publishers.

Freudenthal, Gideon. 2000. "A Rational Controversy over Compounding Forces." In *Scientific Controversies: Philosophical and Historical Perspectives*, ed. Peter Machamer, Marcello Pera, and Aristedes Baltas. Oxford: Oxford University Press.

Gabbey, Alan. 1980. "Force and Inertia in the Seventeenth Century: Descartes and Newton." In *Descartes: Philosophy, Mathematics, and Physics*, ed. Stephen Gaukroger. Sussex: Harvester Press.

Gagné, J. 1969. "Du *Quadrivium* aux *scientiae mediae*." In *Arts libéraux et philosophie au Moyen Âge. Actes du IVe Congrès international de philosophie médiévale*, 975–86. Paris: Vrin.

Garber, Daniel. 1988. "Descartes, the Aristotelians, and the Revolution that Did Not Happen in 1637." *The Monist* 71 (4): 471–86.

Garber, Daniel. 1992. *Descartes' Metaphysical Physics*. Chicago: University of Chicago Press.

Garber, Daniel. 2001. *Descartes Embodied: Reading Cartesian Philosophy through Cartesian Science*. Cambridge: Cambridge University Press.

Garber, Daniel. 2015. "Discussion Paper on the Work of Frédéric De Buzon and John Schuster Presented at the Sixth Annual Séminaire Descartes, Paris, may 23, 2015." https://mathesis.hypotheses.org/files/2015/08/SDGarber_DebuzonSchuster-.pdf.

Garrod, Raphaële and Alexander Marr. 2020. *Descartes and the* Ingenium*: The Embodied Soul in Cartesianism*. Leiden: Brill.

Garrod, Raphaële et al. 2018. *Logodaedalus: Word Histories of Ingenuity in Early Modern Europe*. Pittsburgh. Pennsylvania: University of Pittsburgh Press.

Gassendi, Pierre. 1658. *Opera omnia*. 6 vols. Lyon: Lavrentii Anisson & Ioan. Bapt. Devenet.

Gaukroger, Stephen. 1989. *Cartesian Logic: An Essay on Descartes's Conception of Inference*. Oxford: Clarendon Press.

Gaukroger, Stephen. 1992. "Descartes's Early Doctrine of Clear and Distinct Ideas." *Journal of the History of Ideas* 53 (4): 585–602.

Gaukroger, Stephen. 1995. *Descartes: An Intellectual Biography*. Oxford: Clarendon Press.

Gaukroger, Stephen. 2002. *Descartes's System of Natural Philosophy*. Cambridge: Cambridge University Press.

Gaukroger, Stephen and John A Schuster. 2002. "The Hydrostatic Paradox and the Origins of Cartesian Dynamics." *Studies in History and Philosophy of Science Part A* 33 (3): 535–572.

Gaukroger, Stephen. 2000. "The Foundational Role of Hydrostatics and Statics in Descartes's Natural Philosophy." In *Descartes's Natural Philosophy*, edited by Stephen Gaukroger, John Schuster, and John Sutton, 60–81. London: Routledge.

Gill, Mary Louise. 1989. *Aristotle on Substance: The Paradox of Unity*. Princeton: Princeton University Press.

Gilson, Étienne. 1952. *Jean Duns Scot: Introduction à ses positions fondamentales*. Paris: Vrin.

Gilson, Étienne. 1979. *Index scolastico-cartésien*. Paris: Vrin.

Goclenius, Rodolphus. 1613. *Lexicon philosophicum*. Francofurti.

Goddu, André. 2010. *Copernicus and the Aristotelian Tradition: Education, Reading, and Philosophy in Copernicus's Path to Heliocentrism*. Leiden: Brill.

Goldstein, Bernard R. 1997. "Saving the Phenomena: The Background to Ptolemy's Planetary Theory." *Journal for the History of Astronomy* 28 (1): 1–12.

Goldstein, Bernard R. 2008. "The Status of Models in Ancient and Medieval Astronomy." *Centaurus* 50 (1–2): 168–83.

Gorham, Geoffrey. 2005. "The Metaphysical Roots of Cartesian Physics: The Law of Rectilinear Motion." *Perspectives on Science* 13 (4): 431–51.

Gouhier, Henri. 1962. *La pensée métaphysique de Descartes.* Paris: Vrin.

Grabmann, Martin. 1948. *Die theologische Erkenntnis und Einleitungslehre des Hl. Thomas von Aquin auf Grund seiner Schrift "in Boethium De Trinitate" im Zusammenhang der Scholastik des 13. und Beginnenden 14. Jahrhunderts*. Freiburg: Paulus.

Grajewski, Maurice J. 1944. *The Formal Distinction of Duns Scotus: A Study in Metaphysics*. Washington: Catholic University of America Press.

Gregoric, Pavel. 2007. *Aristotle on the Common Sense*. Oxford: Clarendon Press.

Gueroult, Martial. 1984–1985. *Descartes's Philosophy Interpreted According to the Order of Reasons*. Translated by Roger Ariew. 2 vols. Minneapolis: University of Minnesota Press.

Hacking, Ian. 1975. *Why Does Language Matter to Philosophy?* Cambridge: Cambridge University Press.

Hacking, Ian. 2014. *Why Is There Philosophy of Mathematics at All?* Cambridge: Cambridge Universty Press.

Hamelin, Octave. 1921. *Le système de Descartes*. Paris: Alcan.

Hamou, Phillipe. 1999. *La mutation du visible. Essai sur la portée épistémologique des instruments d'optique au XVIIe siècle. Vol. 1. Du* Sidereus Nuncius *de Galilée à la* Dioptrique *cartésienne*. Villeneuve d'Ascq (Nord): Presses Universitaires du Septentrion.

Hardy, Claude. 1625. *Euclides Data & Marini Philosophi commentarius Graece et Latine. Per Claudium Hardy nunc primum edita*. Paris: M. Mondier.

Hartland-Swann, John. 1947. "Descartes's 'Simple Natures.'" *Philosophy* 22 (82): 139–52.

Hatfield, Gary. 1988. "Science, Certainty, and Descartes." *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 2: 249–62.

Hatfield, Gary. 2008. "Animals." In *A Companion to Descartes*, edited by Janet Broughton and John Carriero, 404–26. Oxford: Blackwell.

Hatfield, Gary. 2012. "Mechanizing the Sensitive Soul." In *Matter and Form in Early Modern Science and Philosophy*, edited by Gideon Manning, 151–86. Leiden: Brill.

Hattab, Helen. 2009. *Descartes on Forms and Mechanisms.* Cambridge: Cambridge University Press.

Heeffer, Albrecht. 2006. "Récréations Mathématiques (1624): A Study on its Authorship, Sources and Influence." *Gibecière* 1 (2): 79–167.

Heeffer, Albrecht. 2017. "Using Invariances in Geometrical Diagrams: Della Porta, Kepler and Descartes on Refraction." In *The Dioptrics of Giambattista Della Porta (ca. 1535–1615): A Reassessment*, ed. Arianna Borelli, Giora Hon, and Yaakov Zik, 145–69. Switzerland: Springer.

Heidegger, Martin. 1962. *Die Frage nach dem Ding: Zu Kants Lehre von den transzendentalen Grundsätzen*. Max Niemeyer: Tübingen.

Heidegger, Martin. 1967. *What Is a Thing?* Translated by W. B. Barton and Vera Deutsch. South Bend: Gateway.

Heider, Daniel. 2016. "The *Notitia Intuitiva* and *Notitia Abstractiva* of the External Senses in Second Scholasticism: Suárez, Poinsot and Francisco De Oviedo." *Vivarium* 54 (2–3): 173–203.

Heider, Daniel. 2017a. "The Internal Sense(s) in Early Jesuit Scholasticism." *Filosofický Časopis (Philosophical Journal)* 65 (Special Issue 2): 133–48.

Heider, Daniel. 2017b. "Suárez on the Functional Scope of the Imaginative Power." *Cauriensia* 12: 135–52.

Heimsoeth, Heinz. 1912. *Die Methode der Erkenntnis bei Descartes und Leibniz*. Giessen: A. Töpelmann.

Henry of Ghent. 1979–2008. *Opera Omnia*, edited by Raymond Macken et al. 38 vols. Ancient and Medieval Philosophy, Series 2. Leuven: Leuven University Press.

Hintikka, Jaakko. 1972. "On the Ingredients of an Aristotelian Science." *Nous* 6 (1):55–69.

Hintikka, Jaakko. 1978. "A Discourse on Descartes' Method." In *Descartes: Critical and Interpretive Essays*, edited by Michael Hooker. Baltimore: Johns Hopkins University Press.

Hintikka, Jaako and Unto Remes. 1972. "On the Ingredients of an Aristotelian Science." *Nous* 6 (1): 55–69.

Hintikka, Jaako and Unto Remes. 1974. *The Method of Analysis: Its Geometrical Origin and its General Significance*. Dordrecht: Reidel.

Hispanus, Gonsalvus. 1935. *Quaestiones disputatae*, edited by Leonis Amoros. Quaracchi: Collegii S. Bonaventurae.

Hoffman, Paul. 1986. "The Unity of Descartes's Man." *Philosophical Review* 95 (3): 339–70.

Husserl, Edmund. [1954] 1970. *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*. Translated by David Carr. Evanston: Northwestern University Press.

Husserl, Edmund. 1976. *Die Krisis der europäischen Wissenschaften und die Transzendentale Phänomenologie*. The Hague: M. Nijhoff.

ibn al-Haytham [Alhazen]. 1572. *Opticae thesaurus Alhazeni arabis libri septem.* Basel.

ibn al-Haytham [Alhazen]. 2006. *Alhacen on the Principles of Reflection: A Critical Edition, with English Translation and Commentary, of Books 4 and 5 of Alhacen's "De Aspectibus," the Medieval Latin Version of Ibn al-Haytham's "Kitab al-Manazir." Volume Two: English Translation*. Translated by Mark A. Smith. *Transactions of the American Philosophical Society* 100 (3): 289–697.

ibn al-Haytham [Alhazen]. 2010. *Alhacen on Refraction: A Critical Edition, with English Translation and Commentary, of Book 7 of Alhacen's "De Aspectibus," the Medieval Latin Version of Ibn al-Haytham's "Kitab al-Manazir." Volume Two. English Translation*. Translated by Mark A. Smith. *Transactions of the American Philosophical Society* 100 (3.2): 213–550.

Irwin, Terence. 1988. *Aristotle's First Principles*. Oxford: Clarendon.

Israel, Giorgio. 1998. "Des *Regulae* à la *Géométrie*." *Revue d'histoire des sciences* 51 (2): 183–236.

Jackson, Stanley W. 1986. *Melancholia and Depression: From Hippocratic Times to Modern Times*. New Haven: Yale University Press.

Janiak, Andrew. 2015. "Mathematics and Infinity in Descartes and Newton." In *Mathematizing Space*, edited by Vincenzo De Risi, 209–30. Basel: Birkhauser.

Jenkins, John I. 1997. *Knowledge and Faith in Thomas Aquinas*. Cambridge: Cambridge University Press.

Joachim, Harold H. 1957. *Descartes's* Rules for the Direction of the Mind. London: George Allen and Unwin.

Kambouchner, Denis. 2005. *Les* Méditations Métaphysiques *de Descartes*. Paris: Presses Universitaires de France.

Kambouchner, Denis. 2006. "Remarques sur la définition cartésienne de la clarté et de la distinction." In *Les facultés de l'âme à l'âge classique: imagination, entendement et jugement*, edited by Chantal Jaquet and Tamás Pavlovits, 159–73. Paris: Publications de la Sorbonne.

Kambouchner, Denis. 2009. "Descartes et le problème de la doctrine." In *Vera Doctrina: Zur Begriffsgeschichte der Lehre von Augustinus bis Descartes/L'idée de doctrine d'Augustin à Descartes*, ed. Philippe Büttgen, 365–81. Wiesbaden: Harrassowitz Verlag im Kommission.

Kambouchner, Denis. 2019. "Descartes: la méthode en pratique." In *La méthode*, edited by Patrick Wotling, 101–30. Paris: Vrin.

Kapp, E. 1975. "Syllogistic." In *Articles on Aristotle*, vol. 2, edited by Jonathan Barnes, Malcolm Schofield, and Richard Sorabji, 35–49. London: Duckworth.

Keeling, S. V. [1934] 1968. *Descartes*. Oxford: Oxford University Press.

Keeling, S. V. 1937. "Le réalisme de Descartes et le rôle des natures simples." *Revue de Métaphysique et de Morale* 44 (1): 63–99.

Kelly, Suzanne, O. S. B. 1965. *The* De Mundo *of William Gilbert*. Amsterdam: M. Hertzberger.

Kenny, Anthony. 1968. *Descartes*. New York: Random House.

Kepler, Johannes. 1604. *Ad Vitellionem paralipomena.* Frankfurt.

Kepler, Johannes. 2000. *Kepler's Dioptrics*, ed. W. Donahue. Santa Fe: Green Lion Press.

Kisiel, Theodore. 1980. "*Ars Inveniendi*: A Classical Source for Contemporary Philosophy of Science." *Revue internationale de philosophie* 34 (131–2): 130–54.

Klein, Jacob. [1934–1936] 1992. *Greek Mathematical Thought and the Origin of Algebra*. New York: Dover.

Köpf, Ulrich. 1974. *Die Anfänge der theologischen Wissenschaftstheorie im 13. Jahrhundert*. Tübingen: Mohr.

Koyré, Alexandre. 1940. *Études galiléennes*. Paris: Hermann.

Koyré, Alexandre. 1956. "The Origins of Modern Science: A New Interpretation." *Diogenes* 4 (16): 1–22.

Koyré, Alexandre. 1957. *From the Closed World to the Infinite Universe*. Baltimore: Johns Hopkins University Press.

Krebs, Engelbert. 1912. *Theologie und Wissenschaft nach der Lehre der Hochscholastik*. Münster: Aschendorff.

Kuhn, Thomas. 1977. *The Essential Tension: Selected Studies in Scientific Tradition and Change*. Chicago: University of Chicago Press.

Laertius, Diogenes. 2018. *Lives of the Eminent Philosophers*, ed. James Miller. Trans. Pamela Mensch. New York: Oxford University Press.

Laird, Walter Roy. 1983. *The* Scientiae Mediae *in Medieval Commentaries on Aristotle's* Posterior Analytics. Ph.D. Dissertation. University of Toronto.

Laporte, Jean. 1945. *Le rationalisme de Descartes*. Paris: Presses Universitaires de France.

Larmore, Charles. 1980. "Descartes's Empirical Epistemology." In *Descartes: Philosophy, Mathematics and Physics*, edited by Stephen Gaukroger, 6–22. Sussex: Harvester Press.

Laudan, Laurens. 1968. "Theories of Scientific Method from Plato to Mach: A Bibliographical Review." *History of Science* 7 (1): 1–63.

Lawn, Brian. 1993. *The Rise and Decline of the Scholastic "Quaestio Disputata."* Leiden: Brill.

Le Blond, J. M. 1937. "Les natures simples chez Descartes." *Archives de philosophie* 13 (2): 163–80.

Lecourt, Dominique. 1968. *L'épistémologie historique de Gaston Bachelard*. Paris: Vrin.

Leff, Gordon. 1975. *William of Ockham: The Metamorphosis of Scholastic Discourse*. Manchester: Manchester University Press.

Leibniz, Gottfried Wilhelm. 1875–1889. *Die Philosophischen Schriften*, edited by C. Gerhardt. 7 vols. Halle.

Lennon, Thomas M. 1974. "The Inherence Pattern and Descartes's Ideas." *Journal of the History of Philosophy* 12 (1): 43–52.

Lennox, James G. 1986. "Aristotle, Galileo, and 'Mixed Sciences.'" In *Reinterpreting Galileo*, edited by W. Wallace, 29–51. Washington, DC: Catholic University of America.

Leurechon, Jean. 1624. *Récréations mathématiques*. Pont-à-Mousson: J. Appier Hanzelet.

Lewis, Frank A. 1995. "Aristotle on the Unity of Substance." *Pacific Philosophical Quarterly* 76 (3–4): 222–65.

Liard, Louis. 1880. "La méthode et la mathématique universelle de Descartes." *Revue philosophique de la France et de l'étranger* 10: 569–600.

Liard, Louis. 1911. *Descartes*. Paris: F. Alcan.

Lindberg, David C. 1968. "The Cause of Refraction in Medieval Dioptrics." *The British Journal for the History of Science* 4 (1): 23–38.

Lipstorp, Daniel. 1653. *Specimina philosophiæ cartesianæ*. Leiden: Elsevier.

Livesey, Steven J. 1982. *Metabasis: The Interrelationship of the Sciences in Antiquity and the Middle Ages*. Ph.D. Dissertation. University of California.

Livesey, Steven J. 1989. *Theology and Science in the Fourteenth Century: Three Questions on the Unity and Subalternation of the Sciences from John of Reading's* Commentary on the Sentences. Leiden: Brill.

Livesey, Steven J. 1990. "Science and Theology in the Fourteenth Century: The Subalternate Sciences in Oxford Commentaries on the *Sentences*." *Synthese* 83 (2): 273–92.

Longeway, John L. 2011. "*Posterior Analytics*, Commentaries on Aristotle's." In *Encyclopedia of Medieval Philosophy: Philosophy between 500 and 1500*, edited by Henrik Lagerlund, 1062–6. Dordrecht: Springer.

Mahnke, Detlef. 1967. *Der Aufbau des philosophischen Wissens nach René Descartes*. München/Salzburg: Anton Pustet.

Mahoney, Michael. 1973. *The Mathematical Career of Pierre de Fermat (1601–1665)*. Princeton: Princeton University Press.

Mancosu, Paolo. 1996. *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*. New York: Oxford University Press.

Mancosu, Paolo. 2008. “Descartes and Mathematics.” In *A Companion to Descartes*, edited by Janet Broughton and John Carriero, 103–23. Oxford: Blackwell.

Manders, K. 1995. “Descartes et Faulhaber.” *Archives de philosophie* 58 (3): 1–12.

Manders, K. 2006. “Algebra in Roth, Faulhaber, and Descartes.” *Historia Mathematica* 33: 184–209.

Marion, Jean-Luc. 1975. *Sur l’ontologie grise de Descartes. Science cartésienne et savoir aristotélicien dans les* Regulae. Paris: Vrin.

Marion, Jean-Luc. 1981. *Sur la théologie blanche de Descartes: Analogie, création des vérités éternelles et fondement*. Paris: Presses Universitaires de France.

Marion, Jean-Luc. 1992. “Cartesian Metaphysics and the Role of the Simple Natures.” In *Cambridge Companion to Descartes*, edited by John Cottingham, 115–40. Cambridge: Cambridge University Press.

Marion, Jean-Luc. 1999. *Cartesian Questions: Method and Metaphysics.* Chicago: University of Chicago Press.

Marion, Jean-Luc. 2013. *Sur la pensée passive de Descartes*. Paris: Presses Universitaires de France.

Marmodoro, Anna. 2014. *Aristotle on Perceiving Objects*. New York: Oxford University Press.

Martinet, Monette. 1974. “Science et hypothèses chez Descartes.” *Archives internationales d’histoire des sciences* 24: 319–39.

Maurer, Armand A. 1990. *Being and Knowing: Studies in Thomas Aquinas and Later Medieval Philosophers*. Toronto: Pontifical Institute of Medieval Studies.

McKirahan, Richard D. 1978. “Aristotle’s Subordinate Sciences.” *The British Journal for the History of Science* 11 (3): 197–220.

McKirahan, Richard D. 1992. *Principles and Proofs: Aristotle’s Theory of Demonstrative Science*. Princeton: Princeton University Press.

McLaughlin, Peter. 2000. “Force, Determination and Impact.” In *Descartes’s Natural Philosophy*, edited by Stephen Gaukroger, John Schuster, and John Sutton, 81–113. London: Routledge.

McMullin, Ernan. 1968. *Empiricism and the Scientific Revolution*. Baltimore: Johns Hopkins University Press.

McRae, Robert. 1957. “The Unity of the Sciences: Bacon, Descartes, and Leibniz.” *Journal of the History of Ideas* 18 (1): 27–48.

McRae, Robert. 1965. “‘Idea’ as a Philosophical Term in the Seventeenth Century.” *Journal of the History of Ideas* 26 (2): 175–90.

McRae, Robert. 1972. “Innate Ideas.” In *Cartesian Studies*, edited by Ronald Joseph Butler, 32–55. Oxford: Basil Blackwell.

Mehl, Édouard. 2001. *Descartes en Allemagne 1619–1620: le contexte allemand de l’élaboration de la science cartésienne*. Strasbourg: Presses Universitaires de Strasbourg.

Mehl, Édouard. 2005. “Descartes critique de la logique pure.” *Les Études Philosophiques* 75 (4): 485–500.

Meier-Oeser, Stephan. 2015. “The Intersubjective Sameness of Mental Concepts in Late Scholastic Thought.” In *Intentionality, Cognition, and Mental Representation in Medieval Philosophy*, edited by Gyula Klima, 287–3. New York: Fordham University Press.

Merrylees, W. A. 1934. *Descartes: An Examination of Some Features of His Metaphysics and Method*. Melbourne: Melbourne University Press.

Mersenne, Marin. 1625. *La vérité des sciences*. Paris: Simon Bachelier.

Mersenne, Marin. 1932–1988. *Correspondance du P. Marin Mersenne*, edited by C. de Waard. 17 vols. Paris: Centre National de la Recherche Scientifique.

Michael, Emily and Fred S. Michael. 1989. “Corporeal Ideas in Seventeenth-Century Psychology.” *Journal of the History of Ideas* 50 (1): 31–48.

Milhaud, Gaston. 1907. “Descartes et la loi des sinus.” *Revue générale des sciences* 18: 223–8.

Milhaud, Gaston. 1921. *Descartes Savant*. Paris: F. Alcan.

Miller, David. 2017. “The Parallelogram Rule from Pseudo-Aristotle to Newton.” *Archive for History of Exact Sciences* 71 (2): 157–91.

Millet, J. 1867. *Histoire de Descartes avant 1637*. Paris: Didier.

Montaigne, Michel. 2003. *The Complete Works: Essays, Travel Journal, Letters*. Translated by Donald L. Frame. New York: Knopf.

Murdoch, Dugald. 1993. “Exclusion and Abstraction in Descartes’s Metaphysics.” *The Philosophical Quarterly* 43 (170): 38–57.

Mydorge, Claude. 1630. *Examen du livre des Récréations mathématiques*. Paris: Jean Moreau and Guillaume Loyson.

Mydorge, Claude. 1631. *Prodromi catoptricorum et dioptricorum*. Paris: Ex Tyopgraphia I. Dedin.

Natorp, Paul. 1882. *Descartes’ Erkenntnistheorie: Eine Studie zur Vorgeschichte des Kriticismus*. Marburg: N.G. Elwert.

Nelson, Alan. 2017. “Logic and Knowledge.” In *The Routledge Companion to Seventeenth Century Philosophy*, edited by Dan Kaufman, 224–49. London: Routledge.

Nelson, Alan and Lawrence Nolan. 2006. “Proofs for the Existence of God.” In *The Blackwell Guide to Descartes’s Meditations*, edited by Stephen Gaukroger, 104–21. Oxford: Blackwell.

Nelson, Alan and Brian Rogers. 2015. “Descartes’ Logic and the Paradox of Deduction.” In *The Battle of the Gods and Giants Redux*, edited by Patricia Easton and Kurt Smith, 106–36. Leiden: Brill.

Newman, Lex. 2009. “Ideas, Pictures, and the Directness of Perception in Descartes and Locke.” *Philosophy Compass* 4 (1): 134–54.

Normore, Calvin. 1986. “Meaning and Objective Being: Descartes and His Sources.” In *Essays on Descartes’s Meditations*, edited by Amélie Oskenberg Rorty, 223–43. Berkeley: University of California Press.

Normore, Calvin. 1993. “The Necessity in Deduction: Cartesian Inference and its Medieval Background.” *Synthese* 96 (3): 437–54.

Novikoff, Alex J. 2014. *The Medieval Culture of Disputation: Pedagogy, Practice, and Performance*. Philadelphia: University of Pennsylvania Press.

Ockham, William. 1974. *Ockham’s Theory of Terms. Part I of the Summa logicae*. Translated by Michael J. Loux. Notre Dame: University of Notre Dame Press.

Ockham, William. 1980. *Ockham’s Theory of Propositions. Part II of the Summa logicae*. Translated by A.J. Freddoso and H. Schuurman. Notre Dame: University of Notre Dame Press.

Ockham, William. 1967–1988. *Opera theologica*. St. Bonaventure: The Franciscan Institute.

Ockham, William. 1974–1988. *Opera philosophica*. St. Bonaventure: The Franciscan Institute.

Ockham, William. 1990. *Philosophical Writings*. Translated by Philotheus Boehner, O.F.M. Indianapolis: Hackett.

Ockham, William. 2007. *Demonstration and Scientific Knowledge in William of Ockham: A Translation of* Summa Logicae *III–II*, edited by John L. Longeway. Notre Dame: University of Notre Dame Press.

Ockham, William. 2021. *Prologue of the* Ordinatio *of the Venerable Inceptor.* Trans. Peter L.P. Simpson. https://www.aristotelophile.com/Books/Translations/Ockham%20Prologue.pdf

O'Connor, Edward. 1968. "The Scientific Character of Theology According to Scotus." In *De Doctrina Ioannis Duns Scoti*, edited by Comissio Scotistica, 3–50. Rome: Societas Internationalis Scotistica.

Olivo, Gilles. 2005. *Descartes et l'essence de la vérité*. Paris: Presses Universitaires de France.

O'Neil, Brian, E. 1972. "Cartesian Simple Natures." *Journal of the History of Philosophy* 10 (2): 161–79.

O'Neil, Brian, E. 1974. *Epistemological Direct Realism in Descartes's Philosophy*. Albuquerque: University of New Mexico Press.

Ott, Walter R. 2017. *Descartes, Malebranche, and the Crisis of Perception*. Oxford: Oxford University Press.

Panaccio, Claude. 2014. "Ockham: Intuition and Knowledge." In *Rational Intuition: Philosophical Roots, Scientific Investigations*, edited by Lisa M. Osbeck and Barbara S. Held, 55–75. Cambridge: Cambridge University Press.

Panaccio, Claude and David Piché. 2009. "Ockham's Reliabilism and the Intuition of Non-Existents." In *Rethinking the History of Skepticism: The Missing Medieval Background*, edited by Henrik Lagerlund, 97–118. Leiden: Brill.

Pasnau, Robert. 2011. *Metaphysical Themes, 1274–1671*. Oxford: Oxford University Press.

Pasnau, Robert. 2014. "Science and Certainty." In *Cambridge History of Medieval Philosophy*, edited by Robert Pasnau, 357–68. Cambridge: Cambridge University Press.

Pasnau, Robert. 2017. *After Certainty: A History of our Epistemic Ideals and Illusions*. Oxford: Oxford University Press.

Pelletier, Jenny E. 2013. *William Ockham on Metaphysics: The Science of Being and God*. Leiden: Brill.

Penchèvre, E. 2004. "L'oeuvre algébrique de Johannes Faulhaber." *Oriens-Occidens, Cahiers du Centre d'Histoire des Sciences et des Philosophies Arabes et Médiévales* 5: 187–222.

Pereira, Benito. 1576. *De communibus omnium rerum naturalium principiis et affectionibus*. Rome: F. Zanettus and B. Tosius.

Perler, Dominik. 2014. "Suárez on Consciousness." *Vivarium* 52 (3–4): 261–86.

Perler, Dominik. 2018. "Suárez on the Metaphysics of Habits." In *The Ontology, Psychology and Axiology of Habits (*Habitus*) in Medieval Philosophy*, edited by Nicolas Faucher and Magali Roques, 365–84. Cham: Springer.

Pfeiffer, Christian. 2018. *Aristotle's Theory of Bodies*. Oxford: Oxford University Press.

Piccolomini, Alessandro. 1547. *In Mechanicas Quaestiones Aristotelis*. Rome: A. Bladus Asulanus.

Poisson, Nicolas-Joseph. 1670. *Commentaire ou remarques sur la méthode de René Descartes*. Vendôme: S. Hip.

Polya, George. 1954. *Mathematics and Plausible Reasoning*. 2 vols. Princeton: Princeton University Press.

Popkin, Richard. 2003. *The History of Scepticism: From Savonarola to Bayle*. Oxford: Oxford University Press.

Popper, Karl. [1935] 2002. *The Logic of Scientific Discovery*. London: Routledge.

Proclus. 1970. *A Commentary on the First Book of Euclid's Elements*, edited by Glenn Morrow. Princeton: Princeton University Press.

Ptolemy. 1984. *Ptolemy's Almagest*, edited by G. J. Toomer. London: Duckworth.

Putallaz, François-Xavier. 1991. *Le sens de la réflexion chez Thomas d'Aquin*. Paris: Vrin.

Putnam, Hilary. 1981. *Reason, Truth, and History*. Cambridge: Cambridge University Press.

Rabouin, David. 2009. *Mathesis universalis: l'idée de mathématique universelle d'Aristote à Descartes*. Paris: Presses Universitaires de France.

Rabouin, David. 2010. "What Descartes Knew of Mathematics in 1628." *Historia Mathematica* 37 (3): 428–59.

Rabouin, David. 2020. "*Ingenium*, Phantasia and Mathematics in Descartes's *Regulae ad directionem ingenii*." In *Descartes and the* Ingenium, edited by Raphaële Garrod and Alexander Marr, 64–90. Leiden: Brill.

Reed, Edward S. 1982. "Descartes's Corporeal Ideas Hypothesis and the Origin of Scientific Psychology." *The Review of Metaphysics* 35 (4): 731–52.

Robinet, André. 1996. *Aux sources de l'esprit cartésien. L'axe La Ramée-Descartes: de la* Dialectique *des 1555 aux* Regulae. Paris:Vrin.

Robinet, André. 1999. *Descartes. La lumière naturelle: intuition, disposition, complexion*. Paris: Vrin.

Rochemonteix, Camille de. 1889. *Un collège de jésuites aux XVIIe & XVIIIe siècles: Le Collège Henri IV de la Flèche*. 4 vols. Le Mans: Leguicheux.

Röd, Wolfgang. 1971. *Descartes' erste Philosophie: Versuch einer Analyse mit besonderer Berücksichtigung der Cartesianischen Methodologie*. Bonn: Bouvier.

Rode, Christian. 2015. *Zugänge zum Selbst: Innere Erfahrung in Spätmittelalter und früher Neuzeit*. Münster: Aschendorff Verlag.

Rodis-Lewis, Geneviève. 1950. *L'individualité selon Descartes*. Paris: Vrin.

Rodis-Lewis, Geneviève. 1998. *Descartes: His Life and Thought*. Translated by Jane Marie Todd. Ithaca: Cornell University Press.

Rorty, Richard. 1979. *Philosophy and the Mirror of Nature*. Princeton: Princeton University Press.

Roth, Léon. 1937. *Descartes'* Discourse on Method. Oxford: Clarendon Press.

Roy, Jean H. 1944. *L'imagination selon Descartes*. Paris: Gallimard.

Rozemond, Marleen. 1998. *Descartes's Dualism*. Cambridge: Harvard University Press.

Rozemond, Marleen. 2003. "Descartes, Mind-Body Union, and Holenmerism." *Philosophical Topics* 31 (1–2): 343–67.

Rozemond, Marleen. 2014. "The Faces of Simplicity in Descartes's Soul." In *Partitioning the Soul: Debates from Plato to Leibniz*, edited by Dominik Perler and Klaus Corcilius, 219–44. Berlin: De Gruyter.

Rudolff, Christoff. 1525. *Behend und hübsch Rechnung durch die kunstreichen Regeln Algebre, so gemeincklich die Coss genennt werden* […]. Strasbourg: Jung.

Russell, Daniel C. 2014. "Phronesis and the Virtues (*NE* VI 12–13)." In *The Cambridge Companion to Aristotle's Nicomachean Ethics*, edited by Roland Polansky, 203–20. Cambridge: Cambridge University Press.

Sabra, A. I. 1981. *Theories of Light from Descartes to Newton*. Cambridge: Cambridge University Press.

Sarton, George. 1921. "Introduction to the History and Philosophy of Science (Preliminary Note)." *Isis* 4 (1): 23–31.

Sarton, George. 1924. "The New Humanism." *Isis* 6 (1): 2–42.

Sasaki, Chikara. 2003. *Descartes's Mathematical Thought*. Dordrecht: Springer.

Schmaltz, Tad M. 2008. *Descartes on Causation*. New York: Oxford University Press.

Schmaltz, Tad M. 2017. *Early Modern Cartesianisms*. New York: Oxford University Press.

Schmaltz, Tad M. 2020. "The Indefinite in the Descartes-More Correspondence." *British Journal for the History of Philosophy* 29 (3): 453–71.

Schneider, I. 1993. *Johannes Faulhaber (1580–1635): Rechenmeister in einer Welt des Umbruchs*. Basel: Birkhauser.

Schneider, I. 2008. "Trends in German Mathematics at the Time of Descartes's Stay in Southern Germany." In *Mathématiciens Français du XVIIe siècle: Descartes, Fermat, Pascal*, edited by Michel Serfati and Dominique Descotes, 45–68. Clermont-Ferrand: Presses Universitaires Blaise Pascal.

Schook[ius], Martin[us]. 1643. *Admiranda methodus novæ philosophiæ Renati Des Cartes*. Utrecht: J. Waesberge.

Schouls, Peter A. 1980. *The Imposition of Method: A Study of Descartes and Locke*. Oxford: Clarendon Press.

Schouls, Peter A. 2000. *Descartes and the Possibility of Science*. Ithaca: Cornell University Press.

Schuster, John A. 1977. *Descartes and the Scientific Revolution, 1618–1634: An Interpretation*. Ph.D. Dissertation. Princeton University.

Schuster, John A. 1980. "Descartes' *Mathesis Universalis*, 1619–28." In *Descartes: Philosophy, Mathematics, and Physics*, edited by Stephen Gaukroger, 41–97. Sussex: Harvester Press.

Schuster, John A. 1986. "Cartesian Method as Mythic Speech: A Diachronic and Structural Analysis." In *The Politics and Rhetoric of Scientific Method*, ed. John Schuster and Richard Yeo, 33–95. Dordrecht: Reidel.

Schuster, John A. 2000. "Descartes *opticien*: The Construction of the Law of Refraction and the Manufacture of Its Physical Rationale, 1618–29." In *Descartes's Natural Philosophy*, ed. Stephen Gaukroger, John Schuster, and John Sutton, 258–313. London: Routledge.

Schuster, John A. 2013. *Descartes-Agonistes: Physico-Mathematics, Method & Corpuscular-Mechanism 1618–33*. Dordrecht: Springer.

Schuster, John A. and Richard R. Yeo, eds. 1986. *The Politics and Rhetoric of Scientific Method: Historical Studies*. Dordrecht: Reidel.

Scotus, John Duns. 1639. *Opera omnia*. 12 vols. Lyon: L. Durand.

Scotus, John Duns. 1891–1895. *Opera omnia*, ed. Wadding. 26 vols. Paris: Vivès.

Scotus, John Duns. 1950–2013. *Opera omnia*, ed. C. Balic. 21 vols. Civitas Vaticana: Typis Polyglottis Vaticanis.

Scotus, John Duns. 1966. *A Treatise on God as First Principle*, edited by Alan B. Wolter. Chicago: Forum Books.

Scotus, John Duns. 1998. *Questions on the Metaphysics of Aristotle*. Translated by Girard J. Etskorn and Allan B. Wolter, O. F. M. 2 vols. St. Bonaventure: The Franciscan Institute.

Scotus, John Duns. 2004–2008. *The Examined Report of the Paris Lecture:* Reportatio *1.-A: Latin Text and English Translation*, edited by Alan B.Wolter, O. F. M. and Oleg V. Bychkov. 2 vols. St. Bonaventure: The Franciscan Institute.

Scotus, John Duns. 2012. *Ordinatio, Prologue*. Translated by Peter Simpson. https://www.aristotelophile.com/Books/Translations/Scotus%20Prologue.pdf.

Secada, Jorge. 2000. *Cartesian Metaphysics: The Late Scholastic Origins of Modern Philosophy*. New York: Cambridge University Press.

Segond, Joseph. 1932. *La sagesse cartésienne et la doctrine de la science*. Paris: Vrin.

Sepper, Dennis L. 1996. *Descartes's Imagination: Proportion, Images, and the Activity of Thinking*. Berkeley: University of California Press.

Serfati, Michel. 1998. "Descartes et la constitution de l'écriture symbolique mathématique." *Revue d'histoire des sciences* 51 (2): 237–89.

Serfati, Michel. 2005. *La révolution symbolique: la constitution de l'écriture symbolique mathématique*. Paris: Éditions Pétra.

Serfati, Michel. 2010. “Symbolic Revolution, Scientific Revolution: Mathematical and Philosophical Aspects.” In *Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics*, edited by Albrecht Heeffer and Maarten Van Dyck, 103–25. London: College Publications.

Serjeanston, Richard. 2013. “Descartes before Dualism: New Evidence.” Paper presented at the Centre for Research in the Arts, Social Sciences, and Humanities, Cambridge University, January 21, 2013.

Serrus, Charles. 1933. *La méthode de Descartes et son application à la métaphysique*. Paris: Alcan.

Shea, William R. 1991. *The Magic of Numbers and Motion: The Scientific Career of René Descartes*. Canton: Science History Publications.

Shapin, Steven. 2018. *The Scientific Revolution*. 2nd ed. Chicago: University of Chicago Press.

Simmons, Alison. 1999. “Jesuit Aristotelian Education: De Anima Commentaries.” In *The Jesuits: Culture, Learning and the Arts, 1540–1773*, edited by John W. O’Malley, S. J. et al., 522–38. Toronto: Toronto University Press.

Singmaster, David. 1994. “Recreational Mathematics.” In *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, ed I. Grattan-Guinness. Vol. 2, 1568–1676. London: Routledge.

Sirven, J. 1928. *Les années d’apprentissage de Descartes (1596–1628)*. Imprimerie coopérative du sud-ouest: Albi.

Smith, Kurt. 2010. *Matter Matters: Metaphysics and Methodology in the Early Modern Period*. Oxford: Oxford University Press.

Smith, Mark A. 1987. *Descartes’s Theory of Light and Refraction: A Discourse on Method*. *Transactions of the American Philosophical Society* 77 (3).

Smith, Mark A. 2014. *From Sight to Light: The Passage from Ancient to Modern Optics*. Chicago: Chicago University Press.

Smith, Norman Kemp. 1902. *Studies in the Cartesian Philosophy*. London: Macmillan.

Smith, Norman Kemp. 1966. *New Studies in the Philosophy of Descartes*. London: Macmillan.

Societas Jesu. 1616. *Regulae Societatis Jesu*. Rome.

Soncinas, Paulus. 1505. *Quaestiones metaphysicales*. Venice.

South, James. 2012. “Suarez, Immortality, and the Soul’s Dependence on the Body.” In *The Philosophy of Francisco Suárez*, edited by Benjamin Hill and Henrik Lagerlund, 121–37. Oxford: Oxford University Press.

Spade, Paul Vincent. 1972. “The Unity of a Science According to Peter Auriol.” *Franciscan Studies* 32: 203–17.

Stock, Hyman. 1931. *The Method of Descartes in the Natural Sciences*. New York: The Marion Press.

Suárez, Francisco. 1856–1878. *Opera omnia*, edited by M. André and C. Berton. 28 vols. Paris: Vivès.

Suárez, Francisco. 1991. *Commentaria una cum quaestionibus in libros Aristotelis De anima*, edited by Salvador Castellote. 2 vols. Madrid: Sociedad de Estudios y Publicaciones.

Suárez, Francisco. 1995. *On Beings of Reason: Metaphysical Disputation LIV*. Translated by John P. Doyle. Milwaukee: Marquette University Press.

Suárez, Francisco. 2000. *On the Formal Cause of Substance: Metaphysical Disputation XV*. Translated by John Kronen and Jeremiah Reedy. Marquette: Marquette University Press.

Tannery, Paul. 1896. “Descartes physicien.” *Revue de Métaphysique et de Morale* 4 (4): 478–88.

Toletus, Franciscus. 1615. *Introductio in universam Aristotelis logicam*. Cologne: H. Mylii.

Torrell, Jean-Pierre. 2003. *Saint Thomas Aquinas*. Translated by Robert Royal. Vol. 2. Washington: The Catholic University of America Press.

van Berkel, Klaas. 2013. *Isaac Beeckman on Matter and Motion: Mechanical Philosophy in the Making*. Baltimore: Johns Hopkins University Press.
van de Pitte, Frederick. 1979. "Descartes' *Mathesis Universalis*." *Archiv für Geschichte der Philosophie* 61 (2): 154–74.
van de Pitte, Frederick. 1988. "Intuition and Judgment in Descartes's Theory of Truth." *Journal of the History of Philosophy* 26 (3): 453–70.
van de Pitte, Frederick. 1991. "The Dating of Rule IV-B in Descartes's *Regulae ad directionem ingenii*." *The Journal of the History of Philosophy* 29 (3): 375.
van Roomen, Adriaan. 1597. *In Archimedis circuli dimensionem expositio et analysis*. Geneva: De Candolle.
van Schooten, Frans. 1651. *Principia matheseos universalis, seu introductio ad geometriæ methodum Renati des Cartes*. Laon: Elsevier.
Verbeek, Theo. 1992. *Descartes and the Dutch: Early Reactions to Cartesian Philosophy, 1637–1650*. Carbondale: Southern Illinois University Press.
Viète, François. 1591. *In artem analyticem Isagoge*. Turin: I. Mettayer.
Vinciolo, Federico de. 1587. *Les singuliers et nouveaux pourtraicts* […]. Paris: Jean le Clerc.
Voetius, Gisbertus. 1648–1669. *Selectarum disputationum theologicarum*. 5 vols. Utrecht: J. Waesberge.
Vuillemin, Jules. 1960. *Mathématiques et métaphysique chez Descartes*. Paris: Presses Universitiares de France.
Weber, Jean-Paul. 1964. *La constitution du texte des* Regulae. Paris: Société d'édition d'enseignement supérieur.
Webering, Damascene. 1953. *Theory of Demonstration According to William Ockham*. St. Bonaventure: The Franciscan Institute.
Wells, Norman J. 1984. "Material Falsity in Descartes, Arnauld, and Suárez." *Journal of the History of Philosophy* 22 (1): 22–50.
Wells, Norman J. 1965. "Descartes and the Modal Distinction." *The Modern Schoolman* 43 (1): 1–22.
Wells, Norman J. 1966. "Descartes on Distinction." In *The Quest for the Absolute*, edited by Frederick J. Adelmann, 104–34. Boston: Springer.
Wengert, R. G. 1965. "The Development of the Doctrine of the Formal Distinction in the *Lectura Prima* of John Duns Scotus." *The Monist* 49 (4): 571–87.
Whewell, W. [1840] 2014. *The Philosophy of the Inductive Sciences*. 2 vols. Cambridge: Cambridge University Press.
White, Michael J. 2009. "Aristotle on the Infinite, Space, and Time." In *A Companion to Aristotle*, edited by Gergios Anagnostopoulos, 260–77. Sussex: Wiley-Blackewell.
Wietrowsky, Maximilianus. 1697. *Philosophia disputata in qua comprehenduntur ex universa philosophia Aristotelis*. Pragae: Typis Univers. Car. Ferd. in Coll. Soc. Jesu ad S. Clem.
Wilson, Margaret Dauler. 1982. *Descartes*. London: Routledge.
Wittgenstein, Ludwig. 1958. *Philosophical Investigations*. Translated by G. E. M. Anscombe. Oxford: Blackwell.
Wittgenstein, Ludwig. 1978. *Remarks on the Foundations of Mathematics*, edited by G. H. von Wright, Elizabeth Anscombe. London: Basil-Blackwell.
Wolter, Alan B. 1965. "The Formal Distinction." In *John Duns Scotus 1265–1965*, edited by Bernardino M. Bonansea and John K. Ryan, 45–60. Washington: Catholic University of America Press.
Wong, David B. 1982. "Cartesian Deduction." *Philosophy Research Archives* 8: 1–19.

Yolton, John W. 1984. *Perceptual Acquaintance from Descartes to Reid*. Minneapolis: University of Minnesota Press.

Yrjönsuuri, Mikko. 2000. “Self-Knowledge and the Renaissance Sceptics.” In *Acta Philosophica Fennica: Ancient Scepticism and the Sceptical Tradition*, edited by J. Sihvola, 225–54. Helsinki: Philosophical Society of Finland.

Yrjönsuuri, Mikko. 2011. “Matthew of Aquasparta.” In *Encyclopedia of Medieval Philosophy: Philosophy between 500 and 1500*, edited by Henrik Lagerlund, 729–30. Dordrecht: Springer.

Zepeda, Joseph. 2016. “Exclusion in Descartes’s *Rules for the Direction of the Mind*: The Emergence of the Real Distinction.” *Intellectual History Review* 26 (2): 203–19.

Index

Note: Figures and tables are indicated by an italic "*f* ", "*t*" and notes are indicated by "n" following the page numbers.

For the benefit of digital users, indexed terms that span two pages (e.g., 52–53) may, on occasion, appear on only one of those pages.